May-June 2020
ISSN: 0193-4120 Page No. 10695 - 10702

Ordinary Differential Equation in Funnel Design for Fastest Bottling Process

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Abstract:
Most problems common in science and engineering can be modeled. The funnel is more used in industrial and home applications to transfer a material from one container using a funnel. The shape of the funnel according to the concept can produce precise stability. This study can also determine the shortest time to empty the funnel. The correct solution can be analyzed using the method in the software Matlab. Numerical methods are the methods used to obtain accurate, efficient and stable estimates. Methods such as the Heun method, implicit midpoint method, explicit midpoint method and Euler method can be solved by differential equations problems which used to determine the appropriate parameters for short time analysis. The stability of each method can determine the appropriate funnel design based on the graph produced for each parameter. The method will be discussed and compared in terms of accuracy and efficiency to solve differential equations to determining the appropriate parameters for optimum design of funnel. Each method is used to determine the most appropriate method. At the end of the study, numerical results can be obtained indicate that implicit midpoint methods provide greater stability and accuracy than other methods. In conclusion, the proper selection of the funnel design can minimize the cost of the funnel design and optimum funnel designs also control excess wastage or spillage.

Keywords: *Numerical Method; Explicit Midpoint; Parameter; Implicit Midpoint; Stability*

INTRODUCTION

Most problems common in science and engineering can be modeled. The funnel is more used in industrial and home applications to transfer a material from one container using a funnel. The funnel is the device used in most procedures in factories and laboratories. The funnel has also been used in industrial and home applications for the transfer of liquids or sand from one container using the pour the liquid into the same container (Barrocas 1980). The funnel structure includes a funnel body that has a larger opening diameter at the upper than the bottom. The purpose of this funnel is to reduce the risk of spillage when filling out any containers (Milan & Peal 2013). This funnel can make it easy for users to save time. The funnel design usually involves determining the configuration for the use of the included liquid, such as the angle and length of the funnel part. This funnel is intended to avoid material from one container to another being overflowing or spilled. A stable structure plays a key role in fitting the funnel with the bottle. To enhance channel funnel tool. The majority of funnels are produced of glass or glass and are marketed in various sizes. This funnel specially allows for easier use of filling in containers such as jugs, bottles and so on. The funnel is one of the selective instruments used to insert or move the water solution with the stream of the material into a container from one location to another and then

performance and effectiveness, the field of manufacturing and processing is presently focused on technology optimization and control. The sector shows a clear need in a extremely competitive market and improves the process of bottling a drink bottle. The filling bottle process important to design the type funnel that easier he liquid transfer process into the bottle is also measured directly in terms of spills and limits to fill each bottle (Chakraborty et al. 2015). The process filling bottling for funnel shown in the figure 1.

May-June 2020
ISSN: 0193-4120 Page No. 10695 - 10702

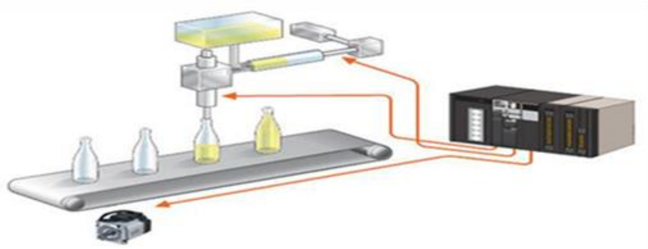

FIGURE 1. Filling bottle process
(Source: Jaymin Patel et al. 2015)

The time taken by using the Bernoulli equation to determine the quantity of liquid such as air through the funnel. The funnel shape that has the reduced portion to drain down the curve slowly. Higher pressure is due to the narrow shape of the tapered funnel. Water that produces high pressure is forced through a tapered funnel at a time with the same energy. The presence of a bernoulli solution with a fundamental angle of the funnel through a funnel with liquid pressure (Ianke & Ilson 1999).

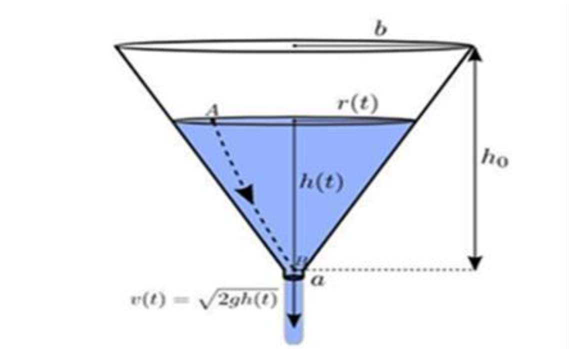

FIGURE 2. Illustration of the tapered funnel to drain water (Source: Diego Assencio 2012)

The funnel system has an adapter with a closure that can be mounted to an opening of the container. A funnel body has a cover that pivotally connected to the funnel body and used to open and close an inlet end of the funnel. The funnel body is also assembled in the adapter, thereby permitting different size funnel bodies to be assembled with the same container closure. In addition, the funnel minimizes leakage of liquid waste in the event that the waste container and its attached funnel are accidently tipped over. The funnel of the present invention permits with different size funnels to be easily assembled with different container closure, thereby substantially reducing cost of acquiring and storing.

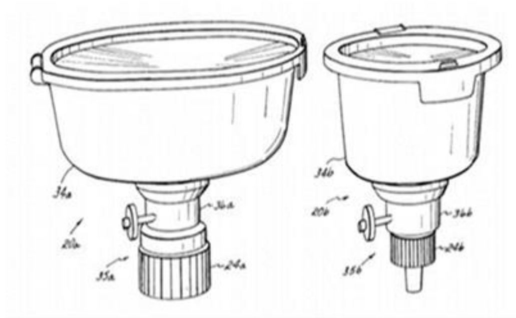

FIGURE 3. Funnel with adjusted at opening funnel exit (Source: Kim et al. 2004)

METHODOLOGY

1. Formulation on water level in tapered funnel

The design of tapered funnels includes determining the configuration for example the tapered angle as well as the diameters D and lengths of the funnel segments for the scheduled water content. It is also required the determination on the time required to empty the contained liquid.

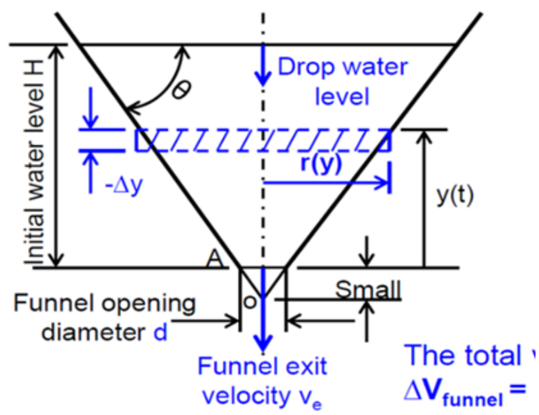

FIGURE 4. The instantaneous water level in a tapered funnel (Source: Hsu Tai-ran 2018)

Using the formulated on water level in tapered funnel:

$$\Delta t\ (\Delta V_{Exit}) = \Delta t(\Delta V_{Funnel})$$

$\Delta t\ (\Delta V_{Exit})$
$=$ The total volume of water leaving the funnel
$\Delta t\ (\Delta V_{Funnel})$
$=$ The total volume of liquid by the funnel

TABLE 1. Formulation on water

May-June 2020
ISSN: 0193-4120 Page No. 10695 - 10702

level in tapered funnel

Initial level, H
Opening diameter, d
Angle for inclide side, θ
Function : $y' = -\frac{d^2}{4}\sqrt{2g}y_o^{-\frac{3}{2}}tan^2\theta$ Yexact : $y(t) = \left[\left(-\frac{5}{8}d^2\sqrt{2g}t + y_o^{\frac{5}{2}}\right)\tan^2\theta\right]^{\frac{2}{5}}$
Time to empty the funnel $t_e = \frac{8H^{\frac{5}{2}}}{5d^2\sqrt{2g}}$

NUMERICAL METHOD

The construction of numerical methods for initial value problems as well as the fundamental nature of such methods should be explained in advance for the simplest method. Types of numerical method used are:

1) Euler's Method

$$Y_1 = y_{n-1} \quad (1)$$

$$y_n = y_{n-1} + hf(x_{n-1}, y_{n-1}) \quad (2)$$

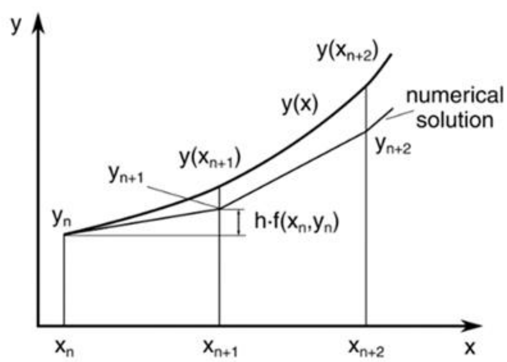

FIGURE 5. Nstep of Euler's Method
(Source: Leif Rune Hellevik 2018)

2) Implicit Midpoint Method

$$Y_1 = y_{n-1} \quad (3)$$

$$Y_2 = y_{n-1} + \frac{h}{2}f\left(x_{n-1} + \frac{h}{2}, y_{n-1}\right) \quad (4)$$

$$Y_n = y_{n-1} + hf[x_{n-1} + \frac{h}{2}, y_{n-1} + \frac{h}{2}f\left(x_{n-1} + \frac{h}{2}, y_{n-1}\right)] \quad (5)$$

3) Explicit Midpoint Method

$$Y_1 = y_{n-1} \quad (6)$$

$$Y_2 = y_{n-1} + \frac{h}{2}f(x_{n-1}, y_{n-1}) \quad (7)$$

$$Y_n = y_{n-1} + hf[x_{n-1} + \frac{h}{2}, y_{n-1} + \frac{h}{2}f(x_{n-1}, y_{n-1}) \quad (8)$$

4) Heun's Method

$$y_{n+1} = y_n \quad (9)$$

$$y_{n+1}^p = y_n + h \cdot f(x_n, y_n) \quad (10)$$

$$y_{n+1} = y_n + \frac{h}{2} \cdot [f(x_n, y_n + f(x_{n+1}, y_{n+1}^p) \quad (11)$$

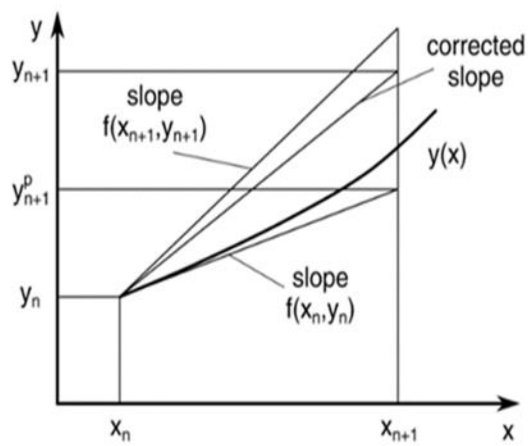

FIGURE 5. Nstep of Heun's Method
(Source: Leif Rune Hellevik 2018)

RESULTS AND DISCUSSION

May-June 2020
ISSN: 0193-4120 Page No. 10695 - 10702

The results show that the design tapered funnels involves different parameter which is

a. Initial water level in funnel, H = 250 mm

b. Funnel opening diameter, d = 6 mm

c. Angle, $\theta = 45°$

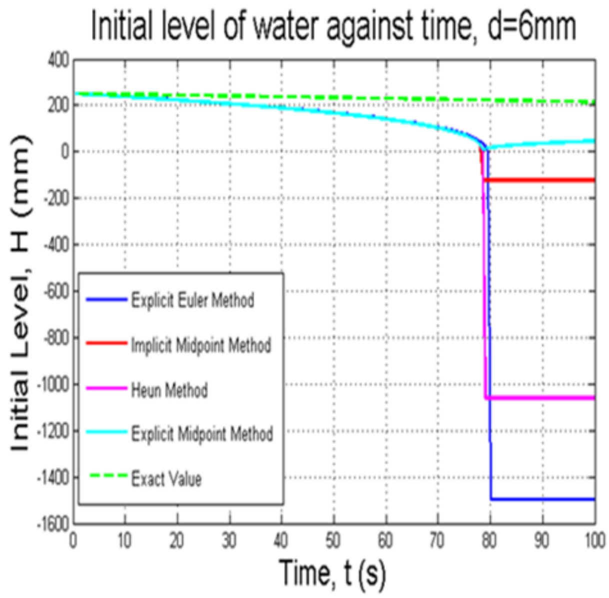

FIGURE 6. Time for each different method

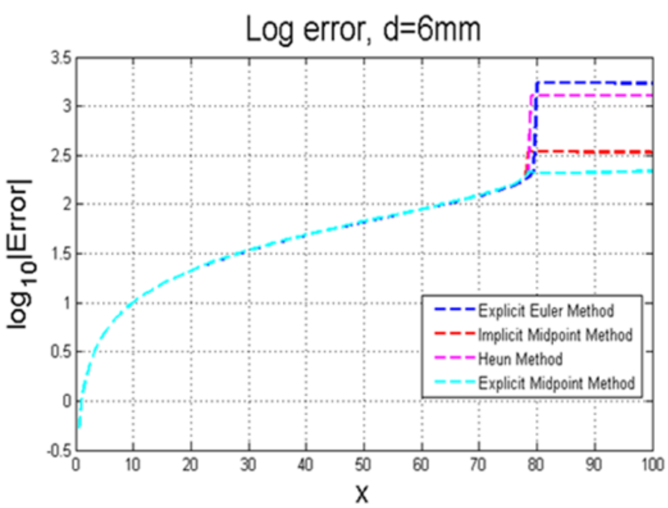

FIGURE 7. Error for each different method

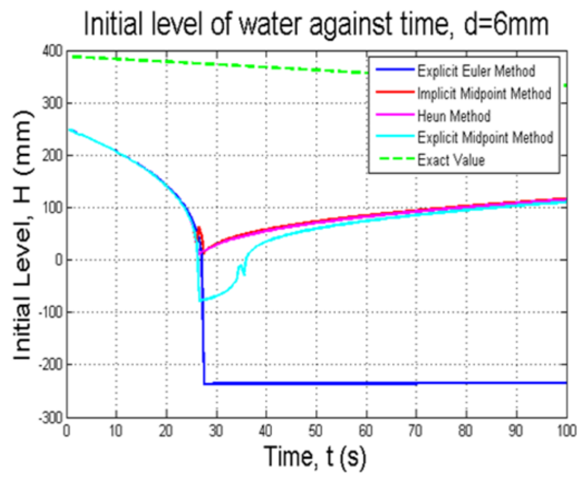

FIGURE 8. Time for each different method

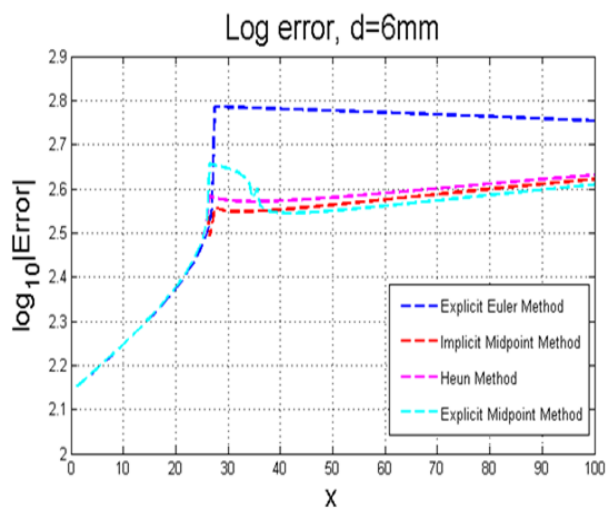

FIGURE 9. Error for each different method

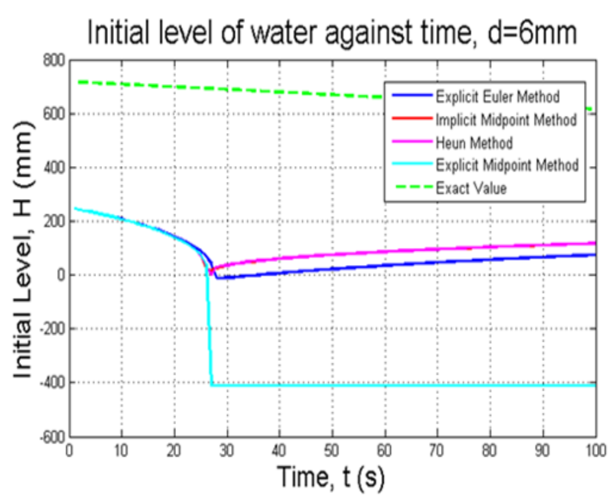

FIGURE 10. Time for each different method

May-June 2020
ISSN: 0193-4120 Page No. 10695 - 10702

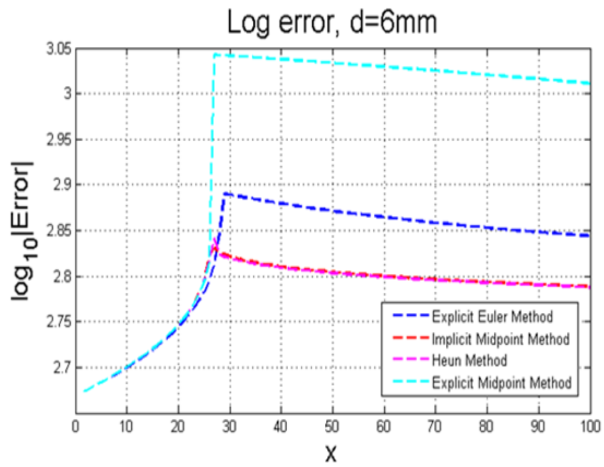

FIGURE 11. Error for each different method

Figure 6, 8 and 10 shows the difference in each method for the height of the initial water level of 250 mm. The graph is plotted to determine the most appropriate method for the appropriate funnel design. The graph approaches the height of the initial water level at 0 mm and the shortest time to empty the funnel is the accurate method. The graph shows the explicit midpoint method is the fastest due to the short time to empty the funnel with 28 s compared to other methods.

Figure 7, 9 and 11 shows the error of the difference in each method used for the initial height of water of 250 mm. In terms of efficiency, the graph plotted for the least error is the most accurate method. The graph at the bottom shows the highest accuracy and can be seen that all the methods are efficient.

Funnel Opening Diameter, d (mm)	Initial water level, H (mm)	
	150	250
4	196.73 s	705.51 s
6	87.44 s	313.56 s
12	21.86 s	78.39 s

TABLE 2 Time with different parameter

For explicit methods, the midpoint method is more accurately compared to the Euler method because the error value is smaller and below with another line method. Furthermore, for implicit method, the midpoint method is the most efficient compared to the Heun method because the value is below and approximates the error value 0. Overall from the graph indicates that the choice of method for reducing error is the explicit midpoint method compared to other methods to avoid too much error.

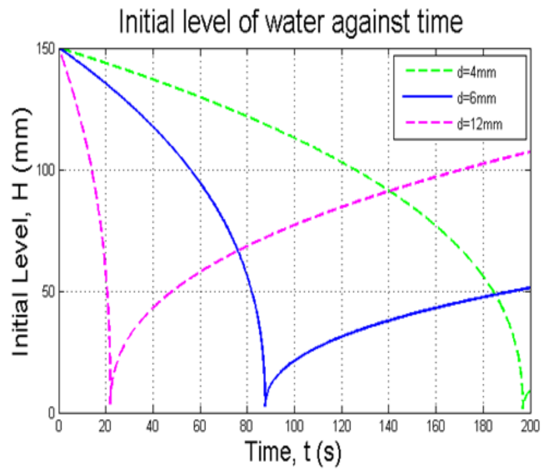

FIGURE 12. Time for each different funnel opening diameter

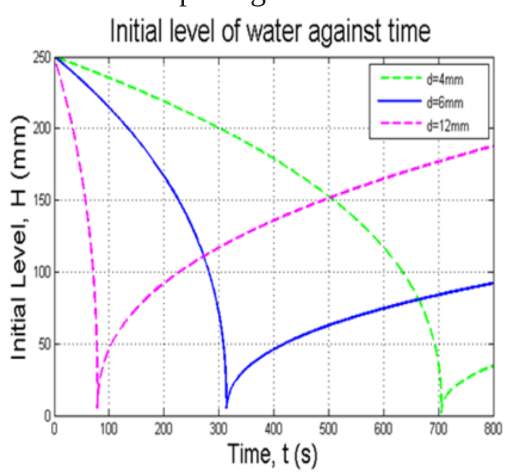

FIGURE 13. Time for each different funnel opening diameter

Table 2 shows the difference in parameters to determine the shortest time to empty the funnel using equations

$$t_e = \frac{8H^{\frac{5}{2}}}{5d^2\sqrt{2g}}$$

Based on figure 12 and 13 above the bigger of funnel opening diameter and the lower the water level in funnel shows that the time required to empty the funnel more fastest. The funnel opening diameter with 12mm and the initial water level with 150mm shows the fastest drainage of water through the tapered funnel.

STABILITY REGION

There are several important stability areas that are standard tools in the analysis of numerical formulas for initial problems of differential equations. Based on Figure 14, a small stability area indicates that a very small time step should be used in solving differential equations to obtain a greater stability area.

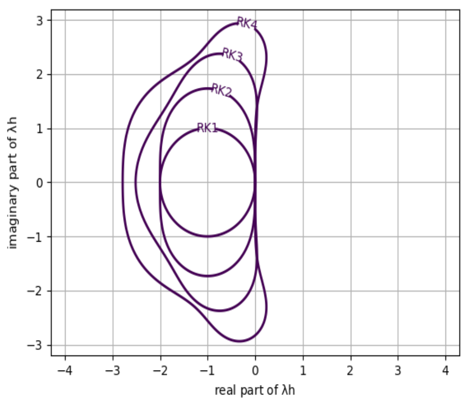

FIGURE 14. Stability region for Numerical Method

Based on Figure below, for each stabily region with colour show that the most stable region than not colour region.

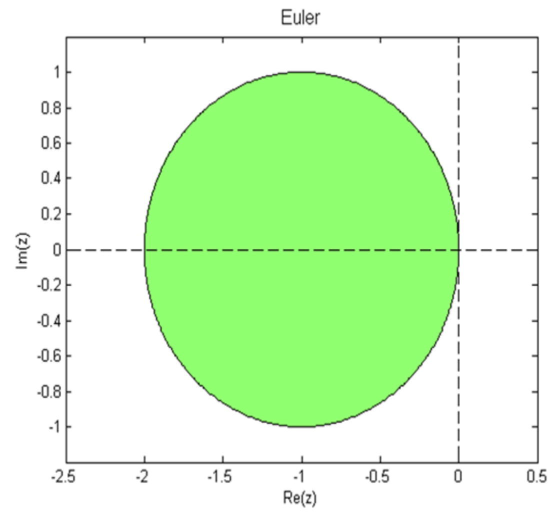

FIGURE 15. Stability region of Explicit Euler method

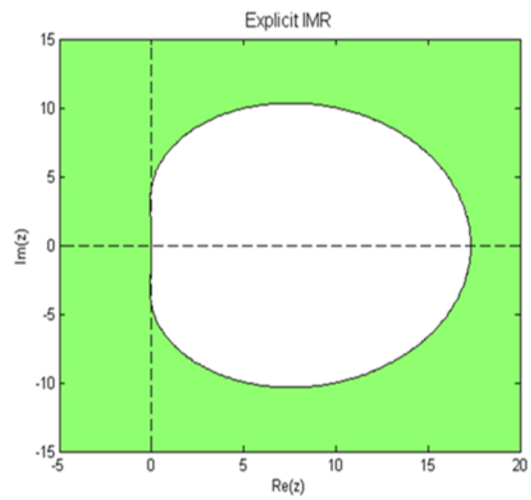

FIGURE 16. Stability region of Explicit Midpoint method

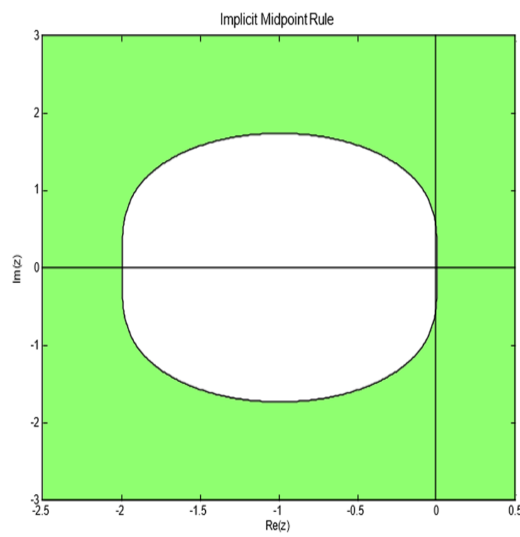

FIGURE 17. Stability region of Implicit Midpoint method

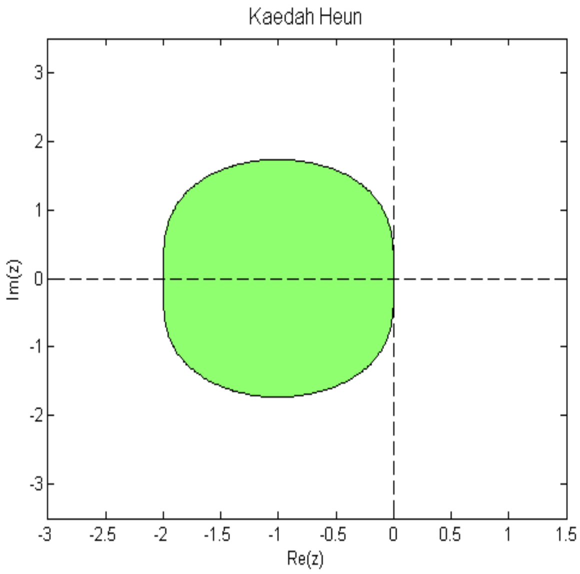

FIGURE 18. Stability region of Heun method

CONCLUSION

In conclusion, based on the analysis that the time plotted using distinct parameters indicates that it takes a short time to clear the funnel for the greater diameter. The original water level height can also influence the effectiveness and time-consuming velocity to empty the funnel. Additionally, the design of funnel with biggest angle takes the shortest time to liquid out of the funnel. Based on the results of the study, the method that shows the highest accuracy is the method of the implicit midpoint method because of the small error value and takes the shortest time compared to the other method. This implicit midpoint method can minimize the selection price of the funnel layout and make it easier to use the funnel. The precise choice of the funnel model can minimize the price of the funnel design as well as the volume in the funnel for the water. Furthermore, optimum funnel designs also regulate excess waste or spillage. The numerical method used in this research is to obtain precise, effective and stable estimates.

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TEST Engineering & Management
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