May-June 2020
ISSN: 0193-4120 Page No. 9611 – 9617

Some Aspects of Total Time Minimization Transportation Problem

B.Mallia1 - 1Department of Mathematics, ITER, Siksha 'O' Anusandhan University, Bhubaneswar,Odisha, India
M. Das2- 2Centre for Applied Mathematics and Computing, Siksha 'O' Anusandhan University, ITER, Bhubaneswar, Odisha,India
C.Das3- 3Department of Mathematics,NIT, Rourkela,Odisha, India

Article Info
Volume 83
Page Number: 9611 – 9617
Publication Issue:
May - June 2020

Article History
***Article Received**: 19 November 2019*
***Revised**: 27 January 2020*
***Accepted**: 24 February 2020*
***Publication**: 18 May 2020*

Abstract:
Purpose - we consider a new type of total time minimization transportation problem along with the total cost minimization transportation problem.
Design / Approach -We have designed an active transportation route and minimize the total transportation time. It leads to zero one integer programming problem. Different objective functions are formulated subject to original constraints in transportation problem.
Findings and concluding remarks -The time transportation problem has been defined from the original transportation problem ,paying importance to time. We analyze different variety of transportation problems and exhibit mathematically the importance of time transportation problem.
Research implication-The paper may be extended for other variety of transportation problemssuch as cargo handling in airport , transportation of perishable marine product etc.
Utility of the Paper - Application of this type of transportation problem is best fitted to combat zones for supplying ammunition to soldiers in advance fighting zones. This problem can be applied to marooned people and disaster management system for cyclone, flood earthquake affected people etc. We have quoted a numerical example and solved it with the algorithm we have developed.
***Keywords**: Transportation problem, Total time, multiple optimal solutions, active transportation route, longest transportation operation.*

1. Introduction

The problem of minimizing total cost of transporting goods from sources to destinations is called Transportation Problem.Transportation problem was first formulated by Hitchcock (1941). It is a liner programming problem. So the Simplexmethod can be used to solve transportation problem. Instead of using simplex method, simplextype of iterative methods is used to solve total cost transportation problem[3]. Sometimes timeis relevant in variety of transportation system, such as supplying rations (food commodities) to soldiers in advance army camp, food and essential commodities to marooned people in cyclone and flood affected areas in fire services too. We consider here the following types of transportation problem:

Total transportation time minimization (Linear function)and minimization of total time along with the longest route [2].We have considered thevariants of total transportation time along the longest active route in time transportation, without giving importance to all destinations that lead to three classes of time transportation problems, one is called single criteria transportation time and other two aremulti-criteria transportation time problems[3]. There are other transportation problem such as bi-criteria transportation problem, multi - objective transportation problem and goalprogramming problem[2].Nikolic, I, and Petric. J.(2007), have minimized total transportation time by using the GoalProgramming (G.P.) approach [7]. Various algorithms for solving time minimization TP have been developed by many researchers [3, 4, 5, 9, and

12]. The transportation time has its relevance in a variety of real transportation problems.
G. Sharma, Abbas S.H. and Gupta V. have proposed a zero -point method to solve the time minimization transportation problem. And also compared with regular method of time minimization problem. This method previously used by P. Pandian and G. Natrajan to get the fuzzy optimal solution and optimal solution in cost minimization transportation problem [11].Again another method of minimizing time in transportation problem has been developed by Swati Agarwal and ShambhuSharma,"An open loop method for time minimizing problem with mixed constraints." [1]. Here an open loop has been used to improve the IBFS by changing the basic cells to other basic cells with a very less time.The paper is designed as follows. The mathematical formulation of the problem is given in section 2 .solution procedure is given in Section 3. In Section 4 numerical example is illustrated.Last sectioncontains a detail discussion andconclusion.

2. Mathematical formulation of the problem.

We formulated the following transportation problem:

Let I={1,2,.........m}and J={1,2,..........n,} represent the index sets of sources and destinations respectively.
Lets_i , $i \in I and d_j, j \in J$be the amount goods available at i[th] origin and amount of goods requiredatj[th]destination respectively. We formulate transportation problem asfollows:

$$Minimize Z = \sum_{i\in I}\sum_{j\in J} c_{ij}\,x_{ij} \quad (1)$$

$$subject to the constraint \sum_{j\in J} x_{ij} = s_i for all i \in I \quad (2)$$

$$\sum_{i\in I} x_{ij} = d_j \quad for\ all\ \ j \in J \quad (3)$$

$$and x_{ij} \geq 0 \quad for all (i,j) \in I \times J \quad (4)$$

Moreover if $\sum_{i\in I} s_i = \sum_{j\in J} d_j$, then the problem is called balanced transportation problem .Instead of minimization of total cost the unit time t_{ij} for route (i,j) is relevant ,becausethe goods $x_{ij}$are supplied along the route (i,j)in some catastrophic situation like cyclone,flood,earthquake, military operations etc. for marooned people. Thus transportation efficiency is necessary at that time. We, therefore minimize the following objective function.

$$f(x) = \sum_{i\in I}\sum_{j\in J} t_{ij}\,x_{ij} \quad (5)$$

$$subject to \sum_{j\in J} x_{ij} = s_i \forall i \in I$$

$$\sum_{i\in I} x_{ij} = d_j \forall j \in J$$

$$and x_{ij} \geq 0 \quad for all (i,j)$$

Moreover if we consider the active transportation route(i,j) defined below and minimize the total time , the corresponding problem becomes :

$$minimize T(x) = \sum_{i\in I}\sum_{j\in J} t_{ij}\,s_{ij} \quad (6)$$

and the variables_{ij} is defined as :

$$s_{ij} = \begin{cases} 1 & if x_{ij} > 0 \\ 0 & if x_{ij} = 0 \end{cases} \quad (7)$$

This type of transportation problem is found in supplying ammunition in advance fighting zone, if military operation will start in all combat zones ,simultaneously atthe earliest possible instant for that it is necessary that the request amount of ammunitions are available in all combat zones at a time . In other words theoperation cannot start before the last shipments of ammunition has arrived and the problem is to schedule the shipment, so that the operation can start as soon as possible.

3. Solution Procedure

We start with twobasis neighbouring feasible solution$x^{(k)}$ and $x^{(k+1)}$ differing only two basic variables$x_{ij}^{(k)} and x_{is}^{(k)}$, $where x_{ij}^{(k)}$ entersand

TEST Engineering & Management

May-June 2020
ISSN: 0193-4120 Page No. 9611 – 9617

variable $x_{is}^{(k)}$ is leaves. The basic solution $x^{(k+1)}$ is calculated from $x^{(k)}$ using stepping stone method.

The change in total time T(x) given in (6) is calculated as follows

$$r_{ij}^{k} = t_{ij} - t_{is} \qquad (8)$$

$$and T^{(k+1)} = T^{(k)} + r_{ij}^{(k)} \qquad (9)$$

$$where T^{(k+1)} = \begin{cases} > T^{(k)} if r_{ij}^{(k)} > 0 \\ = T^{(k)} if r_{ij}^{(k)} = 0 \\ < T^{(k)} if r_{ij}^{(k)} < 0 \end{cases} \qquad (10)$$

In this situation different cases exist and are calculated as follows.

Let's x^* is the optimal solution of (6), T^* is the minimal value of T(x), and a set of(Multiple Optimal Solution)MOS is X_T of (6)

$$T^* = \min_{X}\left\{T(x) = \sum_{i\in I}\sum_{j\in J} t_{ij}s_{ij}\right\} \qquad (11)$$

$$x_T^* = \left\{x \mid T^* = \min_{X}\left\{T(x) = \sum_{i\in I}\sum_{j\in J} t_{ij}s_{ij}\right\}\right\} \qquad (12)$$

$$X_T^* = \left\{x_T^*\right\} \qquad (13)$$

Above discussion gives rise to develop algorithm for solvingTP (6). If this problem has MOS (13) then new criteria is required.

Minimize the total transportation time (5)

$$\underset{T(x)=T^*}{Min}\left\{f(x) = \sum_{i\in I}\sum_{j\in J} t_{ij}x_{ij}\right\} \qquad (14)$$

And minimizing the time of the longest active transportation operation

$$\underset{T(x)=T^*}{Min}\left\{t(x) = \underset{x_{ij}>0}{Max}\ t_{ij}\right\}$$

Minimizing the bulk quantity of goods along longest timetransportation operation

$$\underset{T(x)=T^*}{Min}\left\{R(x) = \sum_{t_{ij}=t(x)} x_{ij}\right\}$$

Lastly, we minimize the total transportation cost

$$\underset{T(x)=T^*}{Min}\left\{f(x) = \sum_{i\in I}\sum_{j\in J} c_{ij}x_{ij}\right\}$$

Where $c_{ij} = the\ units\ transportation\ cost$

We present new algorithms to solve the total time transportation problem as follows

Algorithm-1

Step 0: Start number of iteration k=1. Determine the initial basic feasible solution $x^{(1)}$.

Step 1: For active transportation routes (i,j), $x_{ij}^{(k)} > 0$, the indicators $s_{ij}^{(k)}$ and $T^{(k)} = T(x^{(k)})$ is the total time determine along with

$$s_{ij}^{(k)} = \begin{cases} 1, & if\ x_{ij}^{(k)} > 0 \\ 0, & if\ x_{ij}^{(k)} = 0 \end{cases}$$

$$T^{(k)} = \sum_{i\in I}\sum_{j\in J} t_{ij}s_{ij}^{(k)}$$

Step2: For all non-basic variables $x_{ij}^{(k)} = 0$ determine the characteristics $r_{ij}^{(k)}$ using(8). By using the changing path of the basic solution and corresponding leaving basic variable $x_{is}^{(k)} > 0$ becomes $x_{is}^{(k+1)} = 0$, if entering basic variable is $x_{ij}^{(k+1)} > 0$.

May-June 2020
ISSN: 0193-4120 Page No. 9611 – 9617

Step 3: Use (6) and (10) for total time optimization. If each of $r_{ij}^{(k)} \geq 0$,then the optimal solution x*is found .End. Else go to the next stride.
Step 4: Obtain the following basic solution, by using x_{ij}as entering variablehaving minimum $r_{ij}^{(k)}$ concerning $r_{ij}^{(k)} < 0$ .Set k=k+1 and go to the first step.

If in step 3, we get the optimal solution x* having $r_{ij}^{(k)} = 0$ for non-basic variables $x_{ij}^{(k)} = 0$, then,there is no unique optimal solution. Rather it provides sets of MOSs XT^*.All the variables provide an alternate optimal solution for (6), Go ahead foroptimizing other criteria.

Algorithm 2. (For multiple optimal)
Step 0: Select a criteria from (14) - (17) and estimate the increase Δ_{ij}^{k} forevery non-basic variable $x_{ij}^{(k)} = 0$ with $r_{ij}^{(k)} = 0$ in MOS at the end of theAl 1. For Δ_{ij}^{k} use the well known dual variable solving process of dual variable.

Step 1: If there is any negative increment in $\Delta_{ij}^{k} < 0$, for the concerned criteria, choosethe least of them and go to minimize this criteria inthe MOSsetfor (6).

Step 2: Iterate Step 1 havingevery negative increase for concerned criteriathen select solution having minimum criteria value.

4. Numerical Illustration
We haveconsidered below transportation problem with sources and destinations - m = 4 , n = 5 respectively . TABLE -I contains the initialdata for calculation, rows and columns are used for supply and demand points respectively. We treated a balanced TP with total supply andtotal demand 65 units each.The time $t_{ij}$of transporting goods for source i to destination j is represented in left top corner of the TP , TABLE -I .We have shown x_{ij}, the basic variables inside the circle of basic cells.r_{ij} is represented on bottom left nook of each non-basic cell (i,j).TABLE-I represents the (Optimal Solution)OS $x^{(1)}$ for f(x) and indicators r_{ij} for T(x).

TABLE-IOptimal Solution $x^{(1)}$ for the following problem:

Destination

Origin	D1	D2	D3	D4	D5	Origin Availability
O1	9	6	5 5	10 3	6	15
O2	3 6	15 4	8	10	7	18
O3	15	6	15 5	9	5 8	20
O4	7 6	8	4	10	5 6	12
Destination Requirements	10	15	20	10	10	65

Calculation: $f^{(1)} = 320$
$s_{ij}^{(1)} = 1$ for basic cells {(1,3) (1,4) (2,1) (2,2) (3,3) (3,5) (4,1) (4,5)}
and$T^{(1)} = 43$
We calculated the (InitialBasic Feasible Solution) IBFSusing Step 0 of Al.1 by using row-minima method . Below in Table-I we calculated optimal solution $x^{(1)}$ and minimum value $f^* = f^{(1)} = 320$ .The increase $d_{ij}^{(1)}$ is presented inTable-II.In Step 1 we have calculatedthe corresponding total time $T^{(1)}$=43with theindexs$s_{ij}^{(1)}$.(The indicators with the value 1 for active transportation routes are shownin Table-I).

May-June 2020
ISSN: 0193-4120 Page No. 9611 – 9617

Using Step 2 we have calculated increaser$_{ij}^{(1)}$ of the total transportation time T(x) fornon-basic cells. First non-basic cell (1, 1) is used to verify the process below.

$x_{11}^{(1)}=0$, $L_{11}^{(1)}=\{(1,1)(1,3)(3,3)(3,5)(4,5)(4,1)\}$

$d_{11}^{(1)}=t_{11}-t_{13}+t_{33}-t_{35}+t_{45}-t_{41}=9-5+5-8+6-6=1$

$x_{11}^{(2)}>0$, $x_{11}^{(2)}$ $=\min\{x_{13}^{(1)}, x_{35}^{(1)}, x_{41}^{(1)}\}=\min\{5,8,6\}=5= x_{13}^{(1)}$

$r_{11}^{(1)}=t_{11}-t_{13}=9-5=4$

.

For changing path $L_{43}^{(1)}=\{(4,3)(4,5)(3,5)(3,3)\}$for $x_{43}^{(2)}=\min\{x_{45}^{(1)},x_{33}^{(1)}\}=\min\{5,5\}= x_{33}^{(1)}$. We calculate $q_{43}^{(1)}=t_{43}^{(1)}-t_{33}^{(1)}= -1$. This implies that $x^{(1)}$ has no optimal solution for T(x). The entering of basic variable $x^{(1)}_{43,}$ the new solution $x^{(2)}$decrease $T^{(1)}=43$ to $T^{(2)}=T^{(1)}+t_{43}=43-2=41$ (TABLE-III).
Again $X^{(3)}$ is not a optimal solution as because , the changing path of non-basic variable $r_{32}^{(2)}$ =0 (TABLE-IV) .Keeping

TABLE-II

Non-basic variable $x_{ij}^{(1)}=0$	$X_{11}^{(1)}$ =0	$X_{12}^{(1)}$ =0	$X_{15}^{(1)}$ =0	$X_{23}^{(1)}$ =0	$X_{24}^{(1)}$ =0	$X_{25}^{(1)}$ =0	$X_{31}^{(1)}$ =0	$X_{32}^{(1)}$ =0	$X_{34}^{(1)}$ =0	$X_{42}^{(1)}$ =0	$X_{43}^{(1)}$ =0	$X_{44}^{(1)}$ =0
Indicators $d_{ij}^{(1)}$ for F(x)	$d_{11}^{(1)}$ =1	$d_{12}^{(1)}$ =4	$d_{15}^{(1)}$ =2	$d_{23}^{(1)}$ =5	$d_{24}^{(1)}$ =9	$d_{25}^{(1)}$ =1	$d_{31}^{(1)}$ =7	$d_{32}^{(1)}$ =0	$d_{34}^{(1)}$ =6	$d_{42}^{(1)}$= 4	$d_{43}^{(1)}$= 1	$d_{44}^{(1)}$= 1
Indicator $r_{ij}^{(1)}$ for T(x)	$r_{11}^{(1)}$ =4	$r_{12}^{(1)}$ =1	$r_{15}^{(1)}$ =1	$r_{23}^{(1)}$ =3	$r_{24}^{(1)}$ =7	$r_{25}^{(1)}$ =1	$r_{31}^{(1)}$ =9	$r_{32}^{(1)}$ =2	$r_{34}^{(1)}$ =6	$r_{42}^{(1)}$= 4	$r_{43}^{(1)}$= -1	$r_{44}^{(1)}$= 7

The cell$r_{43}^{(1)}$=-1 indicates that the solution is not optimal.Analyzing the longest time on the separate active transportation routes (15) and corresponding transported units (16)

$t(x)=\max t_{ij}$ $=\max(t_{12},t_{13},t_{21},t_{22},t_{33},t_{35},t_{41},t_{45})$

Xij>0

$=\max(5, 3, 6, 4, 5, 8, 6, 6) =8=t_{35}$

R(x) = 5

TABLE- III: Solution x^2, T^*

9	6	5 **5**	3 **10**	6
6 **3**	4 **15**	8	10	7
15	6	5 **10**	9	8 **10**
67	8	4 **5**	10	4

$T^{(2)}=T^{(1)}+t_{43}=43-2=41$

TABLE-IV: Solution x^3, T^*

May-June 2020
ISSN: 0193-4120 Page No. 9611 – 9617

9	6	5 **5**	3 **10**	6
6 **10**	4 **8**	8	10	7
15	6 **7**	5 **3**	9	8 **10**
6	8	4 **12**	10	6

$T^{(3)}=41$

TABLE-V: Solution x^4, T^*

9 **0**	6	5 **15**	3	6
6	4	8	10 **10**	7 **8**
15	6 **15**	5 **5**	9	8
6 **10**	8	4	10	6 **2**

$T^{(4)}=54$

TABLE-VI : Solution x^5, T^*

9	6	5 **15**	3	6
6	4	8	10 **10**	7 **8**
15	6 **15**	5 **5**	9	8 **0**
6 **10**	8	4	10	6 **2**

The cost matrix

$$C=\begin{array}{|c|c|c|c|c|}\hline 5 & 8 & 1 & 6 & 5 \\ \hline 7 & 9 & 16 & 10 & 8 \\ \hline 7 & 4 & 6 & 10 & 7 \\ \hline 5 & 8 & 9 & 10 & 6 \\ \hline \end{array}$$

TABLE- VII: Optimal solutions single criteriaTransportation Problem

Minimization Criteria	Solution				
	$X^{(1)}$ $\begin{bmatrix} 0 & 0 & 5 & 10 & 0 \\ 3 & 15 & 0 & 0 & 0 \\ 0 & 0 & 15 & 0 & 5 \\ 7 & 0 & 0 & 0 & 5 \end{bmatrix}$	$X^{(2)}$ $\begin{bmatrix} 0 & 0 & 5 & 10 & 0 \\ 10 & 8 & 0 & 0 & 0 \\ 0 & 7 & 3 & 0 & 10 \\ 0 & 0 & 12 & 0 & 0 \end{bmatrix}$	$X^{(3)}$ $\begin{bmatrix} 0 & 0 & 5 & 10 & 0 \\ 3 & 15 & 0 & 0 & 0 \\ 0 & 0 & 10 & 0 & 10 \\ 7 & 0 & 5 & 0 & 0 \end{bmatrix}$	$X^{(4)}$ $\begin{bmatrix} 0 & 0 & 15 & 0 & 0 \\ 0 & 0 & 0 & 10 & 8 \\ 0 & 15 & 5 & 0 & 0 \\ 10 & 0 & 0 & 0 & 2 \end{bmatrix}$	$X^{(5)}$ $\begin{bmatrix} 0 & 0 & 15 & 0 & 0 \\ 0 & 0 & 0 & 10 & 8 \\ 0 & 15 & 5 & 0 & 0 \\ 10 & 0 & 0 & 0 & 2 \end{bmatrix}$
F(x)	320	$F_{min}=318$	325	418	418
T(x)	43	$T_{min}=41$	$T_{min}=41$	54	53
t(x)	$t_{min}=8$	$t_{min}=8$	$t_{min}=8$	10	10
R(x)	$R_{min}=5$	10	10	10	10
C(x)	411	431	431	$C_{min}=331$	$C_{min}=331$

5. Discussion and Conclusion

For single criteria optimal solution of cost and time is given in Table- VII along with other criteria optimal solution and time. For solution X^3, the total cost C (x), the minimum time T, and other parameters are exhibited in Table - VII. However solution X^4,$X^{(5)}$ exhibit multiple solution with same total cost C=331 and same time T=10. Also the solution X^3 exhibits the total minimum time T= 41 and the longest route t=8.In the present timedifferent variety of transportation problems are formulated andanalysed. Thetime transportation problem is relevant for shipping the goods (ammunition) to different consumer zones. The efficiency of the system is determined as thesum total of time, bulk of goods on active transportation system, the longest time on transportation operation, total quantity of goods along the longest time route.In this paperwe have considered the single objective of total time minimization problem and given algorithm for the solution. We have also discussed MOS and avoided lexicographic order or Pareto Optimal solution for multiple objective function and numerical example is given for illustration.

References

1. Agarwal S. and Sharma S. ,(2010) " An open loop method for time minimizing transportation problem for mixed constraints." International Conference on “Innovative Approach in Applied Physical, Mathematical/Statistical, Chemical Sciences and Emerging Energy Technology for Sustainable Development ISBN: 978-93-83083-98-5
2. Aneya, Y.P., and Nair, K.P.K.,(1979) "Bicriteria transportation problem", Management Science, 5 73-78.
3. Barasov, A.S., (1961)Linear Programming,Fizmatgiz, Moskva,. (in Russian)
4. BhatiaH.L., SwaroopKanti and PuriM.C. ,(1974), A procedure for time minimization transportation problem, Presented in 7th Annual Conference of ORSI at Kanpur,.
5. ChandraS. and SaxenaP.K.,(1987) , Time Minimizing Transportation Problem with impurities, Asia- Pacific J. Op. Res. 4, 19-27.
6. Hammer, P.L.,(1969),“Time-Minimizing transportation problem”, Naval Research Logistics Quarterly,16(3) 345-347.
7. Nikolic, I.,(1986) "Minimization of total transportation time ", SYM-OP-IS '86, Hereg-Novi, Yugoslavia,October, , 519-520. (in Serbian)
8. Nikolic, I., and Petric, J.,(2007), "Minimization of transportation time - multicriteria approach", Thirdmini EURO Conference - Operations Research Methods in Transport Planning and TrafficControl,Herceg Novi, Yugoslavia, June 16-19, 1987, 1-12.
9. Nikolic, I.,(1993), "Bicriteria transportation total cost - total time problem ", SYM-OP-IS '93, Belgrade,Yugoslavia, October, 387-390. (in Serbian)
10. Sharma, J.K. & Swarup, K. (1977) , “Time minimization transportation problem”, Proceeding of Indian Academy of Sciences-Mathematical Sciences 86, 513-518.
11. Sharma G. , Abbas S. H. , Gupta V. (2015) , "Solving Time Minimizing Transportation Problem by Zero Point Method." PP -23-26
12. Tanino, T., Tanaka, T., and Inuiguchi, M., (eds.), (2003),Multi-Objective Programming and Goal Programming,SpringerVerlag, Berlin.
13. Vukadinovic, S., (1985),Transportation Problem in Linear Programming,Naucnaknjiga, Belgrade,Yugoslavia.