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Relation of β-Laplace Integral Transform with other Integral Transforms

Amit Gaur[1],Department of Mathematics and Statistics,Manipal University Jaipur, India
Garima Agarwal[2,*],Department of Computer Applications, Manipal University Jaipur, India

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Abstract:
Recently we define generalization of the Laplace Transform, named β-Laplace Integral Transform. This transform is not only generalization of Laplace Transform but also many other Integral Transforms like Sumudu, Kamal, Natural, Polynomial, Tarig, Elzaki, Aboodh, Laplace-Carson (Mahgoub), Mohand, Sawi, Sadik integral transform are particular case of β-Laplace Integral Transform and aim of this paper is to prove this claim.

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INTRODUCTION

Many sciences, engineering, and real-life process and phenomenon can be expressed by mathematical tools and also can be solved by using integral transforms. The problems arise in the field of signal processing, digital communication, probability theory, thermal science, medical sciences, fractional calculus, nuclear physics, aerodynamics, civil engineering, control theory, cardiology, quantum mechanics, marine science, space science, biological science, gravitation field theory, nuclear magnetism, the theory of resonance, heat conduction, economics, detection of diabetes, potential theory, chemical science, stress analysis, electrical engineering, deflection of beams, the science of defense, Brownian motion, the vibration of plates and many other fields have a wide range of application of integral transforms. [5,23,26]

Till now many Integral Transforms such as Laplace Transform, Fourier Transform, Hankel Transform, Mellin Transform, Radon Transform, Gabor Transform, Hilbert Transform, Weiestrauss Transform, Abel Transform, Sumudu Transform,Kamal Transform, Mohand Transform, NaturalTransform, Polynomial Transform, Sawi Transform, ZZ-Transform, Sadik Transform, Tarig Transform, Aboodh Transform, Mahgoub Transform etc. has been defined by many mathematicians. Every Integral Transform has significance due to its unique applicability in the field of science and engineering [2,6,12,15,21,24,27]. All the Transforms have shown many characteristic properties and lookvery promising to solve advanced problems in the field of science and engineering.

Recently Gaur et.al. has introduced a new integral Transform, a new form of generalized Laplace Integral Transform named β-Laplace Integral Transform on the function $f(t), t \geq 0$ defined by [11] $\mathcal{L}_\beta\{f(t)\}_{(s)} = \int_0^\infty \beta^{-st} f(t)dt, \quad \beta > 1$Where, s is a complex parameter independent to t.

This transform has wide application in solving various type of Differential equations [10].

All the above-mentioned transforms are the particular case of β-Laplace Integral Transform and aim of this paper to prove the same.

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DEFINITIONS

(i) LAPLACE INTEGRAL TRANSFORM

Laplace Integral Transform of a suitable function $f(t), t \geq 0$ is defined by [20]

$$\mathcal{L}\{f(t)\}_{(s)} = \int_0^\infty e^{-st} f(t)dt = L(s)$$
$$s \in \mathbb{C}, \qquad Re(s) > 0$$

Where suitable function means a function for which improper integral of right side converges.

(ii) SUMUDU INTEGRAL TRANSFORM

Sumudu Integral Transform of the function $f(t), t \geq 0$ is defined by [25]

$$\mathcal{S}\{f(t)\}_{(s)} = \int_0^\infty e^{-t} f(st)dt = S(s), \qquad s > 0$$

(iii) FOURIER INTEGRAL TRANSFORM

The Fourier transform of the function $f(t)$is denoted by $\mathcal{F}\{f(t)\}_{(k)} = F(k)$, $k \in \mathbb{R}$, and defined by the integral [5]

$$\mathcal{F}\{f(t)\}_{(k)} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^\infty e^{-ikt} f(t)dt = F(k)$$

(iv) HANKEL INTEGRAL TRANSFORM

Hankel Transform of order ν, is defined by the integral for the function $f(t)$, $t \geq 0$ [5]

$$\mathcal{H}_\nu\{f(t)\}_{(s)} = \int_0^\infty t J_\nu(st) f(t)dt, \qquad s > 0$$

(v) MELLIN INTEGRAL TRANSFOR

Mellin transform of the function $f(t), t \geq 0$ is defined by [5]

$$\mathcal{M}\{f(t)\}_{(s)} = \int_0^\infty t^{s-1} f(t)dt, \qquad s \in \mathbb{C}$$

(vi) NATURAL INTEGRAL TRANSFORM

The natural transform of thefunction *f*(*t*), defined for all real numbers *t* ≥ *0*, is the function $\mathcal{N}\{f(t)\}_{(u,s)}$, defined by [14]

$$\mathcal{N}\{f(t)\}_{(u,s)} = \int_0^\infty e^{-st} f(ut)dt = N(u,s), \quad u > 0,$$
$$Re(s) > 0$$

(vii) KAMAL INTEGRAL TRANSFORM

Kamal integral transform of the function $f(t), t \geq 0$ is defined by [13]

$$\mathcal{K}\{f(t)\}_{(s)} = \int_0^\infty e^{-\frac{t}{s}} f(t)dt = K(s), \qquad Re(s) > 0$$

(viii) ABOODH INTEGRAL TRANSFORM

Aboodh integral transform of the function $f(t), t \geq 0$ is defined by [1]

$$\mathcal{A}\{f(t)\}_{(s)} = \frac{1}{s} \int_0^\infty e^{-st} f(t)dt = A(s), \quad Re(s) > 0$$

(ix) MAHGOUB TRANSFORM (LAPLACE-CARSON) INTEGRAL TRANSFORM

Mahgoub integral transform of the function $f(t), t \geq 0$ is defined by [18]

$$\mathcal{M}_A\ \{f(t)\}_{(s)} = s \int_0^\infty e^{-st} f(t)dt = M_A\ (s)$$
$$Re(s) > 0$$

(x) MOHAND INTEGRAL TRANSFORM

Mohand Integral Transform of the function $f(t), t \geq 0$ is defined by [17]

$$\mathcal{M}_o\{f(t)\}_{(s)} = s^2 \int_0^\infty e^{-st} f(t)dt = M_o(s)$$
$$Re(s) > 0$$

(xi) POLYNOMIAL INTEGRAL TRANSFORM

Polynomial Integral Transform of the function $f(t)$, then the integral [4]

$$\mathcal{P}\{f(t)\}_{(s)} = \int_1^\infty z^{-s-1} f(lnz)dz = P(s)$$
$$Re(s) > 0$$

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(xii) TARIG INTEGRAL TRANSFORM

Tarig Integral Transform of the function $f(t), t \geq 0$ is defined by [7]

$$\mathcal{T}\{f(t)\}_{(s)} = \frac{1}{s}\int_0^\infty e^{-\frac{t}{s^2}} f(t)dt = T(s),$$
$$Re(s) \neq 0$$

(xiii) ZZ-INTEGRAL TRANSFORM

Tarig Integral Transform of the function $f(t), t \geq 0$ is defined by [28]

$$\mathcal{Z}\{f(t)\}_{(u,s)} = s\int_0^\infty e^{-st} f(ut)dt = Z(u,s)$$
$$u > 0, \qquad Re(s) > 0$$

(xiv) SAWI INTEGRAL TRANSFORM

Sawi Integral Transform of the function $f(t), t \geq 0$ is defined by [19]

$$\mathcal{S}_A\{f(t)\}_{(s)} = \frac{1}{s^2}\int_0^\infty e^{-\frac{t}{s}} f(t)dt = S_A(s),$$
$$Re(s) > 0$$

(xv) ELZAKI INTEGRAL TRANSFORM

Elzaki Integral Transform of the function $f(t), t \geq 0$ is defined by [9]

$$\mathcal{E}\{f(t)\}_{(s)} = s\int_0^\infty e^{-\frac{t}{s}} f(t)dt = E(s), \quad Re(s) > 0$$

(xvi) SADIK TRANSFORM

Sadik Integral Transform of the function $f(t), t \geq 0$ is defined by [22]

$$\mathcal{S}_D\{f(t)\}_{(s^a,b)} = \frac{1}{s^b}\int_0^\infty e^{-s^a t} f(t)dt = S_D(s^a, b)$$
$$a(>0), b \in \mathbb{R}, \qquad Re(s) > 0$$

MAIN RESULTS

(i) Relation between Laplace and $\boldsymbol{\beta}$-Laplace Integral Transform

Let a function $f(t), t \geq 0$ has Laplace transform $\mathcal{L}\{f(t)\}_{(s)}$ and β-Laplace integral transform $\mathcal{L}_\beta\{f(t)\}_{(s)}$ then by the definition of β-Laplace Integral Transform

$$\mathcal{L}_\beta\{f(t)\}_{(s)} = \int_0^\infty \beta^{-st} f(t)dt$$

Replacing s with $\frac{s}{ln\beta}$ $(\beta > 1)$, we have

$$\mathcal{L}_\beta\{f(t)\}_{\left(\frac{s}{ln\beta}\right)} = \int_0^\infty \beta^{-\left(\frac{st}{ln\beta}\right)} f(t)dt$$
$$= \int_0^\infty e^{-st} f(t)dt$$
$$\mathcal{L}_\beta\{f(t)\}_{\left(\frac{s}{ln\beta}\right)} = \mathcal{L}\{f(t)\}_{(s)}$$

$$\mathcal{L}\{f(t)\}_{(s)} = \mathcal{L}_\beta\{f(t)\}_{\left(\frac{s}{ln\beta}\right)} \quad (1)$$

(ii) Relation between Sumudu and $\boldsymbol{\beta}$-Laplace Integral Transform

Let a function $f(t), t \geq 0$ has Sumudu Transform $\mathcal{S}\{f(t)\}_{(s)}$, $s > 0$ and β-Laplace Integral Transform $\mathcal{L}_\beta\{f(t)\}_{(s)}$ then by the definition of β-Laplace Integral Transform

$$\mathcal{L}_\beta\{f(t)\}_{(s)} = \int_0^\infty \beta^{-st} f(t)dt = \int_0^\infty e^{-stln\beta} f(t)dt$$

Let substitute $st\, ln\beta = u \Rightarrow s\, ln\beta dt = du$

$$\mathcal{L}_\beta\{f(t)\}_{(s)} = \frac{1}{sln\beta}\int_0^\infty e^{-u} f\left(\frac{u}{sln\beta}\right)dt$$
$$= \frac{1}{sln\beta}\mathcal{S}\{f(t)\}_{\left(\frac{1}{sln\beta}\right)}, \qquad s > 0$$

or, we can write

$$\mathcal{S}\{f(t)\}_{(s)} = \frac{1}{s}\mathcal{L}_\beta\{f(t)\}_{\left(\frac{1}{sln\beta}\right)} \quad (2)$$

(iii) Relation between Fourier and $\boldsymbol{\beta}$-Laplace Integral Transform

Let f be a real valued function defined as

$f = \begin{cases} f(t) & if \;\; t \geq 0 \\ 0 & if \;\; t < 0 \end{cases}$ and β-Laplace, Fourier Integral Transforms are $\mathcal{L}_\beta\{f(t)\}_{(s)}$ and $\mathcal{F}\{f(t)\}_{(k)}$ respectively and when $s = k$, where $k \in \mathbb{R}$ then

$$\mathcal{L}\{f(t)\}_{(k)} = \sqrt{2\pi}\,\mathcal{F}\{f(t)\}_{(k)}$$

By equation (1)

$$\mathcal{L}\{f(t)\}_{(k)} = \mathcal{L}_\beta\{f(t)\}_{\left(\frac{k}{ln\beta}\right)} = \sqrt{2\pi}\,\mathcal{F}\{f(t)\}_{(k)}$$

Or we can rewrite

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$$\mathcal{F}\{f(t)\}_{(k)} = \frac{1}{\sqrt{2\pi}}\mathcal{L}_\beta\{f(t)\}_{\left(\frac{k}{ln\beta}\right)}$$

(iv) Relation between Hankel and $\boldsymbol{\beta}$-Laplace Integral Transform

Preposition ([3]): If f and $\mathcal{H}_\nu\{f\}_{(\xi)}$belongs to $L(0,\infty)$ and if $Re(\mu) > -1, Re(\mu+\nu) > -1, Re(s) > 0$, then

$\mathcal{L}\{t^\mu f(t)\}_{(s)} = \int_0^\infty k(s,\xi)\mathcal{H}_\nu\{f\}_{(\xi)}d\xi(3)$

Where

$$k(s,\xi) = \Gamma(\mu++1)\,\xi(s^2 + \xi^2)^{-\frac{1}{2}(\mu+1)}P_\mu^{-\nu}\left(\frac{s}{\sqrt{(s^2+\xi^2)}}\right)$$

Replace s by$s\,ln\beta$, we get

$$\mathcal{L}\{t^\mu f(t)\}_{(sln\beta\,)} = \int_0^\infty k(sln\beta,\xi)\mathcal{H}_\nu\{f\}_{(\xi)}d\xi$$

Where $k(sln\beta,\xi) = \Gamma(\mu+\nu+1)\xi((sln\beta)^2 +$ $\xi 2-12\mu+1P\mu-\nu sln\beta(sln\beta)2+\xi 2$

By applying equation (1), we obtain

$$\mathcal{L}_\beta\{t^\mu f(t)\}_{(s)} = \mathcal{L}\{t^\mu f(t)\}_{(sln\beta\,)} = \int_0^\infty k(sln\beta,\xi)\mathcal{H}_\nu\{f\}_{(\xi)}d\xi \quad (4)$$

(v) Relation between Mellin and $\boldsymbol{\beta}$-Laplace Integral Transform

Preposition ([16]): If $\mathcal{L}\{f(t)\}_{(s)}$ converges absolutely on $Re(s) > -c$ for some $c > 0$, then $\mathcal{M}\{f(t)\}_{(z)}$ converges absolutely on $Re(z) > a$ for some $a \le 1$, and

$\mathcal{M}\{f(t)\}_{(z)} = \frac{1}{2\pi i}\Gamma(z)\int_{-c-i\infty}^{-c+i\infty}(-p)^{-z}\mathcal{L}\{f(t)\}_{(p)}dp$

on $Re(z) > 1$.

Substitute$p = sln\beta$, we have

$$\mathcal{M}\{f(t)\}_{(z)} = \frac{ln\beta}{2\pi i}\Gamma(z)\int_{-c-i\infty}^{-c+i\infty}(-sln\beta)^{-z}\mathcal{L}\{f(t)\}_{(sln\beta\,)}ds$$

$$\mathcal{M}\{f(t)\}_{(z)} = \frac{ln\beta}{2\pi i}\Gamma(z)\int_{-c-i\infty}^{-c+i\infty}(-sln\beta)^{-z}\mathcal{L}_\beta\{f(t)\}_{(s)}ds \quad (5)$$

(vi) Relation between Natural and $\boldsymbol{\beta}$-Laplace Integral Transform

Let f be a function defined on $[0,\infty)$ such that the β-Laplace and Natural Integral Transform are $\mathcal{L}_\beta\{f(t)\}_{(s)}$ and $\mathcal{N}\{f(t)\}_{(u,s)}$ respectively, then

$$\mathcal{L}_\beta\{f(t)\}_{(s)} = \int_0^\infty \beta^{-st}f(t)dt = \int_0^\infty e^{-stln\beta}\,f(t)dt$$

Substitute $t = uz \Rightarrow dt = udz$, we obtain

$$\mathcal{L}_\beta\{f(t)\}_{(s)} = u\int_0^\infty e^{-suln\beta z}\,f(uz)dz = u\mathcal{N}\{f(t)\}_{(u,usln\beta\,)}$$

$$\mathcal{L}_\beta\{f(t)\}_{(s)} = u\,\mathcal{N}\{f(t)\}_{(u,usln\beta\,)}$$

Or we can write

$$\mathcal{N}\{f(t)\}_{(u,s)} = \frac{1}{u}\mathcal{L}_\beta\{f(t)\}_{\left(\frac{s}{uln\beta}\right)} \quad (6)$$

(vii) Relation between Kamal and $\boldsymbol{\beta}$-Laplace Integral Transform

Let function $f(t), t \ge 0$ has Kamal and β-Laplace Integral Transform $\mathcal{K}\{f(t)\}_{(s)}$ and $\mathcal{L}_\beta\{f(t)\}_{(s)}$ respectively, then

$$\mathcal{L}_\beta\{f(t)\}_{(s)} = \int_0^\infty \beta^{-st}f(t)dt$$

Replacing s with $\frac{1}{sln\beta}$ $(\beta > 1)$

$$\mathcal{L}_\beta\{f(t)\}_{\left(\frac{1}{sln\beta}\right)} = \int_0^\infty e^{-\left(\frac{t}{s}\right)}f(t)dt = \mathcal{K}\{f(t)\}_{(s)}$$

$$\mathcal{K}\{f(t)\}_{(s)} = \mathcal{L}_\beta\{f(t)\}_{\left(\frac{1}{sln\beta}\right)} \quad (7)$$

(viii) Relation between Aboodh and $\boldsymbol{\beta}$-Laplace Integral Transform

Let function $f(t), t \ge 0$ has Aboodh and β-Laplace Integral Transform $\mathcal{K}\{f(t)\}_{(s)}$, and $\mathcal{L}_\beta\{f(t)\}_{(s)}$ respectively, then

$$\mathcal{L}_\beta\{f(t)\}_{(s)} = \int_0^\infty \beta^{-st}f(t)dt$$

Replace s with $\frac{s}{ln\beta}$, we get

$$\mathcal{L}_\beta\{f(t)\}_{\left(\frac{s}{ln\beta}\right)} = \int_0^\infty \beta^{-\left(\frac{s}{ln\beta}\right)t}f(t)dt = \int_0^\infty e^{-st}f(t)dt$$

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$$\frac{1}{s}\mathcal{L}_\beta\{f(t)\}_{\left(\frac{s}{ln\beta}\right)} = \frac{1}{s}\int_0^\infty e^{-st} f(t)dt = \mathcal{A}\{f(t)\}_{(s)}$$

$$\mathcal{A}\{f(t)\}_{(s)} = \frac{1}{s}\mathcal{L}_\beta\{f(t)\}_{\left(\frac{s}{ln\beta}\right)} \qquad (8)$$

(ix) Relation between Mahgoub and $\boldsymbol{\beta}$-Laplace Integral Transform

Let function $f(t), t \geq 0$ has Mahgoub and β-Laplace Integral Transform $\mathcal{M}_A\{f(t)\}_{(s)}$, $\mathcal{L}_\beta\{f(t)\}_{(s)}$ respectively, then

$$\mathcal{L}_\beta\{f(t)\}_{(s)} = \int_0^\infty \beta^{-st} f(t)dt$$

$$s\mathcal{L}_\beta\{f(t)\}_{\left(\frac{s}{ln\beta}\right)} = s\int_0^\infty e^{-st} f(t)dt$$

$$= \mathcal{M}_A\{f(t)\}_{(s)}$$

$$\mathcal{M}_A\{f(t)\}_{(s)} = s\,\mathcal{L}_\beta\{f(t)\}_{\left(\frac{s}{ln\beta}\right)} \quad (9)$$

(x) Relation between Mohand and $\boldsymbol{\beta}$-Laplace Integral Transform

Let function $f(t), t \geq 0$ has Mohand and β-Laplace Integral Transform $\mathcal{M}_o\{f(t)\}_{(s)}$, $\mathcal{L}_\beta\{f(t)\}_{(s)}$ respectively, then

$$\mathcal{L}_\beta\{f(t)\}_{(s)} = \int_0^\infty \beta^{-st} f(t)dt$$

$$s^2\mathcal{L}_\beta\{f(t)\}_{\left(\frac{s}{ln\beta}\right)} = s^2\int_0^\infty e^{-st} f(t)dt$$

$$= \mathcal{M}_o\{f(t)\}_{(s)}$$

$$\mathcal{M}_A\{f(t)\}_{(s)} = s^2\int_0^\infty e^{-st} f(t)dt$$

$$\mathcal{M}_o\{f(t)\}_{(s)} = (s)^2\mathcal{L}_\beta\{f(t)\}_{\left(\frac{s}{ln\beta}\right)} \quad (10)$$

(xi) Relation between Polynomial and $\boldsymbol{\beta}$-Laplace Integral Transform

Let function $f(t), t \geq 0$ has polynomial andβ-Laplace Integral Transform $\mathcal{P}\{f(t)\}_{(s)}$, $\mathcal{L}_\beta\{f(t)\}_{(s)}$ respectively, then

$$\mathcal{L}_\beta\{f(t)\}_{(s)} = \int_0^\infty \beta^{-st} f(t)dt$$

Substitute $e^t = z \Rightarrow t = lnz$, we have

$$\mathcal{L}_\beta\{f(t)\}_{(s)} = \int_1^\infty z^{-sln\beta\,-1} f(lnz)dz$$

$$= \mathcal{P}\{f(t)\}_{(sln\beta\,)}$$

$$\mathcal{P}\{f(t)\}_{(s)} = \mathcal{L}_\beta\{f(t)\}_{\left(\frac{s}{ln\beta}\right)} \quad (11)$$

(xii) Relation between Tarig and $\boldsymbol{\beta}$-Laplace Integral Transform

Let function $f(t), t \geq 0$ has Tarig and β-Laplace Integral Transform $\mathcal{T}\{f(t)\}_{(s)}$, $\mathcal{L}_\beta\{f(t)\}_{(s)}$ respectively, then

$$\mathcal{L}_\beta\{f(t)\}_{(s)} = \int_0^\infty \beta^{-st} f(t)dt$$

Replace s with$\frac{1}{s^2 ln\beta}$, and divide by s both sides, we get

$$\frac{1}{s}\mathcal{L}_\beta\{f(t)\}_{\left(\frac{1}{s^2 ln\beta}\right)} = \frac{1}{s}\int_0^\infty e^{-\frac{t}{s^2}} f(t)dt = \mathcal{T}\{f(t)\}_{(s)}$$

$$\mathcal{T}\{f(t)\}_{(s)} = \frac{1}{s}\mathcal{L}_\beta\{f(t)\}_{\left(\frac{1}{s^2 ln\beta}\right)} \qquad (12)$$

(xiii) Relation between ZZ-Transform and $\boldsymbol{\beta}$-Laplace Integral Transform

Let f be a function defined on $[0,\infty)$ such that the β-Laplace and ZZ-integral transform of the function f is $\mathcal{L}_\beta\{f(t)\}_{(s)}$ and $\mathcal{Z}\{f(t)\}_{(u,s)}$ respectively, then

$$\mathcal{L}_\beta\{f(t)\}_{(s)} = \int_0^\infty \beta^{-st} f(t)dt = \int_0^\infty e^{-stln\beta}\, f(t)dt$$

By equation (6)

$$\mathcal{N}\{f(t)\}_{(u,s)} = \frac{1}{u}\mathcal{L}_\beta\{f(t)\}_{\left(\frac{s}{uln\beta}\right)}$$

Multiplying both side with s

$$s\mathcal{N}\{f(t)\}_{(u,s)} = \frac{s}{u}\mathcal{L}_\beta\{f(t)\}_{\left(\frac{s}{uln\beta}\right)}$$

$$\mathcal{Z}\{f(t)\}_{(s)} = \frac{s}{u}\mathcal{L}_\beta\{f(t)\}_{\left(\frac{s}{uln\beta}\right)} \qquad (13)$$

(xiv) Relation between Sawi and $\boldsymbol{\beta}$-Laplace Integral Transform

Let function $f(t), t \geq 0$ has Sawi and β-Laplace Integral Transform $\mathcal{S}_A\{f(t)\}_{(s)}$, $\mathcal{L}_\beta\{f(t)\}_{(s)}$ respectively.

$$\mathcal{L}_\beta\{f(t)\}_{(s)} = \int_0^\infty \beta^{-st} f(t)dt$$

Replacing s with $\frac{1}{sln\beta}$

$$\mathcal{L}_\beta\{f(t)\}_{\left(\frac{1}{sln\beta}\right)} = \int_0^\infty e^{-\left(\frac{t}{s}\right)} f(t)dt$$

Divide by s^2, both sides, we have

$$\frac{1}{s^2}\mathcal{L}_\beta\{f(t)\}_{\left(\frac{1}{sln\beta}\right)} = \frac{1}{s^2}\int_0^\infty e^{-\left(\frac{t}{s}\right)} f(t)dt$$
$$= \mathcal{S}_A\{f(t)\}_{(s)}$$

$$\mathcal{S}_A\{f(t)\}_{(s)} = \frac{1}{s^2}\mathcal{L}_\beta\{f(t)\}_{\left(\frac{1}{sln\beta}\right)} \quad (14)$$

(xv)Relation between Elzaki and $\boldsymbol{\beta}$-Laplace Integral Transform

Let function $f(t), t \geq 0$ has Elzaki and β-Laplace Integral Transform $\mathcal{E}\{f(t)\}_{(s)}$, $\mathcal{L}_\beta\{f(t)\}_{(s)}$ respectively, then

$$\mathcal{L}_\beta\{f(t)\}_{(s)} = \int_0^\infty \beta^{-st} f(t)dt$$

Replacing s with $\frac{1}{sln\beta}$

$$\mathcal{L}_\beta\{f(t)\}_{\left(\frac{1}{sln\beta}\right)} = \int_0^\infty e^{-\left(\frac{t}{s}\right)} f(t)dt$$

Multiply with s both sides, we have

$$s\mathcal{L}_\beta\{f(t)\}_{\left(\frac{1}{sln\beta}\right)} = s\int_0^\infty e^{-\left(\frac{t}{s}\right)} f(t)dt$$

$$s\mathcal{L}_\beta\{f(t)\}_{\left(\frac{1}{sln\beta}\right)} = s\int_0^\infty e^{-\left(\frac{t}{s}\right)} f(t)dt = \mathcal{E}\{f(t)\}_{(s)}$$

$$\mathcal{E}\{f(t)\}_{(s)} = s\mathcal{L}_\beta\{f(t)\}_{\left(\frac{1}{sln\beta}\right)} \quad (15)$$

(xvi) Relation between Sadik and $\boldsymbol{\beta}$-Laplace Integral Transform

Let function $f(t), t \geq 0$ has Sadik and β-Laplace Integral Transform $\mathcal{S}_D\{f(t)\}_{(s^a,b)}$, $\mathcal{L}_\beta\{f(t)\}_{(s)}$ respectively.

By the definition of β-Laplace integral transform

$$\mathcal{L}_\beta\{f(t)\}_{(s)} = \int_0^\infty \beta^{-st} f(t)dt$$

Replace s with $\frac{s^a}{ln\beta}$ and divide by s^b both sides then we have

$$\frac{1}{s^b}\mathcal{L}_\beta\{f(t)\}_{\left(\frac{s^a}{ln\beta}\right)} = \frac{1}{s^b}\int_0^\infty e^{-s^a t} f(t)dt$$
$$= \mathcal{S}_D\{f(t)\}_{(s^a,b)}$$

$$\mathcal{S}_D\{f(t)\}_{(s^a,b)} = \frac{1}{s^b}\mathcal{L}_\beta\{f(t)\}_{\left(\frac{s^a}{ln\beta}\right)} (16)$$

CONCLUSION

This new form of generalized Integral Transform i.e. β-Laplace Integral Transform is aneffective and powerful transform among the above described transform and all these transforms are particular case of β-Laplace integral transform. So we can conclude that we can solve all the problems by β-Laplace integral transform which can be solved by the above mentioned Integral Transform. Now there is no need to recall all the transforms separately.

By only one transform now we can get all the transforms which its self-shows its importance.

CONFLICTS OF INTERESTS

The authors declare no conflict of interests.

REFERENCES

1. Aboodh, K., The New Integral Transform Aboodh Transform, Global Journal of Pure and Applied Mathematics, Vol. 9(1), pp. 35 – 43. (2013)
2. Bellman, R. E., and Roth, R. S., The Laplace Transform, World Scientific, Singapore. (1984)
3. Bhonsle, B.R., A Relation Between Laplace and Hankel Transforms, Glasgow Mathematical Journal, Cambridge University Press, Vol. 5(3), pp. 114-115. (1962).
4. Chaudhary, P., Chanchal P., Khandelwal, Y., Singh, Y., Duality of "Some Famous Integral Transform" from the Polynomial Integral Transform, International Journal of Mathematics Trends and Technology, Vol. 55(5), pp. 345-349. (2018)
5. Debnath, L., Bhatta, D., Integral Transforms and Their Applications, 3rd edition, Chapman and Hall/CRC. (2015)
6. Doetsch, G., Introduction to the Theory and Applications of the Laplace Transformation. Springer-Verlag, New York. (1970)

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ISSN: 0193-4120 Page No. 8653 - 8659

7. Elzaki T.M., Elzaki S.M., On the Relationship Between Laplace and New Integral Transform “Tarig Transform”, Applied Mathematics, Vol. 36, pp. 3230-3233. (2011)
8. Elzaki T.M., The New Integral Transform “Elzaki Transform”, Global Journal of Pure and Applied Mathematics, Vol. 7(1), pp. 57-64. (2011)
9. Elzaki, T.M., The New Integral Transform “Elzaki Transform”, Global Journal of Pure and Applied Mathematics, Vol. 7(1), pp. 57-64. (2011)
10. Gaur, A., Agarwal, G., Application Of β-Laplace Integral Transform, Talent Development and Excellence, Vol. 12(3s), pp. 1058-1068. (2020)
11. Gaur, A., Agarwal, G., On β-Laplace Integral Transforms and Their Properties, Inter-national Journal of Advanced Science and Technology, Vol. 29, pp. 1481-1491. (2020)
12. Jaeger, J. C., and Newstead, G., An Introduction to the Laplace Transformation with Engineering Applications, (Third Edition), Methuen Company Ltd., London. (1969)
13. Kamal, A., Sedeeg, H., The New Integral Transform Kamal Transform, Advances in theoretical and applied mathematics, ISSN 0973-4554, Vol. 11(4), pp. 451-458. (2016)
14. Khan, Z.H., Khan, W.A., "N-Transform-Properties and Applications" NUST Journal of Engineering Sciences, Vol. 1, 127–133. (2008)
15. Laplace, P. S., Th´eorie Analytique des Probabiliti´es, Vol. I, Part 2, Lerch, Paris. (1820)
16. Lew, J. S., "On Some Relations between the Laplace and Mellin Transforms," in IBM Journal of Research and Development, vol. 19, no. 6, pp. 582-586, doi: 10.1147/rd.196.0582. (1975)
17. Mohand M., Mahgoub, A., The New Integral Transform "Mohand Transform", Advances in Theoretical and Applied Mathematics, Vol. 12(2), pp. 113-120. (2017)
18. Mohand, M., Mahgoub, A., The New Integral Transform “Mahgoub Transform”, Advances in Theoretical and Applied Mathematics, Vol. 11(4), pp. 391-398. (2016)
19. Mohand, M., Mahgoub, A., The New Integral Transform "Sawi Transform", Advances in Theoretical and Applied Mathematics, Vol. 14(1), pp. 81-87. (2019)
20. Podlubny, I., Fractional Differential Equations, Academic Press, United States, pp.103. (1999)
21. Schiff J. L., The Laplace Transforms, Springer, New York. (1999)
22. Shaikh S.L., Introducing A New Integral Transform: Sadik Transform, American International Journal of Research in Science, technology, Engineering and Mathematics, vol. 22(1), pp. 100-102. (2018)
23. Sneddon, I. N., The Use of Integral Transforms, McGraw-Hill Book Company, New York, United States, pp.103. (1972)
24. Watson, E. J., Laplace Transforms and Applications, Van Nostrand Reinhold, New York. (1981)
25. Watugala, G. K., Sumudu transform: A New Integral Transform to Solve Differential Equations and Control Engineering Problems, International Journal of Mathematical Education in Science and Technology, Vol. 24(1), pp. 35–43. (1993)
26. Widder, D. V., An Introduction to Transform Theory, Academic Press, Boston. (1971)
27. Wyman, M., The Method of Laplace. Trans. Roy. Soc. Canada, **2,** 227–256. (1964)
28. Zafar Z.U.A., ZZ-Transform Method, International Journal of Advanced Engineering and Global Technology, Vol. 4(1), pp. 1605-1610. (2016)

AUTHORS:
Amit Gaur: Department of Mathematics and Statistics, School of Basic Sciences, Manipal University Jaipur, India
Email Address: gaur84amit@gmail.com

Garima Agarwal: Department of Computer Applications, School of Basic Sciences, ManipalUniversity Jaipur, India
Email Address:garima.agarwal@jaipur.manipal.edu,Garima.msc.phd@gmail.com
(*Corresponding Author)