TEST Engineering & Management

March-April 2020
ISSN: 0193-4120 Page No. 8311 - 8318

Thermal Post-buckling of Composite Plates Subjected to Uniform Thermal Loading using Finite Element Method

[1]Z. Yusof, [2]Z. A. Rasid, [3]M. Z. Hassan
[1]Malaysia-Japan International Institute of Technology, Universiti Teknologi Malaysia, Kuala Lumpur
[2]Malaysia-Japan International Institute of Technology, Universiti Teknologi Malaysia, Kuala Lumpur
[3]Razak Faculty of Technology and Informatics, Universiti Teknologi Malaysia, Kuala Lumpur
[1] zarinayusof91@gmail.com [2]arzainudin.kl@utm.my, [3]mzaki.kl@utm.my

Article Info
Volume 83
Page Number: 8311 - 8318
Publication Issue:
March - April 2020

Article History
***Article Received**: 24 July 2019*
***Revised**: 12 September 2019*
***Accepted**: 15 February 2020*
***Publication**: 09 April 2020*

Abstract

The non-linear buckling of laminated plates exposed to evenly increase in temperature has been investigated numerically. Applying the Newton-Raphson scheme via the total Lagrangian approach, the post-bifurcation paths were determined. Initially, the governing equation based on finite element method was developed applying the Mindlin's model for plates with moderate strains and the Hamilton principle. The model was solved using computer program developed within FORTRAN environment and validated with past results. Several factors that influence the non-linear thermal instability behaviour of the plates were investigated on the plate with anti-symmetric and angle ply configuration. The influences of these parameters were found to be substantial. The investigation will thus be useful in applications such as automotive and aerospace vehicles..

Index Terms; *— Mindlin's Shear Deformation Theory, Newton-Raphson method, Total Lagrangian Approach, non-linear Thermal instability*

I. INTRODUCTION

Fibre reinforced composite (FRC) has been popular choices in structural designs that require weight saving [1-4]. As a result, the design of the FRC parts are made geometrically thin which in turn can cause instability problem if loadings applied are compressive. For aerospace and aeronautical structures, consideration on thermal loading is crucial before the problem turns into disaster [5]. As the temperature reaches the critical temperature, T_{cr} huge lateral deflection that will bring catastrophe will occur. However, it is known that plate structures remain stable in the post-buckling region. This fact requires more studies on the behaviour of the thermal instability of plates with lamination scheme in order to optimize the design of such structures.

In their study on the thermal instability of symmetric and quassi-isotropic laminated composite plates, Prabhu, and Dhanaraj [6] found that for composite with symmetric lamination, the change of critical temperature differs from the one typically shown by symmetric composites. In a study by Rasid [7], the serendipity quadrilateral element with 5-degree of freedom per node were used to study thermal buckling analysis on symmetric and antisymmetric composite. The effect of several factors including the level of anisotropy, thickness of the plate, fibre orientation and boundary condition were studied. Furthering this study, shape memory alloy (SMA) was used to enhance the linear and nonlinear thermal instability of plates with lamination schemes [8-11]. The SMA was used to induce recovery stress inside the composite through the so called shape memory effect behaviour. By this, the composite's

March-April 2020
ISSN: 0193-4120 Page No. 8311 - 8318

potential energy was increased and as such linear and nonlinear buckling of the composites subjected to heat increase can be enhanced. Even and non-even heat distribution in the form of parabolic distribution have been considered. It was found that the SMA has greatly affected the thermal buckling behaviour of the composites. Furthermore, carbon nanotubes were embedded within composite to improve the composite linear and non-linear thermal instability [12] and frequency of post-instability matrix cracked laminated plates containing CNT reinforced composite layers resting on elastic foundation [13]. In the latter case, the shear deformation theory of higher order and the nonlinearity of the von Karman type were used in deriving the governing equations while the micro-mechanical model was used to determine the temperature dependent material properties of the composite. Comparison studies were conducted and the present model was shown to demonstrate its accuracy and effectiveness. In a study by Kianni [14], non-uniform rational B-spline (NURBS) based isogeometric finite element method was used to study thermal post-buckling of piece-wise functionally graded laminated plates reinforced with graphene sheets. While the temperature dependent properties were assumed, the third order shear deformation theory was implemented. It was determined that the FG-X pattern of graphene reinforcement has resulted in the lowest critical buckling and the lowest post-buckling deformation. Shen and his co-workers [15-16] conducted studies on post-buckling of functionally graded graphene reinforced composite laminated cylindrical panels subjected to axial compression while being in thermal environments. The model employed the HSDT of laminated composite added with the von Karman strains while the single perturbation technique was employed to determine the critical loads and the post-buckling paths. The critical loads were found to improve with the addition of the functionally graded graphene fillers.

In this paper, finite element governing equation for the thermal post-buckling of laminated composite plates was developed along with its source codes. The equation was solved using the method of Newton-Raphson while applying the total Lagrangian approach. Upon validation, the post-buckling behaviour of the composites subjected to several parameter changes including number of layers and boundary conditions on the thermal post-buckling of composite plates.

II. Development of Governing Equations

A. Dimensions and Properties

The plate with lamination scheme understudied here is shown in Fig. 1. The square plate is 0.1 m x 0.1 m while the side to thickness ratio is usually $\frac{l}{t}$ = 100. The $[45/-45]_2$ lamination scheme is used in this study while 2 types of graphite epoxy properties employed are:

Property 1(P1):

E_1 = 155.0 GPa, E_2 = 8.07 GPa, v_{12} = 0.22, G_{12} = 4.55 GPa, G_{23} = 4.55 GPa, G_{13} = 3.25 GPa, α_1= -0.07E-6,

$\alpha_2 = \alpha_3$ = 30.1E-6

Property 2 (P2):

E_1/E_2 = 40, E_2 = 6.25, G_{12}/E_2 = 0.8, $G_{12} = G_{13}$, G_{23}/E_2 = 0.52, v_{12} = 0.24, α_2/α_1 =10

The boundary condition utilized in this study can be the hinged (HH), simply supported (SS) and clamped (CC) types.

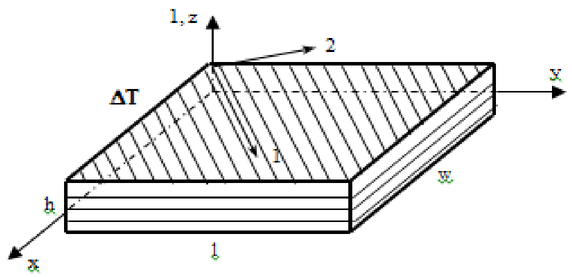


Fig. 1. The schematic diagram of the composite plate understudied

Published by: The Mattingley Publishing Co., Inc.

B. FEM Modelling

The $\sigma - \varepsilon$ relationship for the composite plate employed here in the 1-2-3 coordinate system is

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_{12} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{33} \end{bmatrix} \left[\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix} - \begin{Bmatrix} \alpha 1 \\ \alpha_2 \\ 0 \end{Bmatrix} \Delta T \right] \quad (1)$$

or equivalently,

$$\{\sigma_1\} = [Q]\{\varepsilon_1\} - \{\alpha_1\} \Delta T \quad (2)$$

Here, the well-known reduced stiffness matrix is $[Q]$ [17] and $\{\sigma_1\}$ is the stress vector, $\{\varepsilon_1\}$ and $\{\alpha_1\}$ are the strain and thermal coefficient of expansion vector, respectively. Upon utilizing the transformation of (2), we obtain the stress and strain in x-y-z coordinate system such as,

$$\{\sigma_x\} = [\bar{Q}]\{\varepsilon_x\} + \{\alpha_x\}\Delta T \quad (3)$$

where the well-known transformed reduced stiffness matrix of the composites is $[\bar{Q}]$. The Mindlin's theory [18], was used here where with the application of the non-linear von Karman's strain, the strain vector is

$$\{\varepsilon\} = \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial y} \\ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \end{Bmatrix} + \frac{1}{2} \begin{Bmatrix} \left(\frac{\partial w}{\partial x}\right)^2 \\ \left(\frac{\partial w}{\partial y}\right)^2 \\ 2\left(\frac{\partial w}{\partial x}\right)\left(\frac{\partial w}{\partial y}\right) \end{Bmatrix} + z \begin{Bmatrix} \frac{\partial \theta_x}{\partial x} \\ \frac{\partial \theta_y}{\partial y} \\ \left(\frac{\partial \theta_x}{\partial y} + \frac{\partial \theta_y}{\partial x}\right) \end{Bmatrix} \quad (4)$$

The relationship for vector in transverse direction is

$$\{\gamma\} = \begin{Bmatrix} \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial w}{\partial x} + \theta_y \\ \frac{\partial w}{\partial y} + \theta_y \end{Bmatrix} \quad (5)$$

The constitutive relationship in term of resultant stress is

$$\begin{Bmatrix} \{N\} \\ \{M\} \end{Bmatrix} = \begin{bmatrix} [A] & [B] \\ [B] & [D] \end{bmatrix} \left(\begin{Bmatrix} \{\varepsilon_m\} \\ \{\kappa\} \end{Bmatrix} + \begin{Bmatrix} \{\varepsilon_{nl}\} \\ 0 \end{Bmatrix} \right) + \begin{Bmatrix} \{N_T\} \\ \{M_T\} \end{Bmatrix} \quad (6)$$

and

$$\{Q\} = \begin{Bmatrix} Q_{xz} \\ Q_{yz} \end{Bmatrix} = \begin{bmatrix} A_{44} & A_{45} \\ A_{45} & A_{55} \end{bmatrix} \{\gamma\} = [A'] \{\gamma\} \quad (7a)$$

where [7] is for the out of plane case. The resultant force, $\{N_T\}$ and the resultant moment vectors, $\{M_T\}$ are as in the following.

$$(\{N_T\},\{M_T\}) = \sum_{k=1}^{n} \int_{-\frac{t}{2}}^{\frac{t}{2}} [\bar{Q}]\{\alpha_x\}\Delta T(1,z)dz \quad (7b)$$

The research here employs the isoparametric quadrilateral elements with 8 nodes. The conservation of energy is

$$\delta W = \delta W_{int} - \delta W_{ext} = 0 \quad (8)$$

$$\delta W_{int} = \{\delta q\}^T \int_A \{\delta\varepsilon_m\}^T\{N\} + \{\delta\varepsilon_{nl}\}^T\{N\}$$

$$+ \{\varepsilon_b\}^T\{M\} + \{\varepsilon_s\}^T\{Q\}\, dA \quad (9)$$

Based on the standard procedures for FEM modelling [17],

$$\delta W_{int} =$$

$$\{\delta q\}^T \left([K_L] + [K_s] - [K_T] + [K_G] + \frac{1}{2}[N1] + \frac{1}{3}[N2] \right)\{q\}$$

$$- \{\delta q\}^T \{P_T\} \quad (10)$$

Without any external loads,

$$\delta W_{ext} = 0 \quad (11)$$

So, relating the previous equations with (10), the FEM governing equation is

$$\left([K_L] + [K_s] - [K_T] + [K_G] + \frac{1}{2}[N1] + \frac{1}{3}[N2] \right)\{q\} = \{P_T\} \quad (12)$$

where the linear, shear, geometric and thermal stiffness matrices are $[K_L]$, $[K_s]$, $[K_G]$ and $[K_T]$ are and $[N_1]$ and $[N_2]$. In this study, equation (12) is solved employing the Newton Raphson's scheme. The residual forces, $\{\psi\}$ is

$$\{\psi(q)\} =$$

$$\left([K_L] + [K_s] - [K_T] + [K_G] + \frac{1}{2}[N1] + \frac{1}{3}[N2] \right)\{q\} - \{P_T\} \quad (13)$$

March-April 2020
ISSN: 0193-4120 Page No. 8311 - 8318

Through simplification [17],

$$\psi(q)^{n+1} = \psi(q)^n + \left(\frac{d\psi}{dq}\right)_n \{\delta q\}^n \tag{14}$$

where

$$\left(\frac{d\psi}{dq}\right)_n = [K_{\tan}]_n = ([K_L]+[K_s]-[K_T]+[K_G]+[N_1]+[N_2])_n \tag{15}$$

and

$$[K_{\tan}]\{\delta q\}^n = -\varphi(q)^n \tag{16}$$

while the increment of vector of displacements is

$$\{q\}^{n+1} = \{q\}^n + \{\delta q\}^n \tag{17}$$

C. Solution to the Governing Equation

Equations (15) - (17) were solved using Newton-Raphson scheme and developing source codes. Linear eigen-value buckling is initially conducted to determine the ΔT_{cr}. With this, the eigen-vector, {ϕ} to be employed as the initial value is determined. Fig. 2 shows the flowchart of the algorithm.

III. VALIDATION AND RESULTS

A. Comparison test

For validation purpose, several comparison tests are performed for the critical temperature. While the property used is P2, the lamination schemes of the composite are [30/-30], [30/30]$_2$, and [0/90]$_2$. From Table 1, the model developed is shown to produce outcomes that conforms well to past results.

Table 1:

Comparison test for critical temperature values

	[19]	4x4	5x5	6x6	8x8
(30/-30)$_s$	8.396	9.200	8.735	8.580	8.431
(30/30)$_2$	10.169	10.210	10.178	10.168	10.162
(0/90)$_2$	21.166	24.468	22.219	21.743	21.570

At the same time, validation on the non-linear instability analysis was performed for a qussi-isotropic and an orthotropic plate using the P2 property. In the lamination scheme for the quassi-isotropic plate is [45/-45/0/90], while for the orthotropic plate is [45/-45]$_2$. It can be seen that the comparisons were successful such as shown in Fig. 3 and 4.

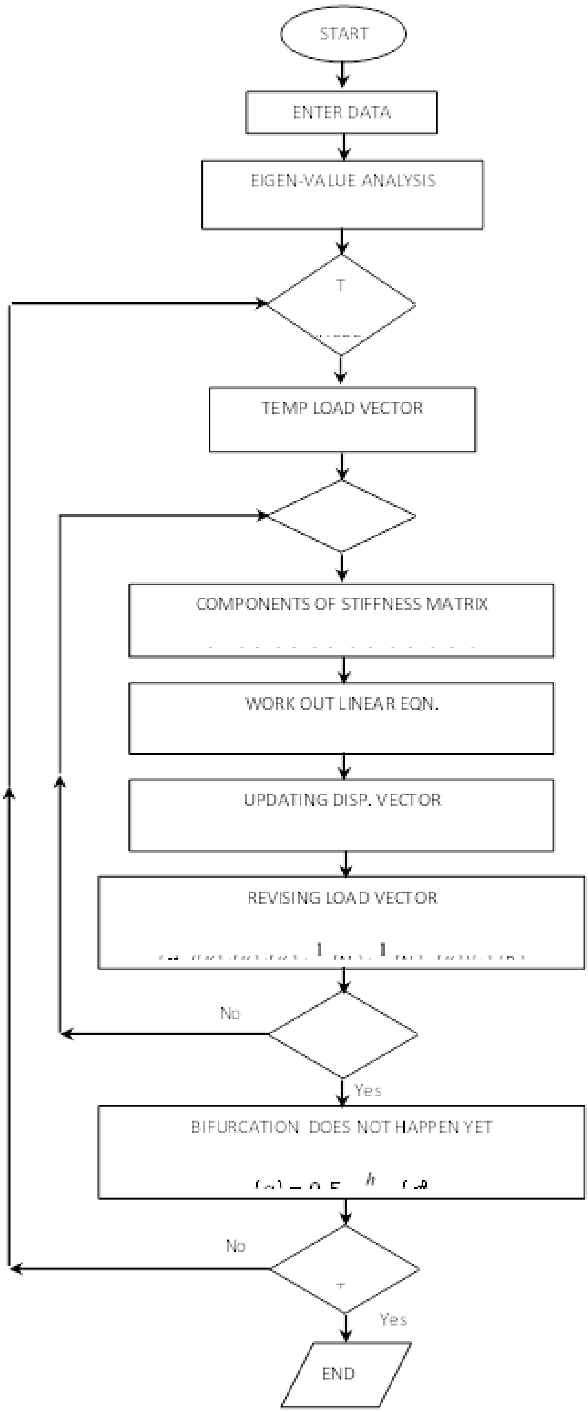


Fig. 2. The flowchart for solving the FEM governing equations

Published by: The Mattingley Publishing Co., Inc.

TEST Engineering & Management

March-April 2020
ISSN: 0193-4120 Page No. 8311 - 8318

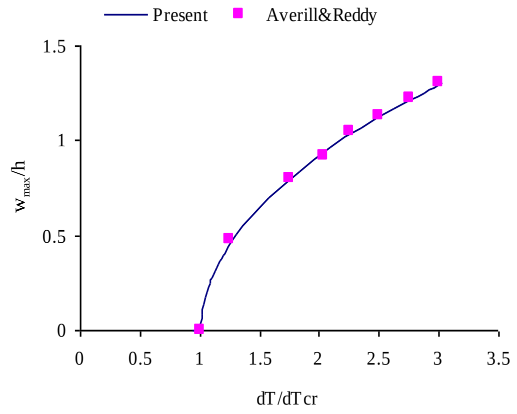


Fig. 3. Validation for the thermal instability of the layered plate

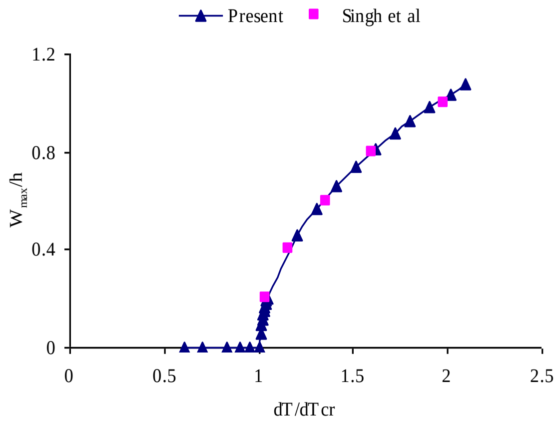


Fig. 4. Validation for the thermal post-buckling of an anti-symmetric composite plate

B. The number of layer parameter

Here, with property P1 being used, the number of layers, n in configuration $[45/-45]_n$ is increased while the thickness is maintained. The results shows that bifurcation occurs at ΔT_{cr} values as predicted. Fig. 5 also shows the positive correlation between the critical temperature (λ_n) and the numbers of layer. One behaviour dictated by bending-extensional coupling effect is the ratio $\frac{\Delta T}{\Delta T_{cr}}$ is decreased as the number of layer is increased.

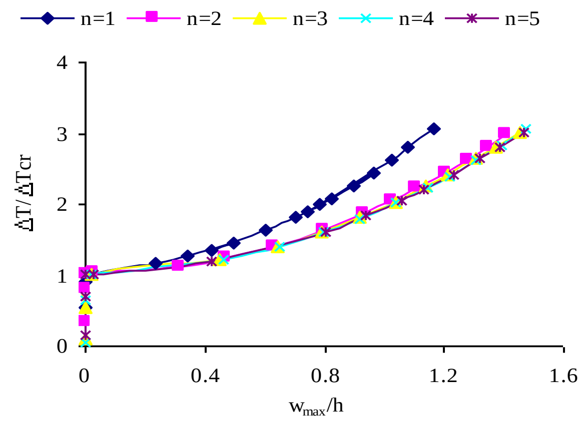


Fig. 5. The post- thermal instability of $[45/-45]_n$ composite plates

(λ_1=55.3°C λ_2= 86.4°C λ_3=90.6 °C λ_4=92.1 °C λ_5=92.7 °C)

C. The plate thickness parameter

Here, the plate thickness, h is altered while the side length is treated as constant. While the $[45/-45]_2$ laminate configuration is used, P1 material property is employed. The results from Fig 6 shows as the thickness is decreased, the critical loads are decreased as predicted. Fig. 6 indicates that the critical temperatures are lowered along with the thickness of plates. Furthermore, the presence of the bending- extension coupling has increased the $\frac{l}{h}$ ratio will the decrease of the $\frac{\Delta T}{\Delta T_{cr}}$ ratio.

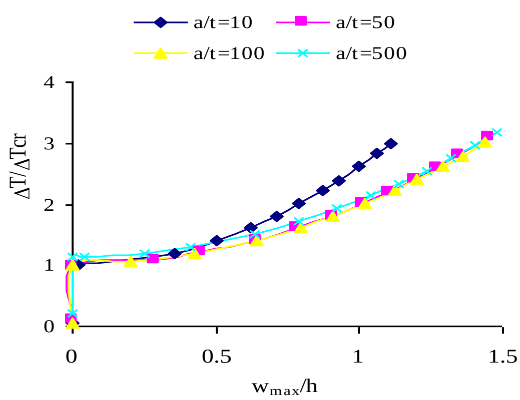


Fig. 6. The influence of length to thickness ratio on the non-linear thermal instability $[45/-45]_2$ composite plates. (λ_{10}=6915.2°C λ_{50}= 453.0°C λ_{100}=115.7 °C λ_{500}=4.8 °C)

Published by: The Mattingley Publishing Co., Inc.

D. The boundary condition parameter

The composite with configuration of $[45/-45]_2$ is given the three types of boundary conditions: HH, CC and SS. Fig. 7 shows that as projected, the smallest critical temperature (λ_{SS}=105.7°C) happens with the application of the SS BC and in opposite the largest critical temperature corresponds to CC boundary condition (λ_{CC}=232.8°C). Again, it can be realized that bifurcation start at the values of ΔT_{cr} predicted by the eigen-value analysis.

E. The aspect ratio parameter

The ratio a/b parameter is differed to examine the behaviour of the non-linear thermal instability of the layered plates. Fig. 8 shows that the critical temperature rises as the escalation of the aspect ratio. Similarly to be seen in Fig. 8, bifurcation point appear at ΔT_{cr} as given by eigen-value analysis.

F. The lamination angle parameter

The influence of lamination angle is investigated by varying the angle for the orientation in $[\theta/-\theta]_2$. θ here is taken as 0°, 15°, 30°, 45°, 60°, 75° and 90°. However, since the plot for θ equals to 60°, 75° and 90° overlap with the plots for θs equal to 30°, 15° and 0° respectively, they will not be shown here. Fig. 10 demonstrated that the orientation of $[45/-45]_2$ provides the topmost critical temperature (λ_3=115.7 °C).

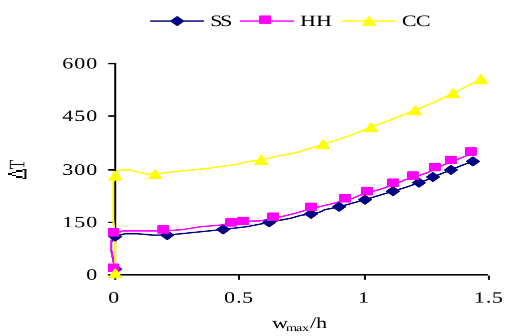


Fig. 7. The impact of BC on the post- thermal instability of $[45/-45]_2$ composites. (λ_{SS}=105.7°C λ_{HH}=115.7°C λ_{CC}=232.77 °C)

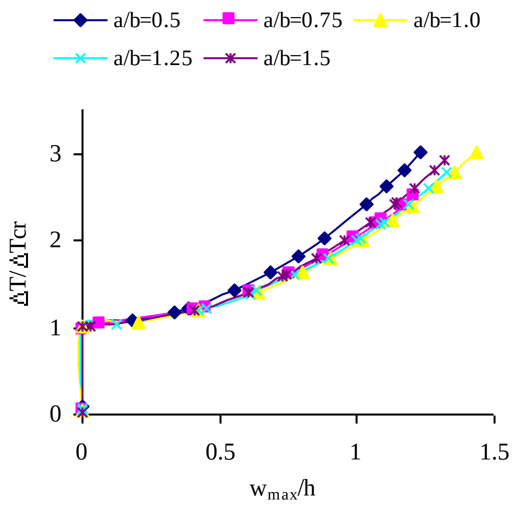


Fig. 8 The impact of aspect ratios on the post-thermal instability of $[45/-45]_2$ composite plates. ($\lambda_{0.5}$=60.3°C $\lambda_{0.75}$=87.1°C λ_1=105.7 °C, $\lambda_{1.25}$=144.4.1°C $\lambda_{1.5}$=173.8 °C)

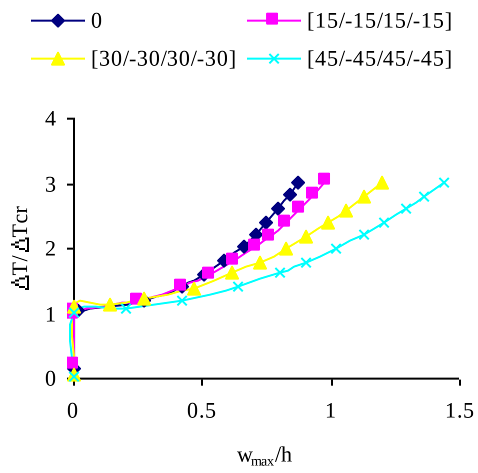


Fig. 9. The impact of lamination angles on the post-thermal instability of $[45/-45]_2$ composite. (λ_1=68.2°C λ_2=71.4°C λ_3=115.7 °C)

IV. CONCLUSION

A numerical model applying the finite element method has been established for the non-linear thermal instability of laminated composite plates. The Mindlin's theory along with non-linearity of von Karman sense were used. The Newton-Raphson scheme was applied to the governing equation and with the developed source-codes, the non-linear thermal instability paths of the laminated composite

Published by: The Mattingley Publishing Co., Inc.

March-April 2020
ISSN: 0193-4120 Page No. 8311 - 8318

plates were able to be discovered. It was found that the post-buckling occurs at critical point as predicted in the linear analysis while the impacts of several parameters on the non-linear thermal instability characteristics of the layered plates have been determined.

V. ACKNOWLEDGEMENTS

Appreciation is acknowledged to the MJIIT Faculty of the University Technology of Malaysia and the Ministry of Education, Malaysia for GUP Grant (Q.K130000.2643.14J90) and PRGS-ICC Grant (Q.K.130000.7756.4J381). The 3rd author is grateful to Universiti Putra Malaysia for supporting this work through the Visiting Scholar (Post-Doctoral) scholarship.

REFERENCES

[1]. M.Z. Hassan, S.M. Sapuan and Z.A. Rasid, "Thermal degradation and mechanical behaviour of banana pseudo-stem reinforced composites," International Journal of Recent Technology and Engineering, vol. 4 (8), pp. 5899-5902, 2019.

[2]. S.A.H. Roslan, Z. Yusof, Z.A. Rasid, M.H. Yahaya, M.Z. Hassan and J. Mahmud, "Dynamic instability response of smart composite material," Materialwissenschaft und Werkstofftechnik, vol. 50 (3), pp. 302-310, 2019.

[3]. Z. Yusof, Z.A. Rasid, M.Z. Hassan, J. Mahmud and H. Yahaya, "Parametric study on dynamic instability of fully anisotropic composite plates," International Journal of Engineering and Technology (UAE), vol. 7 (4), pp. 132-136, 2018.

[4]. Z. Yusof, Z.A. Rasid and J. Mahmud, "The active strain energy tuning on the parametric resonance of composite plates using finite element method," Journal of Mechanical Engineering, vol. 5, pp. 104-119, 2018.

[5]. L. Librescu, W. Lin, M.P. Nemeth, and J.H. Starnes, "Effect of tangential edge constraints on the post-buckling behaviour of flat and curved panels subjected to thermal and mechanical load. : Buckling and post-buckling of composite structures," ASME AD, vol. 41 / PVP, vol. 293, pp. 115-133, 1994.

[6]. M.R. Prabhu, and R. Dhanaraj, "Thermal buckling of laminated composite plates," Computer and Structures, vol. 53 (5), pp. 1193–1204, 1994.

[7]. Z.A. Rasid, "Thermal buckling analysis of laminated composite plates using finite element method, in Proceeding for Conference on Applications and Design in Mechanical Engineering, 25-26 October 2007, Kangar, Perlis, Malaysia.

[8]. Z.A. Rasid, A. Ayob, R. Zahari, F. Mustapha, D.L. Majid and R. Varatharajoo. "Thermal buckling and post-buckling improvements of laminated composite plates using finite element method," Key Engineering Materials, vol. 471-472, pp. 530-535, 2011.

[9]. Z.A. Rasid, R. Zahari, A. Ayob, D.L. Majid and A.S.M. Rafie. "Thermal post-buckling improvements of laminated composite plates using the active strain energy tuning approach." Advanced Materials Research, vol. 311-313, pp 2235-2238, 2011.

[10]. Z.A. Rasid, R. Zahari, A. Ayob, D.L. Majid, and A.S.M. Rafie. "Thermal post-buckling of shape memory alloy composite plates under non-uniform temperature distribution," International Journal of Mechanical and Materials Engineering, vol. 2(1), pp 58-63, 2011.

[11]. Z.A. Rasid, R. Zahari and A. Ayob, "The instability improvement of the shape memory alloy composite plates subjected to in-plane parabolic temperature distribution," Applied Mechanics and Material, vol. 554. pp 32-36, 2014.

[12]. Z.A. Rasid and H. Yahaya. "The thermal instability analysis of functionally graded carbon nanotube composite plates using finite

Published by: The Mattingley Publishing Co., Inc.

March-April 2020
ISSN: 0193-4120 Page No. 8311 - 8318

element method," Applied Mechanics and Materials, vol. 695, pp 285-288, 2015.

[13]. Y. Fan and H. Wang, 2016, "Thermal postbuckling and vibration of postbuckled matrix cracked hybrid laminated plates containing carbon nanotube reinforced composite layers on elastic foundation," Composite Structures, vol. 157, pp. 386–397, 2016.

[14]. Y. Kianni, "NURBS-based isogeometric thermal postbuckling analysis of temperature dependent graphene reinforced composite laminated plates," Thin-Walled Structures, vol. 125, pp. 211–219, 2018.

[15]. H.S. Shen, Y. Xiang and Y. Fan, "Postbuckling of functionally graded graphene-reinforced composite laminated cylindrical panels under axial compression in thermal environments," International Journal of Mechanical Sciences, vol. 135, pp. 398–409, 2018.

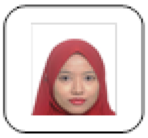
[16]. H.S. Shen and Y. Xiang, 2018, "Postbuckling behaviour of functionally graded graphene-reinforced composite laminated cylindrical shells under axial compression in thermal environments," Comput. Methods Appl. Mech. Engrg., vol. 330, pp. 64–82, 2018.

[17]. J.N. Reddy, Mechanics of Composite Plates and Shells: Theory and Analysis, 2nd Edition, New York: CRC Press, 2002.

[18]. R.D. Mindlin, "Influence of rotary inertia and shear on flexural motions of isotropic elastic plates." Journal of Applied Mechanics, vol.18, pp. 31-38, 1951.

[19]. V.S. Thankam, G. Singh, G.V. Rao and A. K. Rath, "Thermal post-buckling behaviour of laminated plates using a shear-flexible element based on coupled-displacement field," Composite Structures, vol. 59, pp. 351– 359, 2003.

VI. AUTHORS PROFILE



Zarina Yusof studied Master's degree in Mechanical Engineering and Management of Technology from the Universiti Teknologi Malaysia (UTM) and Yamaguchi University and finished her master in 2019. Several conference and journal papers were published during her study. Currently, she is pursuing her study at PhD level at the UTM Kuala Lumpur.

Zainudin A. Rasid finished his degree in Mechanical Engineering in 1996 and in 2013, his study in Aerospace Engineering finished, earning him his PhD. Employed at the UTM, Kuala Lumpur he has published more than 50 papers while being a senior lecturer and a member of the Malaysian Board of Engineers. His research area includes vibration and buckling of laminated composites, smart material and composites and Carbon Nanotubes, rotordynamic of shaft and wind turbine.

Mohamad Zaki Hassan is a member Malaysian's Board of Engineer while has been working at the UTM for more than 30 year. 60 papers has been published under his name where his area of expertise include He has published more than 60 papers in forms of journal and conference proceedings mainly in the subject of composite, sandwich structure and natural fibers including kenaf, jute, hemp and bamboo.