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Mode I SIFs for internal and external surface semi-elliptical crack located on a thin cylinder

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Abstract
Stress intensity factors (SIFs) considered one of the most important parameters in fracture mechanics, SIFs can be used to describe the crack growth as well as fracture behaviour. In this paper, mode I SIFs are calculated by finite element software ANSYS, for a single semi-elliptical circumferential crack in a thin cylinder. The cracks were located either on the internal or external surface of the cylinder and subjected to two different types of loading, internal pressure and tension, applied separately. To produce results in a more comprehensive form, the dimensionless analysis was used, and a wide variety of parameters that define the crack geometry is considered. The ratio of crack depth to crack length ranging from 0.4 to 1.2, the ratio of crack depth to cylinder wall thickness vary between 0.2, 0.5 and 0.8, and the ratio of the cylinder wall thickness to the cylinder internal radius 0.1. Based on the obtained results, the distribution of SIFs found to be symmetric and the position of the maximum SIFs on the crack front strongly affected by the aspect ratio.Overall, external cracks exhibit slightly higher SIFs than those of internal cracks, and transition phenomenon occurs on crack aspect ratio between 0.6 and 0.8. In addition, a significant effect for the relative depth of the crack on SIFs, which is more pronounced in surface points than deep points on crack front.

1. Introduction

Generally, the cylindrical bodies as example pipes and pressure vessels, play an important role in the industry due to their extensive usage. The structural integrity of these structures considered a significant issue in term of cost or safety. The failure of this kind of structures has been triggered by the presence of the surface cracks, where these defects that detected in this kind of elements, which traced to the corrosion or fabrication faults that may yield cracks within the structure.

Basically, cracks can be divided according to its orientation on the pipes or hollow cylinders into three types, they are axial cracks (longitudinal), circumferential (surface cracks), and inclined cracks. All the aforementioned types could be named as external or internal surface cracks depend on its position on the outer and inner surface of the cylinder, respectively. These cracks initiate with an irregular shape, but after few cyclic loadings, takes approximately a semi-elliptical shape, which has been proved by Lin and Smith [1], and for this reason, most of the researchers used this shape in their studies. Semi-elliptical surface cracks considered the very popular shapes of defects in pipes and hollow

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cylinders, as mentioned by Raju and Newman [2], and Li and Yang [3].

In order to estimate the collapse of the cracked pipes or hollow cylinders, it is necessary to precisely calculate the stress intensity factors (SIFs) along the crack front. Therefore, SIFs has huge significance from an engineering view point. Underwood [4] performed the first attempt to solve the problem of a cracked cylinder, where the engineering estimation has been used, but, this study did not take in consideration the wall thickness effect. According to Diamantoud is and Labe as[5]numerical methods considered among the most reliable and accurate methods used to calculate the SIFs. Ismail et al. [6] reviewed the use of the finite element program ANSYS to calculate the fracture mechanics parameters in analyzing the engineering structures containing defects or cracks. Various studies have been conducted to determine SIFs for semi-elliptical cracks, Ramezani et al. [7]produced closed-form solutions to the three types of failure SIFs of a surface crack in a solid-pipe exposed to torsion, where a curve fitting method has been applied to the determined SIFs from the dual boundary element approach.

Ismail et al. [8,9]stated the SIFs for a surface crack in a solid-pipe exposed to bending, torsion, and mixed-mode, loadings. Shin and Cai [10]estimated the SIFs of a semi-elliptical surface crack in a solid-pipe for diverse aspect ratios experimentally and by the use of numerical modeling using the ABAQUS finite element analysis (FEA). Raju and Newman [2] applied 3-D finite element to analyze the SIFs for longitudinal cracks in pressurized cylinders, this analysis was limited to mode I. However, SIFs presented in their study applicable to cases with crack aspect ratio (a/c, crack depth to crack length ratio), $0.2 \leq a/c \leq 1.0$. Wang and Lambert [11] used a 3-D finite element analysis and weight function method to extend the crack aspect ratio to be between 0 to 1.0. Fett [12]modified a procedure to determine SIFs where the weight-function method employed for semi-elliptical cracks, this method was dedicated for 2D surface cracks, and the application limited to a diversity of loading conditions. Another study conducted by Shahani and Habibi [13]used 3-D FEA to calculate SIFs distribution along the crack front for an external circumferential crack subjected to mix-mode loading. Predan et al.[14] applied the FE methods to solve the case of the cracked hollow cylinder under torsional loading.

Carpinteriet al. [15] and, Carpinteri and Brighenti[16]analyzed the problem of a hollow pipe with surface edge crack beneath bending moment and axial loading separately, where the 3-D FE modeling was used. Raju and Newman [17]applied 3-D FEA for a varied range of circumferential semi-elliptical cracks located on pipes and rods. However, despite the available solutions in the literature, but according to Zareei and Nabavi[18],there is no complete solution. Most of the studies dealt with either external or internal crack, considering the two surface cracks, along with a broad variety of crack geometry, seems to be rare.

In this paper, the mode I SIFs are calculated, for a single semi-elliptical circumferential crack located either on the inner or external surface of a thin cylinder subjected to internal pressure and tension separately. Different crack geometries are used to cover wide ranges of crack shapes and depths. For the crack aspect ratio (ratio of crack depth to crack length), ranging between 0.4 to 1.2, while for each value of crack aspect ratio, three values of relative depth ratio (ratio of depth of crack to wall thickness of the cylinder) have been used, varying between 0.2, 0.5, and 0.8. Before proceeding the analysis procedure, the proposed model has been validated with the available models in the literature, and results introduced in the non-dimensional manner of SIFs (or normalized SIFs).

2. Finite element modelling

In this study, ANSYS, finite element software[19]utilized to calculate the mode I SIFs for a semi-elliptical crack located either at internal or exterior surface of a thin cylinder subjected to two separate types of loadings, internal pressure, and tension. A thin cylinder showed in Figure1,with total length, *L*, outside radius, R_o, internal radius,

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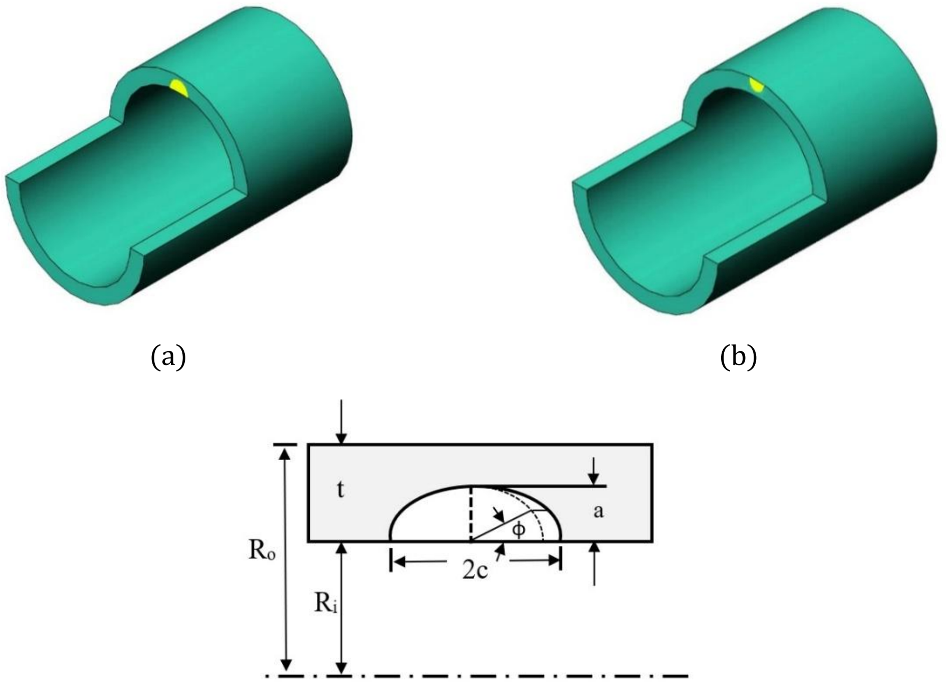
R_i, and wall thickness, t, havinga single semi-elliptical surface crack of depth, a, and length, $2c$, located on the inside and outside surfaces of the cylinder.



(a) (b)

(c)

Figure 1. The layout of the problem (a) cylinder with internal semi-elliptical crack, (b) cylinder with external semi-elliptical crack, (c) cross-sectional geometry of semi-elliptical crack.

Generally, the crack can be defined by two dimensionless parameters, a/c and a/t, the so-called crack aspect ratio and relative depth of the crack, respectively. Any arbitrary point on the crack front is characterized by a non-dimensional coordinate regarded as the normalized coordinate (or normalized crack front position), $2\phi/\pi$, where **ϕ,** is the parametric angle of elliptical crack. To include varied ranges of crack shapes, the crack aspect ratio of the semi-elliptical crack in this study was taken changing from 0.4 to 1.2, and the relative depth of the crack varies between 0.2, 0.5 and 0.8. The ratio of the exterior to the interior radius of the cylinder in all considered cases supposed to be R_o/R_i=1.1 and t/R_i=0.1, the Poisson ratio is assumed to be 0.3, and Young's modulus is 200GPa. Due to the symmetry of the SIFs distribution along the crack front, therefore, SIFs values for half of the crack front were plotted, and the normalized coordinates, ranging from 0 to 1, points (B and A), representing the deepest and outer point (surface point) of the crack front respectively as shown in Figure 2.

The cylinder is modelled, then meshed, and the boundary conditions were applied. As a result of the speedy changes of the geometrical parameters in the region around the crack front, special care taken for the meshing process. Therefore, two types of mesh sizes used for this model, where a very fine mesh size used for the region vicinity to the crack tips, while a coarse mesh size is used for the remaining area elsewhere as shown in Figure 3. As mentioned before, two types of crack positions were examined, interior and exterior semi-elliptical circumferential crack as presented in Figure 2, exposed to internal pressure and tension separately, furthermore, thin cylinder with internal crack examined under internal pressure first, and then under tension, the same procedure followed for the external crack also. All the applied loads kept in the elastic limits in order to avoid large plastic deformations. For tension loading, the load is applied remotely to one end, where this end linked with a remote point and the cylinder loaded through this point, and the other end of the cylinder fixed. While, for internal pressure, the two ends of the cylinder fixed, and the pressure applied directly to the inside surface of the cylinder

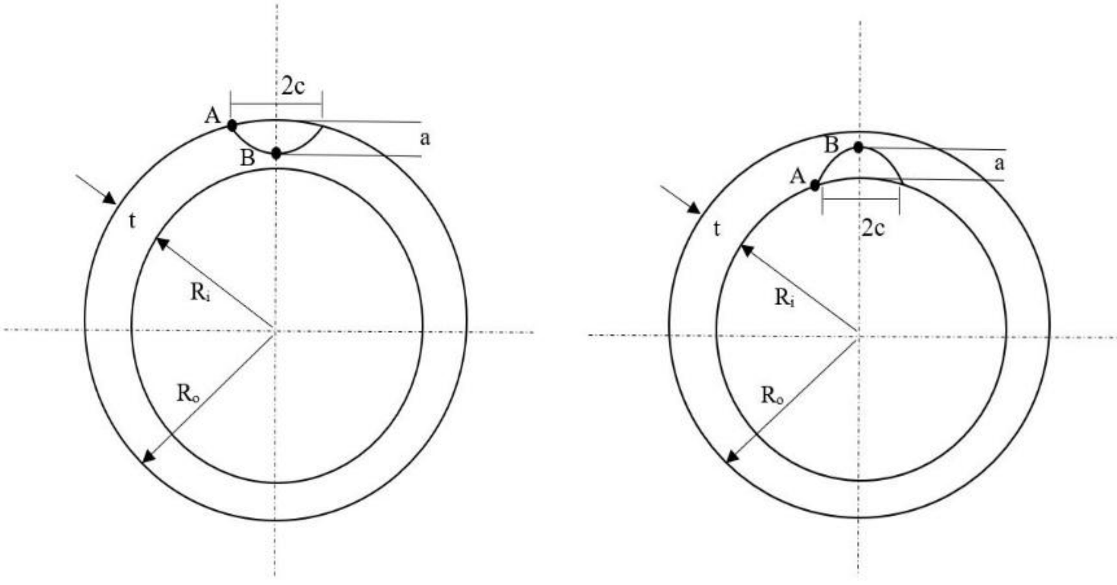


Figure 2. Cross-section of a cylinder with (a) external, and (b) internal circumferential semi-elliptical cracks

The use of ANSYS provides the ability to create number of contours around the crack tips along the crack front, and SIFs calculated for each contour along the crack front. Therefore, a proper selection of the contour considered an important task, this is because of the considerable material shrinkage in the region close to the crack tip as mentioned by Ismail[20].Thus, for this study six contours have been created round the crack tip along the crack front, and the results of the 5th contour are selected. Owing to the benefits that application of non-dimensional analysis provides, this technique utilized here, the calculated stress intensity factors, normalized based on the subsequent relationships, Raju and Newman[2,17]:

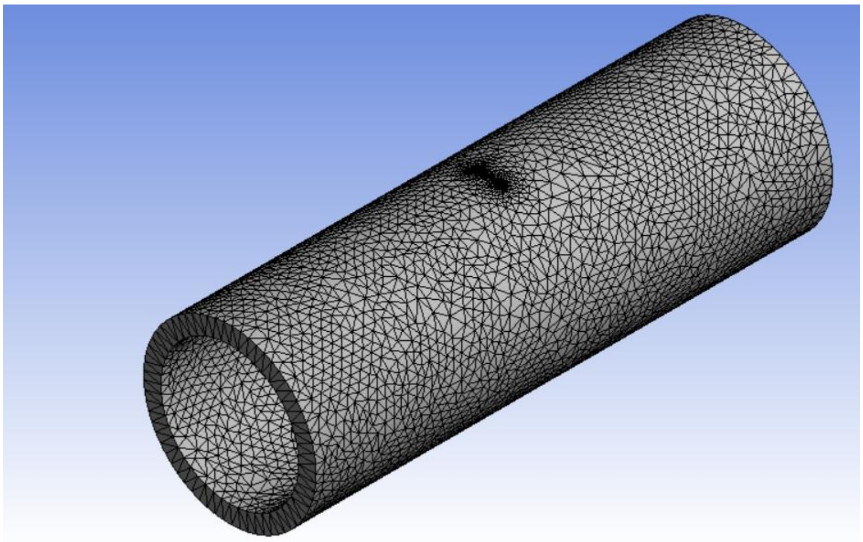
$$F_i = \frac{K_{cal}}{\frac{pR}{t}\sqrt{\pi a/Q}} \tag{1}$$

$$F_t = \frac{K_{cal}}{\sigma_t\sqrt{\pi a/Q}} \tag{2}$$

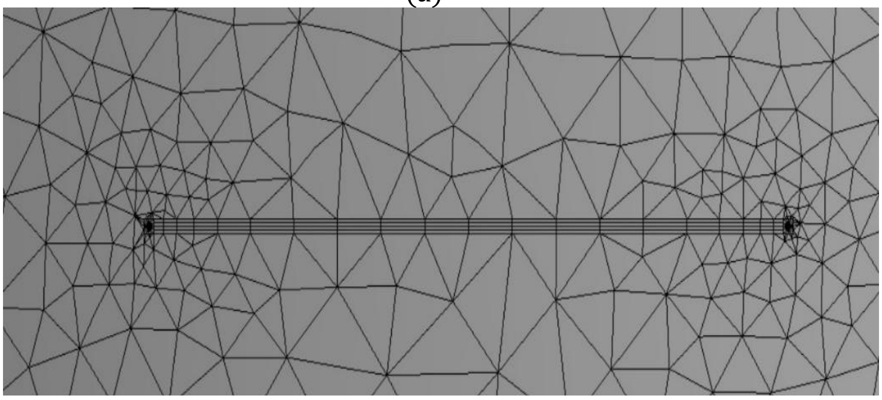
Where:
F_i and F_t = SIFs coefficients for internal pressure and tension loading respectively,
K_{cal} = the calculated SIF,
PR/t = the average hoop stress,
p = applied internal pressure,
σ_t = axial stress = $F/\pi\,(R^2_o - R^2_i)$, where F is the applied remote tension force,
Q = shape factor for elliptical crack, which is calculated by the following equations Raju and Newman[17]:

$$Q = 1 + 1.464(^{a}/_{c})^{1.65} \qquad \text{for } a/c \leq 1 \qquad (3$$

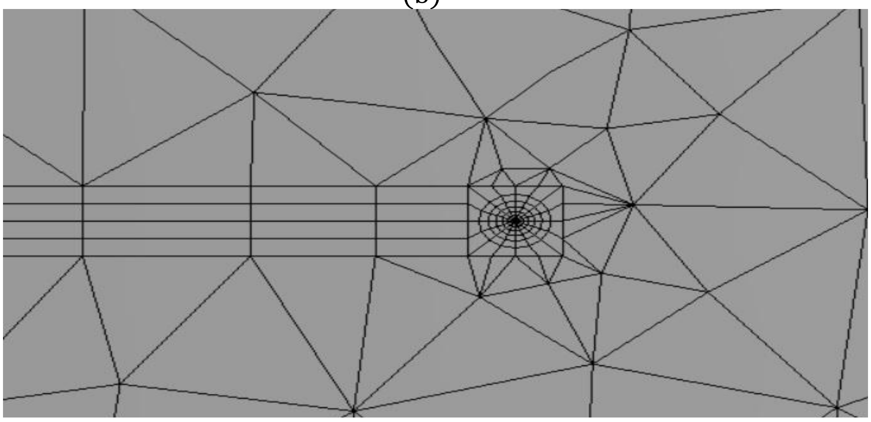
$$Q = 1 + 1.464(c/a)^{1.65} \quad \text{for a/c > 1} \quad (4)$$



(a)



(b)



(c)

Figure 3. Cylinder finite element model (a) the complete cylinder, (b) region around crack tips, and (c)close to the crack tip

3. Result and discussion

It is a crucial task to check the validity of the proposed model with the available models in the literature, Raju and Newman [2].Before proceeding the analysis procedure, F_{axial}, the normalized SIFs (or dimensionless SIFs coefficients) of an axial crack of the current model are compared to those of Raju and Newman [2]. Figure 4 illustrates comparison of the dimensionless SIFs, as a function of the normalized crack front position (or the normalized coordinate system), where same crack geometry parameters were used in both studies. The results showed that the proposed model is in good agreement and it is eligible to pursue further analysis for the other crack configurations.

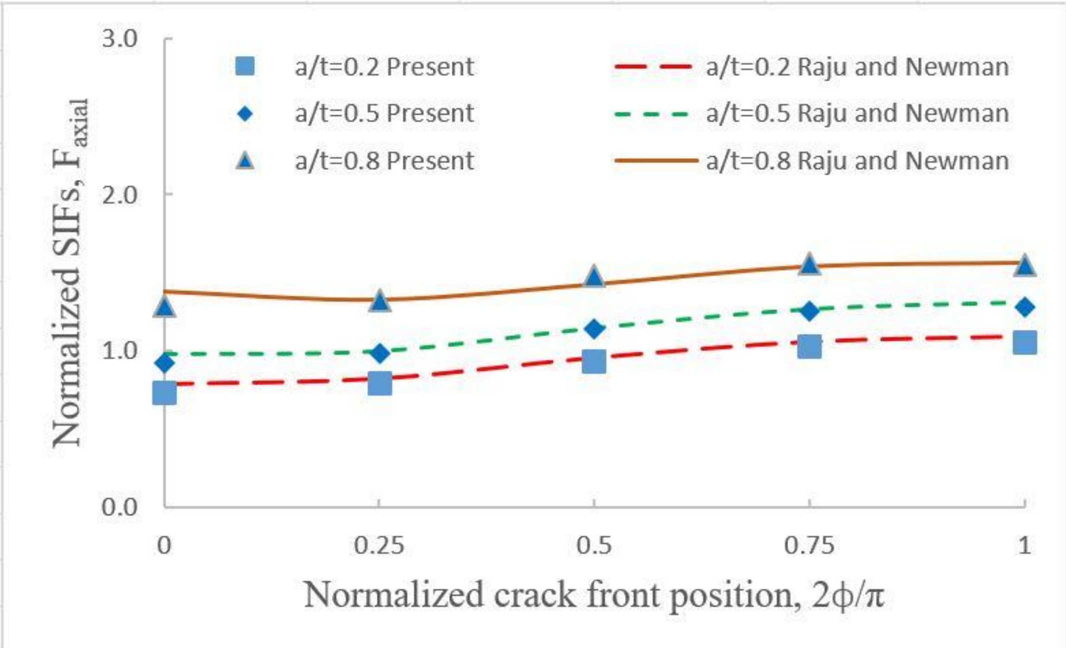


Figure 4. The validation of the proposed model of the present study, for an axial crack.

3.1 Cracked cylinder under internal pressure

A thin cracked cylinder with single circumferential semi-elliptical crack is examined under internal pressure loading condition, where two configurations of crack positions are tested, the first when the crack is located on the inner surface of the cylinder, and the second when the crack positioned on the external surface of the cylinder. The crack aspect ratio used ranging between 0.4 to 1.2, and the relative crack depth varying between 0.2, 0.5 and 0.8, furthermore, for each value of crack aspect ratio, there are three values of the relative depth of the crack are used, which helps to understand the effect of each ratio on the fracture behavior.

Figure 5 shows the distribution of the normalized SIFs under internal pressure, F_i, in term of normalized crack coordinates (2ϕ/π) of the crack front for a circumferential semi-elliptical crack located on the interior surface of the cylinder.All $F_i$values found to be symmetric along the crack front, and due to this symmetry, values of SIFs between the deepest point B, and surface point A on the crack front are plotted. It's found that F_i distribution following different curve shapes, which obviously depends on the value of crack aspect ratio; therefore, crack aspect ratiocould cause dissimilar modes of failure. For $a/c \leq 0.6$, F_i distribution following convex curve shape, where the maximum values of F_i located at the deepest point of the crack front point B, and this value gradually decrease towards the surface point A (surface point). While for $0.6 < a/c < 0.8$, F_i values following an approximately straight line, where the values of SIFs along the crack front from point A until point B almost are equal, which is an introduction to the well-known phenomenon, transition effect as mentioned by Carpinteri [22]. Transition effect means the position (on the crack front) of the maximum value of F_i moves from point B to A, and it occurs when a/c lying between 0.6 and 0.8. Due to this effect, F_i values for $a/c >$ 0.8 following a concave curve shape, hence, the maximum value reached a point near to the surface and slowly decrease towards point B, where minimum value lying. It is essential to say that, *a/t* has a significant effect on the F_i value, where an increase in this ratio leads to a rise in F_i value, which indicate that, deep cracks are generally dangerous and could accelerate the fracture process than cracks with less depth.

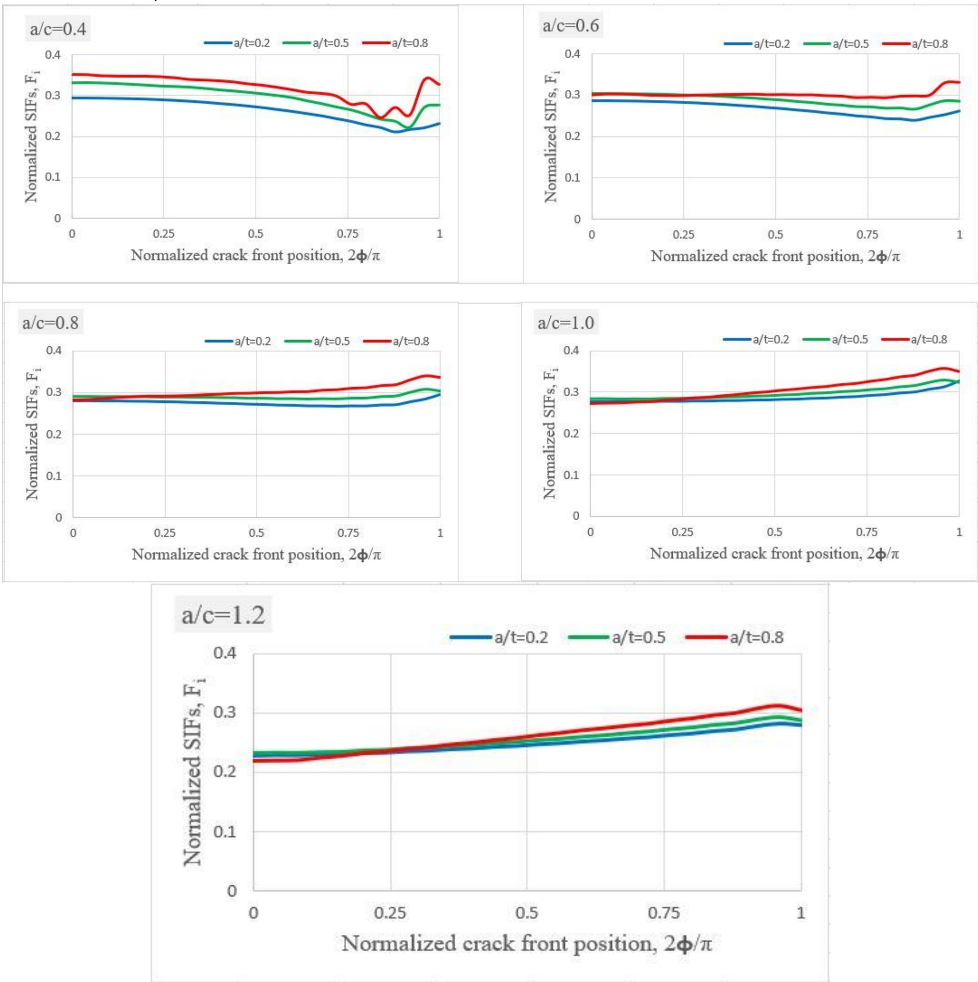


Figure 5. The distribution of the normalized SIFs along the crack front for an internally cracked cylinder subjected to internal pressure loading.

Figure 6 illustrates distribution of the normalized SIFs of an external semi-elliptical crack, subjected to internal pressure F_i, for different crack aspect ratios and relative crack depths. The behavior of the SIFs in term of the normalized crack front position for the external crack found to be similar to those of the internal crack, where crack aspect ratio affecting the shape of the curve also. Which means, depending on the applied aspect ratio, the position of the maximum value of F_i on the crack front can be specified. Besides, the transition effect occurs at the same *a/c* values for the internal crack. Note that, the values of $F_i$for the external crack (under internal pressure loading), generally, found to be slightly higher than those of internal crack. It is noteworthy that *a/c* has a significant effect on F_i, which is the same to what has been found for internal crack.

3.2Cracked cylinder under tension

A thin cylinder containing single semi-elliptical circumferential crack, examined under the application of tension, where tension force applied remotely to the cylinder. The crack aspect ratio was taken to be ranging between 0.4 to 1.2, while the relative depth of the crack was assumed to

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vary between 0.2, 0.5 and 0.8. Because of the symmetry of SIFs distribution along the crack front of the crack, only values between points A and B are plotted, which are the surface point and deepest point on the crack front, respectively.

Figure 7 displays the allocationof the normalized SIFs along the crack front for an internal circumferential crack under remote tension F_t, as a function of the normalized crack front position, where different values of *a/c*and *a/t*are examined. For $a/c \leq 0.6$, the position of the point that the maximum value of F_t is attained, located on the deepest point of the crack front, point B, and decrease in value along the crack front till it reached the minimum value on point A (surface point). While for $0.6 \leq a/c \leq 0.8$, a transition phenomenon can be distinguished through this range of the crack aspect ratio, the F_t values turning from convex curve shape to nearly a straight line then it takes a concave curve shape. Whereas for the high value of $a/c > 0.8$, the maximum F_t situated near to the surface point A on crack front. Furthermore, *a/t* has a significant effect on F_t value, where, as the crack is deeper, results in a high value of F_t, note that this impactnoticeable at surface points than at the deepest point of the crack front. Overall, sharp cracks (lowaspect ratio) always have the highest values of F_t.

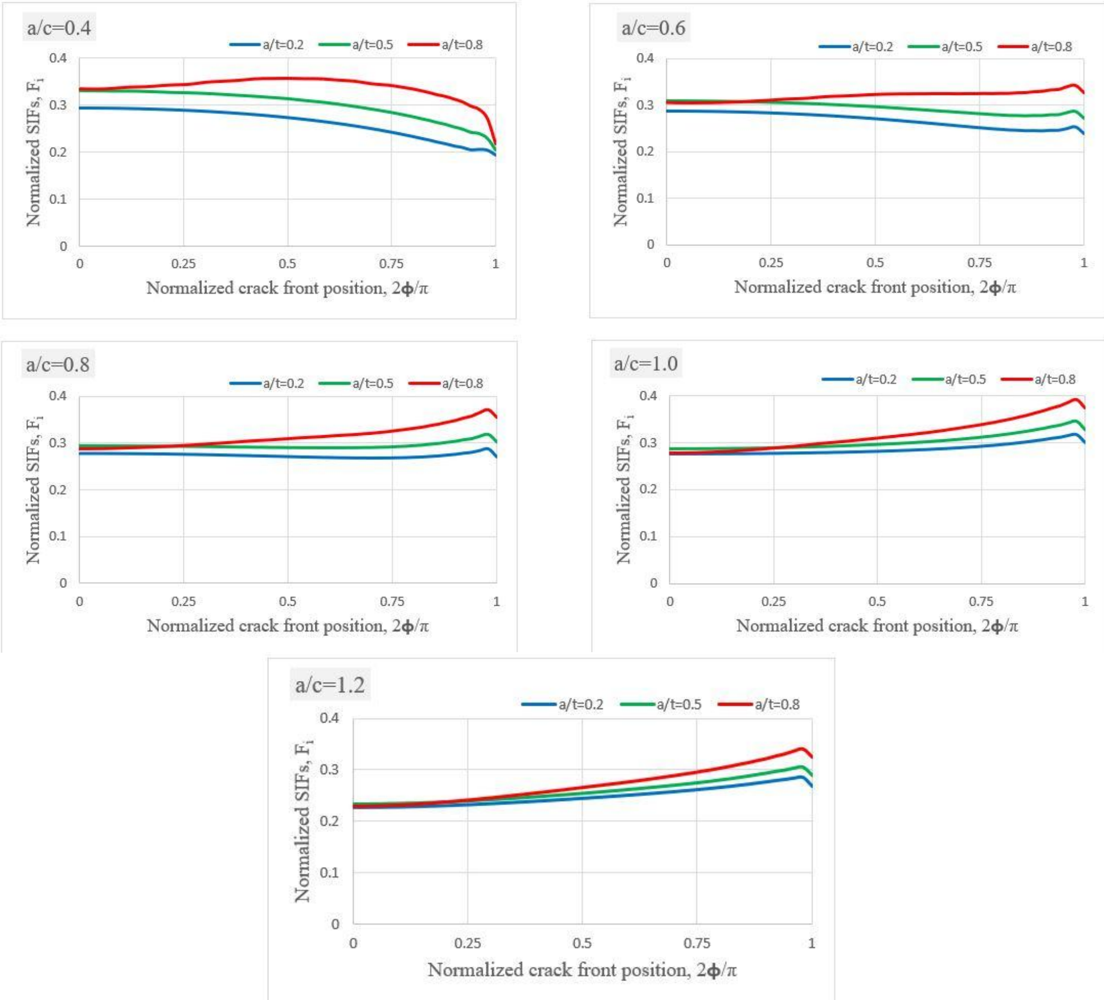


Figure 6. The distribution of the normalized SIFs along the crack front for externally cracked cylinder subjected to internal pressure loading.

Figure 8 shows the distribution of normalized SIFs along the crack front for an external circumferential crack under remote tension loading F_t, similar crack configurations and loading conditionsused for the internal crack are used for the internal crack also. The effect of the *a/c* and *a/t* for the external crack found to be similar to that of internal crack, where for small crack aspect ratios, the position of the maximum F_t lying on the deepest point on the crack front, while for high values of aspect ratios, the maximum value seems to be attained on the surface point. In general, the values of F_t for external crack were slightly higher than those of internal crack, and the effect of the *a/t* diminished on the deepest points on the crack front, but it is more distinct near the surface points. In addition, the transition effect can be observed, and it occurs on a value of *a/c* between 0.6 and 0.8.

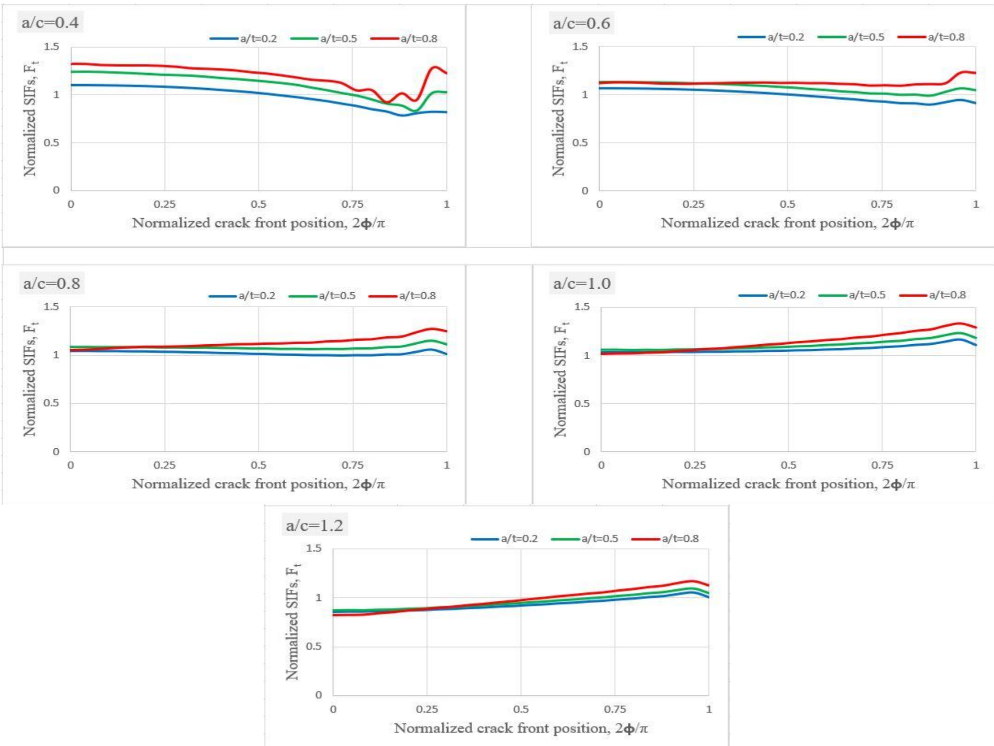
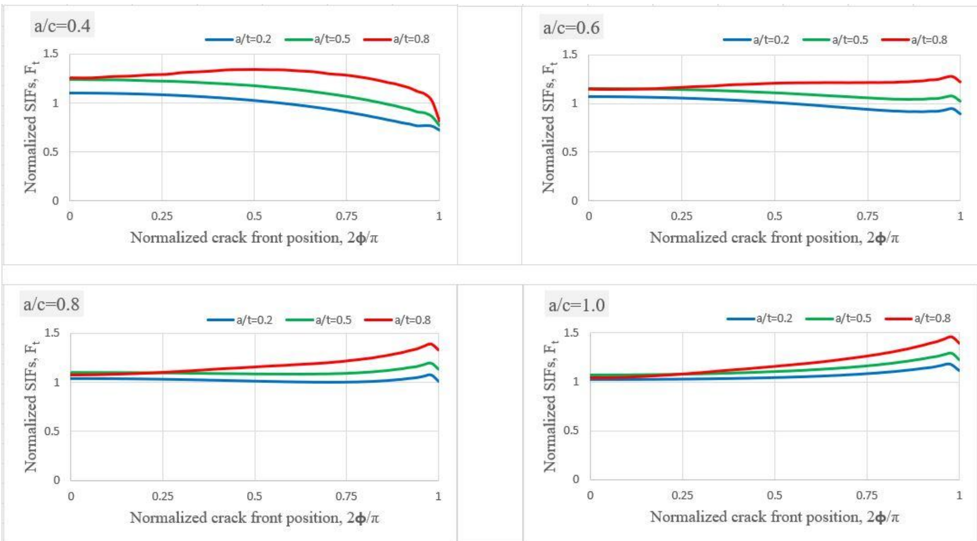


Figure 7. The distribution ofthe normalized SIFs along the crack front for an internally cracked cylinderexposedto remote tension loading.



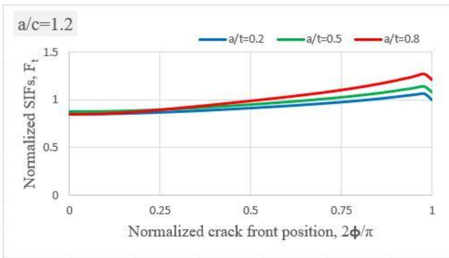


Figure 8. The distribution of normalized SIFs along the crack front for an externally cracked cylinder subjected to remote tension loading.

4. Conclusion

In this paper, the finite element analysis software ANSYS utilized to calculate mode I stress intensity factors (SIFs) for a single semi-elliptical surface crack positioned either on the internal or external surface of a thin cylinder subjected to two types of loading separately, internal pressure and tension. According to the three-dimensional finite element analysis, the distribution of SIFs produced inthe non-dimensionalstyleF_i (normalized SIFs under internal pressure loading),and F_t (normalized SIFs under tension loading). Basedon the numerical simulations, SIFs distribution found to be symmetric along the crack front; therefore, the distribution of half of the crack front is plotted.Thehighest(SIFs) can take place at the deepest point of the crack front for slender cracks, or for cracks with low aspect ratio.While it may attained near the surface point when the crack has ahigh-level of aspect ratio.Thus, the change in aspect ratio couldresult indissimilartypes of failure for the hollow cylinders. Besides, a transition phenomenon can be perceived for crack aspect ratio among 0.6 and 0.8. The relative depth of the crack has a significant effect on SIFs, where any increase in this ratio produces an increase in SIFs, this effect is more pronounced for high aspect ratio than for small aspect ratio. Generally, the external crack in both examined loading cases showed slightly higher values of SIFs compared to those of internal crack except for low aspect ratios where SIFs for the internal crack approaches values more elevated than that of external crack.

References

1. Lin XB, Smith RA. Fatigue growth prediction of internal surface cracks in pressure vessels. Journal of pressure vessel technology. 1998 Feb 1;120(1):17-23.
2. Raju IS, Newman JC. Stress-intensity factors for internal and external surface cracks in cylindrical vessels. Journal of pressure vessel technology. 1982 Nov 1;104(4):293-8.
3. Li CQ, Yang ST. Stress intensity factors for high aspect ratio semi-elliptical internal surface cracks in pipes. International Journal of Pressure Vessels and Piping. 2012 Aug 1;96:13-23.
4. Underwood JH. Stress intensity factors for internally pressurized thick-wall cylinders. InStress Analysis and Growth of Cracks: Proceedings of the 1971 National Symposium on Fracture Mechanics: Part 1 1972 Jan. ASTM International.
5. Diamantoudis AT, Labeas GN. Stress intensity factors of semi-elliptical surface cracks in pressure vessels by global-local finite element methodology. Engineering Fracture Mechanics. 2005 Jun 1;72(9):1299-312.
6. Ismail AE, Jamian S, Kamarudin K, Mohd Nor MK, Ibrahim MN, Choiron MA. An Overview of Fracture Mechanics with ANSYS. Int J Integr Eng. 2018;10(5):59–67.
7. Ramezani, M.K., Purbolaksono, J., Andriyana, A., Ramesh, S. and Putra, I.S., 2018. Empirical solutions for stress intensity factors of a surface

November-December 2019
ISSN: 0193-4120 Page No. 586 - 596

crack in a solid cylinder under pure torsion. Engineering Fracture Mechanics, 193, pp.122-136.
8. Ismail AE, Ariffin AK, Abdullah S, Ghazali MJ. Stress intensity factors for surface cracks in round bar under single and combined loadings. Meccanica. 2012 Jun 1;47(5):1141-56.
9. Ismail AE, Ariffin AK, Abdullah S, Ghazali MJ, Abdulrazzaq M, Daud R. Stress intensity factors under combined bending and torsion moments. Journal of Zhejiang University. Science A. 2012;13(1):1-8.
10. Shin CS, Cai CQ. Experimental and finite element analyses on stress intensity factors of an elliptical surface crack in a circular shaft under tension and bending. International Journal of Fracture. 2004 Oct 1;129(3):239-64.
11. Wang X, Lambert SB. Stress intensity factors and weight functions for longitudinal semi-elliptical surface cracks in thin pipes. International journal of pressure vessels and piping. 1996 Jan 1;65(1):75-87.
12. Fett T. Estimation of stress intensity factors for semi-elliptical surface cracks. Engineering Fracture Mechanics. 2000 Jul 1;66(4):349-56.
13. Shahani AR, Habibi SE. Stress intensity factors in a hollow cylinder containing a circumferential semi-elliptical crack subjected to combined loading. International journal of Fatigue. 2007 Jan 1;29(1):128-40.
14. Predan J, Močilnik V, Gubeljak N. Stress intensity factors for circumferential semi-elliptical surface cracks in a hollow cylinder subjected to pure torsion. Engineering Fracture Mechanics. 2013 Jun 1;105:152-68.
15. Carpinteri A, Brighenti R, Spagnoli A. Fatigue growth simulation of part-through flaws in thick-walled pipes under rotary bending. International Journal of Fatigue. 2000 Jan 1;22(1):1-9.
16. Carpinteri A, Brighenti R. Circumferential surface flaws in pipes under cyclic axial loading. Engineering Fracture Mechanics. 1998 Jul 1;60(4):383-96.
17. Raju IS, Newman JC. Stress-intensity factors for circumferential surface cracks in pipes and rods under tension and bending loads. InFracture mechanics: seventeenth volume 1986 Jan. ASTM International.
18. Zareei A, Nabavi SM. Calculation of stress intensity factors for circumferential semi-elliptical cracks with high aspect ratio in pipes. International Journal of Pressure Vessels and Piping. 2016 Oct 1;146:32-8.
19. ANSYS 19.0 Documentation (2019). ANSYS Company.
20. Ismail, A.E., 2016. Mode I stress intensity factors of sickle-shaped surface cracks in round solid bars under bending moment. International Journal of Automotive & Mechanical Engineering, 13(2).
21. Carpinteri A. Shape change of surface cracks in round bars under cyclic axial loading. International Journal of Fatigue. 1993 Jan 1;15(1):21-6.