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Probability-based Crossover and Mutation in Genetic Algorithms for Vibration-based Damage Detection in Plates

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Abstract

Damage occurred in a structure cause reduce stiffness of the structure. Theoretically, once the stiffness is varied, the vibration characteristics - natural frequencies and mode shapes - of the structure are consequently changed. Therefore, the vibration-based damage detection can be formulated to an optimization problem in which an objective function numerically calculated from the difference between the experimental vibration characteristics and those of predicted damage where this paper employs experimental vibration characteristics that are approximated from the numerical calculation from the actual damage. The damage detection in plates is used as test cases in which a genetic algorithm (GA), a population-based derivative-free approach, is the solution search in the formulated optimization problems. There are 2 test cases of damage detection in plates to be investigated. The first case has one damaged region consisting of 4 damaged elements and the second case has five separately damaged elements. To enhance of performance of GA, this paper proposes a probability-based crossover and mutation embedded in GA. By this proposed idea each decision variable is assigned a probability to be performed GA operators - crossover and mutation. Unlike normal GA which applies the GA operators on all decision variables, by the probability-based crossover and mutation, only some decision variables to be performed the GA operators to avoid the unnecessary variables to be applied by the operators. After simulation runs, solutions obtained from the GA with probability-based crossover and mutation are better than those obtained from the normal GA for both 2 test cases. These results show that the probability-based crossover and mutation can enhance the performance of GA in damage detection in plates.

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1. Introduction

The vibration-based damage detection methods, non-destructive methods, are based on the fact that vibration characteristics such as natural frequencies and mode shapes of the structure are changed due to the occurred damage in a structure. Many applications in civil engineer and mechanical engineer employed these methods [1-3]. This method formulates damage detection as an optimization problem having a minimized objective function numerically calculated from

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the differences between the vibration characteristics of actual damage and those of predicted damage [4,5].

Genetic algorithm (GA) [6,7] is a derivative-free population-based optimization method of which search mechanisms are based on the Darwinian concept of survival of the fittest. Several works employed GAs to solve damage detection in plates such as [8-10]. As the GA is a population-based optimization method, the GA contains a set of solutions or population. Each GA generation, so-called parent solutions are selected from the current population. Consequently, the parent solutions are applied GA operators - crossover and mutation to form newly generated solutions, offspring solutions, which are then the members of the updated population.

Previous works in damage detection such as the focus on algorithms used as optimizers [11-13] or objective identifications [14,15]. However, the previous works had not considered the relation of decision variables. In the damage detection, a decision variable represents damage amount of its corresponding divided element. Since there is coupling between divided elements of plates, which affects to the vibration characteristics of the plates, probability-based crossover and mutation are proposed to improve solutions obtained from a genetic algorithm (GA) to avoid GA operators applied on unnecessary decision variables. By the probability-based crossover and mutation, decision variables representing damage factors of divided elements are assigned probabilities to be performed by the GA operators - crossover and mutation. Unlike normal GA which performs the crossover and mutation on all decision variables, only some decision variables are considered to be performed by the GA operators based on their corresponding probabilities.

2. Objective Calculation in Vibration-based Damage Detection

As previously described, vibration-based damage detection can be formulated to the optimization problem with an objective function to be minimized. It can be consequently explained by the formulation as follows. For free vibration of undamped structures, the equation of motion is given by the following equation

$$[m]\ddot{\vec{x}} + [k]\vec{x} = \vec{0} \quad (1)$$

where $[m]$ and $[k]$ are mass and stiffness matrices respectively. The corresponding eigen value equation for vibration mode j is given by

$$([k] - \omega_j^2[m])\vec{v} = \vec{0} \quad (2)$$

where ω_j^2 and $\vec{v}$ are eigen value, the square of nature frequency, and eigen vector, mode shape, of j^{th} mode of vibration, respectively.

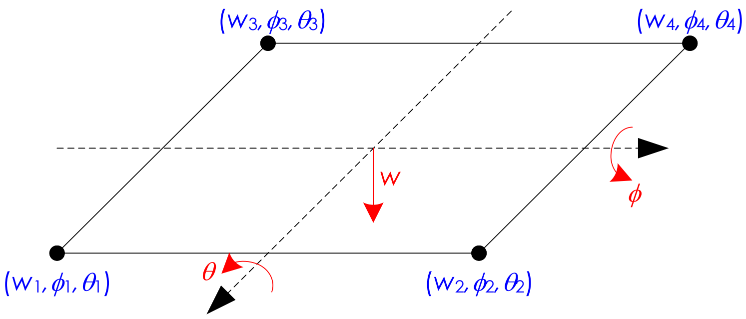

Figure 1 A rectangular divided element for finite element model.

Figure 1 shows a divided rectangular element used in the finite element models for plates [16]. At each nodal point, there are 3 degrees of freedom which are deflection in z direction (w), twist angle about x-axis (ϕ) and twist angle about y-axis (θ). From the element, there are 4 nodal points so that 12 degrees of motion for computing local stiffness matrix $[k]_i$ and local mass matrix $[m]_i$.

In finite element model, the stiffness matrix $[k]$ and mass matrix $[m]$ can be calculated by the sum of their local matrices of all divided elements as the equations (3) and (4).

$$[m] = \sum_{i=1}^{N} [m]_i \quad (3)$$

$$[k] = \sum_{i=1}^{N} [k]_i \quad (4)$$

Once the damage occurs in an element i of a structure, local damaged matrix $[k]_{di}$ is reduced from its local undamaged matrix $[k]_i$ according to damage factor (γ_i) of the element. The damaged local matrix can be computed by the following equation.

$$[k]_{di} = (1-\gamma_i)[k]_i \quad (5)$$

The values of the parameters γ_i fall in the range 0 to 1. The damage factor γ_i = 1 indicates that an completely damaged element and γ_i = 0 or less than 1 implies undamaged or partially damaged elements respectively.

Similar to equation (4) the stiffness matrix of the damaged structure is the sum of their local damaged matrices.

$$[k] = \sum_{i=1}^{N} [k]_{di} = \sum_{i=1}^{N} (1-\gamma_i)[k]_i \quad (6)$$

Moreover, it is assumed that the mass matrix is unchanged due to the occurred damage. By substituting the stiffness matrix [k] from equation (2) into equation (6), approximately experimental vibration characteristics - natural frequencies and mode shapes - could be obtained.

In the optimization process, the decision variables are the predicted damage factors β_i of all divided elements so that the number of decision variables is equal to the number of divided elements. The objective function f is numerically calculated from the difference between natural frequencies and mode shape of true occurred damage and the approximately experimental vibration characteristics [15] as show in the following equation.

$$f = \sum_{i=1}^{NM} w_{\omega_i} \Delta\omega^2 + \sum_{i=1}^{NM} w_{\vec{v}_i} (1 - MAC_i) \quad (7)$$

where NM is the number of vibration modes used in the calculation. w_{ω_i} is a weight factor corresponding to the i^{th} eigen value while $w_{\vec{v}_i}$ is a weight factor corresponding to the i^{th} eigen vector. In this paper, w_{ω_i} and $w_{\vec{v}_i}$ are equal to one dividing NM for all vibration mode i. $\Delta\omega^2$ is a minimum numerical indicator shows the difference between the eigen value of predicted damage and that of actual damage. MAC_i is a maximum index to indicate the difference of the eigen vectors. $\Delta\omega^2$ and MAC_i are given by equations (8) and (9) respectively.

$$\Delta\omega_i = \frac{\left|\omega_i^p - \omega_i^e\right|}{\omega_i^m} \quad (8)$$

$$MAC_i = \frac{\left(\vec{v}_i^{\,p} \cdot \vec{v}_i^{\,e}\right)^2}{\left|\vec{v}_i^{\,p}\right|^2 \left|\vec{v}_i^{\,m}\right|^2} \quad (9)$$

where ω_{ip} and ω_{ie} are natural frequencies of predicted damage and actual damage of vibration mode i, while $\mathbf{v}_{ip}$ and $\mathbf{v}_{ie}$ are mode shapes of measured points of the predicted and actual damage of the vibration mode i. It can be noted that ω_{ip} and $\mathbf{v}_{ip}$ are related to the predicted damage, while ω_{ie} and $\mathbf{v}_{ie}$ are measured natural frequency and mode shape of the actual damaged occurred in the structure. If the damage is correctly predicted, for all vibration mode i $\Delta\omega_i$ and MAC_i are equal to 0 and 1, the objective function f is then equal to 0.

3. Test problem

The damage detection in a cantilever plate (Figure 2) used as the test problem. The plate had the dimensions of 100 cm × 100 cm × 5 cm. The properties of the plate are as follows: modulus of elasticity (E) = 210 GPa, Poisson' ratio (ν) = 0.3, and density (ρ) = 7,800 kg/m^3. In the finite element model, the plate is divided into 64 square elements and each element has 4 nodal points. In experimental approximation, there are 6 measured points as shown in Figure 3 where deflection in the z-direction (w) can be measured. Two different cases of the test problem (Figure 4) are considered – (1) the plate having only one region consisting of elements 27, 28, 35, and 36 damaged partially to an extent of 20%, and (2) the plate having elements 2, 15, 27, 45, and 62 partially damaged of 40%, 30%, 50%, 60%, and 20% respectively. The occurred damage factors γ_i for both cases are numerically described by equations (10) and (11) respectively. The first test case represents a simple case because it

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has only one region so that it has a small number of neighbor undamaged elements that make GA search mechanism difficulty. On the other hand, the second test case represents the harder case which has a larger number of neighbor undamaged elements.

$$\gamma_{27} = \gamma_{28} = \gamma_{35} \quad (10)$$

$$\gamma_{2} = 0.4,\ \gamma_{15} = 0.3,\ \gamma_{27} = 0 \quad (11)$$

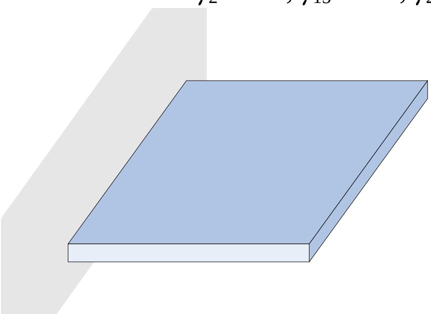


Figure 2 Cantilever plate.

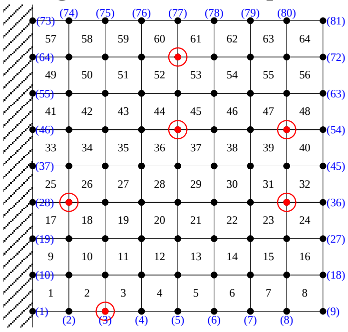
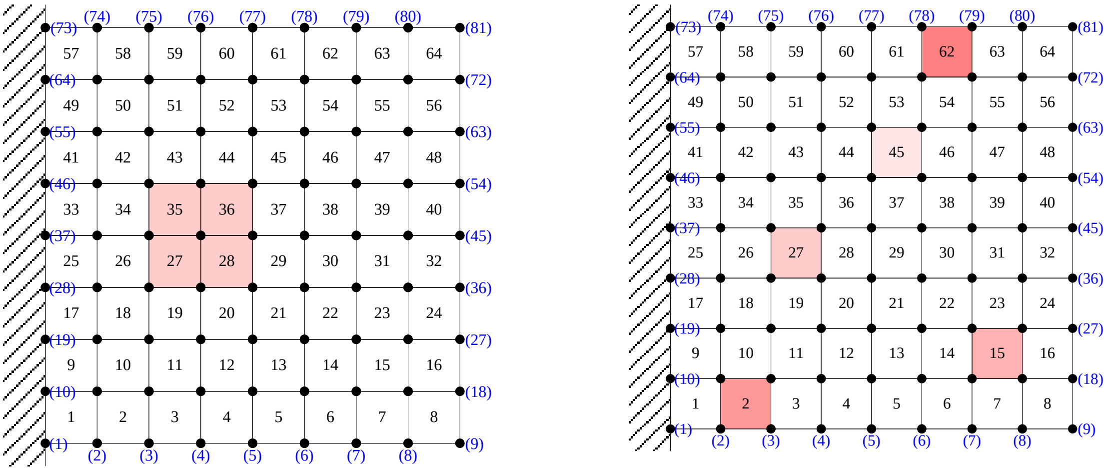


Figure 3 Top view of the cantilever plate with divided elements and 6 measured points in circles.

4. Probability-based crossover and mutation

Probability-based crossover and mutation is proposed in order to assign probabilities on decision variables to be performed GA operators. At first, genetic algorithm (GA), a population-based derivative-free optimization method, will be explained. GA search mechanisms are based on the Darwinian concept of the survival of the fittest. In GA, a population or a set of solutions, which is the input to GA, is randomly generated. The solutions in the population are then evaluated to obtain their objectives, which indicate how fit of solutions to the optimization problem. After objective calculation, elitism operator is applied to pass good solutions including the best solution to be members of updated population. Thereafter fit solutions are then selected with replacement to be parent solutions. The parent solutions are then changed by GA operators, crossover and mutation, to achieve resulting offspring solutions, which will be members of the updated population. The solutions in the population will be evaluated again and then to be selected for the parent populations in order to be performed the crossover and mutation. The process of GA is repeated until termination condition is satisfied. Finally, after the termination, the best solution of the latest population is the output of the algorithm. The process of GA is displayed in Figure 5.



(a) Case 1 : one damaged region (b) Case 2 : five separately damaged elements

Figure 4 Two different cases of the test problem

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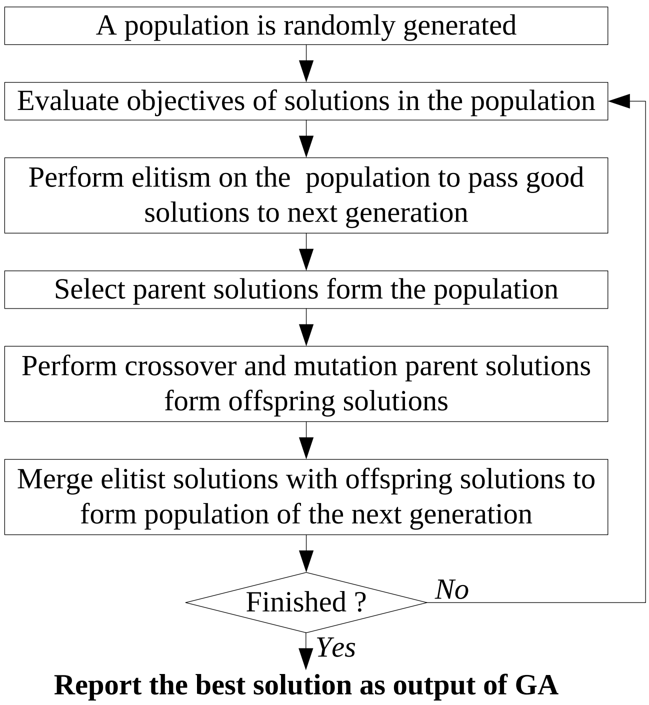

Figure 5 GA process

The GA operators generally applied on the whole of representative chromosome of a solution. For the test problem used in this paper, the representative chromosome of a solution is encoded into a real number string with length of 64, the number of divided elements. After a period of time in GA search, GA probably found the best solution with optimum decision variables, damaged factors, represent for undamaged elements, which is zero, so there is not necessary to perform the crossover and mutation on the representative variables of those elements because the crossover and mutation may change the values of the variables. This paper will propose probability-based crossover and mutation in GA in order to enhance GA performance in vibration-based damage detection in plates. The connectivity of elements are considered in order to assign probabilities for crossover and mutation of divided elements of a plate.

From Figure 1, an element has 4 nodal points at its corners. For instance, element 10 has 4 nodal points - 10, 11, 20, and 21, these 4 nodal points are also connected to other 9 neighbour elements 1-3, 9, 11, and 19-21 which are considered as the neighbour elements of element 10. The displacement coordinates - w, ϕ, and θ - of the nodal points are therefore also used for the finite element model of the neighbour elements. Theoretically this shows unavoidable coupling among the decision variables corresponding damage factors of the elements. If there is much difference between damage factor of an element and those of neighbour elements, this element should be performed the GA operators, crossover and mutation, rather than another element with the less difference between its damage factor and damaged factors of its neighbour elements.

The probability depends on how the predicted damage of a considering element is different from those of the neighbor elements in which the current best solution is used for the probability calculation. At first, the squared summation of the predicted damage of an element i and those of its neighbor elements, s_i, is calculated by using equation (12).

$$s_i = \sqrt{\sum_{j=1}^{nn} \left(\beta_i - \beta_j\right)^2} \qquad (12)$$

where nn is number of neighbor elements surrounding around an element i. Given Ψ is the summation of s_i for all elements as shown in equation (13).

$$\psi = \sum_{i=1}^{N} s_i \qquad (13)$$

The probability of the element i (P_i) to be performed in crossover and mutation is given

$$P_i = P_{\min} + \frac{N s_i (1 - P_{\min})}{2\psi} \qquad (14)$$

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where N is the number of divided elements. In the equation, it shows that the average of P_i is equal to 0.5. In each time of the GA operators, a pair of parent individuals is randomly picked, the probability P_i is then compared with a random number $r \in [0,1]$. If P_i is more that r, a corresponding decision variable of the element i is then picked to be performed the GA operators, crossover and mutation. The picked decision variables are then performed crossover and mutation to obtain a generated sub-individuals. The picked variables of resulting generated full individuals are same as those of the generated sub-individuals, while other variables of the full individuals are same as those of the parent individuals. Figure 6 shows an example of probability-based crossover and mutation on 6 decision variables chromosomes. In this figure, it is assumed that after probabilities are assigned to decision variables and variables 2, 3, and 4 are then picked for the crossover and mutation. Compared to the crossover and mutation in normal GA, computational time in crossover and mutation of GA with the proposed idea is less than that of the normal GA because the crossover and mutation is performed on a smaller number of decision variables.

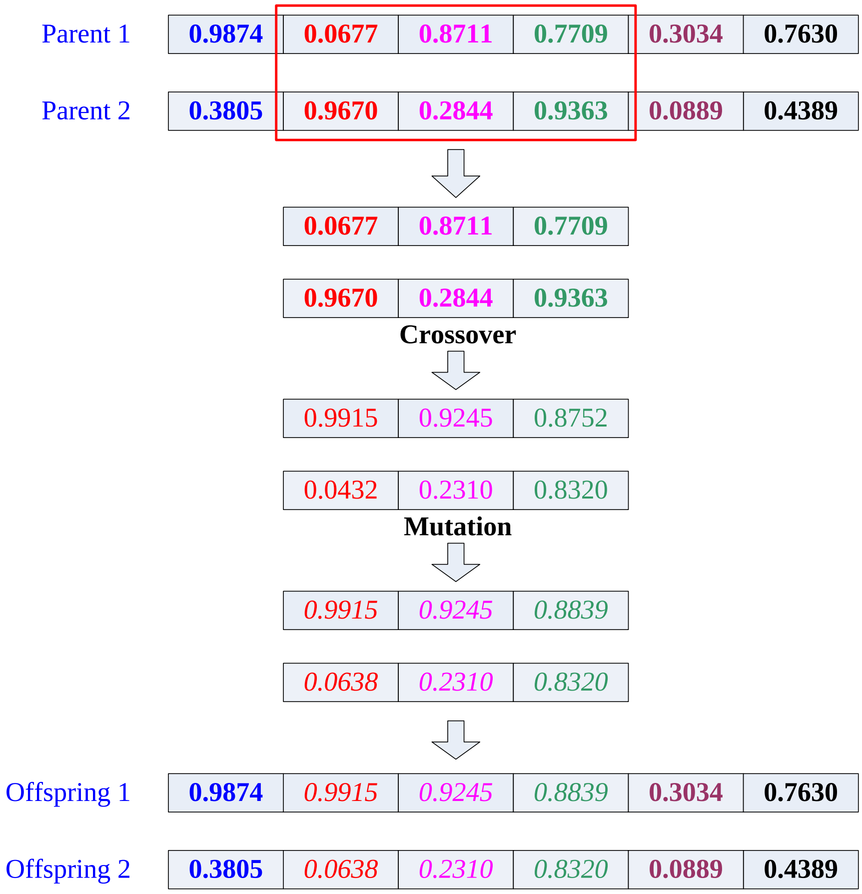

Figure 6 Probability-based crossover and mutation on 6 bit real-coded chromosome.

Table 1. Optimized solutions and its objective function of test case 1

Element	Exact value	Normal GA	GA with probability-based crossover and mutation
27	0.2	0.224904	0.201853
28	0.2	0.108285	0.198778
35	0.2	0.169831	0.199810
36	0.2	0.191527	0.200408
Other elements	0.0	0.000006-0.065870	0.000000-0.001935
Objective function		0.00000416	0.00000002

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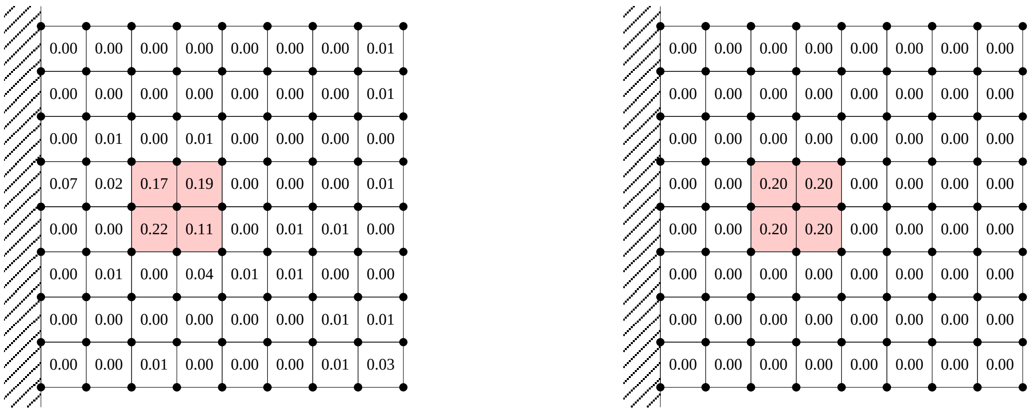
Table 2. Optimized solutions and its objective function of test case 2

Element	Exact value	Normal GA	GA with probability-based crossover and mutation
2	0.4	0.392777	0.385960
15	0.3	0.073004	0.217605
27	0.2	0.164418	0.158516
45	0.1	0.004171	0.041386
62	0.5	0.343915	0.464706
Other elements	0.0	0.000079-0.328162	0.000000-0.065984
Objective function		0.00001500	0.00000171

5. Numerical results

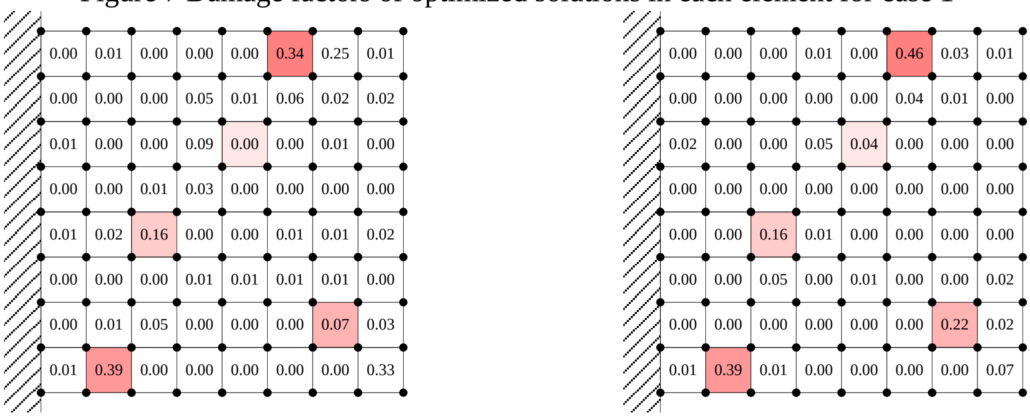
In GA run, a solution is encoded into a real coded chromosome of which length is 64, the number of divided elements, as shown in Figure 3. Population size and number of generations used as a termination condition are 20 and 3200 respectively so that the number of generated solutions is equal to 64,000. Simulated binary crossover (SBX) [17], [18] is used for real-coded crossover in which crossover probability is 1.0. For mutation, variable-wise polynomial mutation [19], of which the probability of 1 divide the number of decision variables of a chromosome to be mutated, is used. The number of repeated runs is 10 where the best solution of all repeated run is reported as the output of an algorithm.

Table *1* and Table 2 show the optimized solutions and their objective function of test cases 1 and 2 respectively. Figure 7 and Figure 8 display damage factors of each element of the solutions for cases 1 and 2 respectively.



(a) Normal GA (b) GA with probability-based crossover and mutation

Figure 7 Damage factors of optimized solutions in each element for case 1



(a) Normal GA (b) GA with probability-based crossover and mutation

Figure 8 Damage factors of optimized solutions in each element for case 2

From Table 1 and Table 2, it is found that the optimized solution by GA with probability-based crossover and mutation is better than that by the normal GA for both objective functions and the predicted damage factors for both cases 1 and 2. By illustrating damaged factors as two decimal digits numbers in Figure 7, the damage factors from the optimized solution obtained from with probability-based crossover and mutation are the same as the exact damage factors in case 1. In Figure 8, GA with probability-based crossover and mutation supplies the optimized solution for case 2 is not good as its optimized solution for case 1. The separation of damaged elements causes difficulty for the GA search mechanism. From the optimized solution by normal GA in Figure 8(a), the predicted damage factor of element 2, 0.39, is close to the actual amount, 0.4, however, the predicted damage factor of element 15, 0.07, is far away from the actual damage factor, 0.3. It can be observed that the pre predicted damaged factor of neighboring element 8, which is undamaged, is 0.33. Similarly, the other damaged elements 45 and 62, also have the neighboring elements 44 and 64 with significant nonzero damage factors. The reason behind this unexpected outcome is that the damage occurred as Figure 8(a) probably contributes to the vibration characteristics close to the vibration characteristics of the actual damage in equations (11). In search mechanism after a number of GA generation, if the solution as Figure 8(a) is obtained as the best solution at that time, thereafter it is very difficult for GA finally meet the actual best solution because no GA operators that exchange values between different decision variables where GA actually has crossover to exchange values of the same decision variable from two parental solutions. The probability-based crossover and mutation can improve GA performance on this difficulty, however, it is not enough to make GA correctly predict the damage factors for case 2. In Figure 8(b) there are neighboring elements 8, 19, 44 having significant nonzero damage factors of actual damage elements 15, 27, 45. By this imperfection should, the studies of the effects of neighboring elements to the vibration characteristics should be investigated.

6. Conclusions

This paper proposed the probability-based crossover and mutation in GA for the damage detection in plates. The damage detection is formulated as an optimization problem having a minimized objective function numerically calculated from the differences between the experimental vibration characteristics and the predicted damage. This paper employs the approximately experimental data of vibration characteristics - natural frequencies and mode shapes - numerically evaluated from the true damage occurred in the plates. By the proposed idea, before the implementation of GA operators - crossover and mutation each decision variable, which represents the damage factor of a divided element, is assigned its corresponding probability. There are two test cases of damage detection in plates - (1) one damaged region having 4 damaged elements and (2) five separately damaged elements - to be investigated. The simulation runs show that GA using the probability-based crossover and mutation obviously outperforms normal GA for both two cases. For the first test case, the damage factors can be correctly predicted by the proposed idea. However, there is a significant difference between the damaged factors obtained from the proposed idea and the exact damage factors. The effects of neighbor elements should be further studied in order to improve solutions obtained from GA for damage detection in plates.

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