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A New Type of Fuzzy Transportation Problem

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Abstract

In the present paper a new method proposed for the solution of transportation problems, where supply and demand are fuzzy normal random variables with mean and variance are triangular fuzzy numbers. Finally one numerical example is given to display the proposed method

Keywords: *Stochastic Programming, Fuzzy Programming, Normal Random Variables, Optimization Techniques.*

I. Introduction

Transportation Problem (TP) is a type of mathematical Programming Problem (PP), where our aim is to minimize the transportation cost, transportation time, etc., when homogeneous products which are initially stored at different sources is transported to different destinations. Kantorovich (1960)had first introduced the transportation model. Greig (1980) has derived different methodologies for solving TP using the table methods when prices and quantities are crisp numbers.

Bellman and Zadeh (1970)had first developed and solved fuzzy optimizaion problem applying fuzzy set theory. Bit et al. (1994) have expressed the multi-objective probabilistic TP. They have used chance constrained technique to handle randomness of the constraints.

However, recently Kikuchi (2000), El-Wahed (2001) and Mahapatra et al. (2010) have obtained the solution of TP using fuzzy programming technique. Acharya et al. (2014a,b) have determined the solution of fuzzy PP problem using Fuzzy Random Variables (FRVs). Dutta et al. (2016) discussed the solution of stochastic TP using genetic algorithm.

Maheswari and Ganesan (2018) discussed solution of fuzzy TP, where parameters are pentagonal fuzzy number. Ranarahu et al. (2017, 2018, 2019) have developed a new technique for solution of different types of fuzzy PP problem, which are multi-objective.

2 Fuzzy Stochastic Transportation Problem involving Fuzzy Normal Distribution

2.1 Mathematical Model

We present mathematical programming models of different Fuzzy Stochastic Transportation (FST) problems where fuzziness and randomness are considered in the constraints as following models.

Model-I:

$$\mathrm{Min} Z = \sum_{i=1}^{n} \sum_{j=1}^{r} c_{ij} x_{ij} \quad (1)$$

Subject to

$$\tilde{P}\left(\sum_{j=1}^{r} x_{ij} \le \tilde{a}_i\right) \ge \tilde{\beta}_i \quad (2)$$

$$\sum_{i=1}^{n} x_{ij} \ge b_j \quad (3)$$

$$x_{ij} \ge 0, \forall i, j$$

where $c_{ij}, b_j \in R$, $\forall i, j$, $(i = 1,2 \cdots, n$, $j = 1,2 \cdots, r)$, $\tilde{\beta}_i$ are Fuzzy Numbers (FNs), $\tilde{a}_i$ are independent Fuzzy Normal Random Variables (FNRVs), x_{ij} are the decision variables.

Model-II:

$$\mathrm{Min} Z = \sum_{i=1}^{n} \sum_{j=1}^{r} c_{ij} x_{ij} \quad (5)$$

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$$\sum_{j=1}^{r} x_{ij} \le a_i \quad (6)$$

$$\tilde{P}(\sum_{i=1}^{n} x_{ij} \ge \tilde{b}_j) \ge \tilde{\gamma}_j \quad (7)$$

$$x_{ij} \ge 0, \forall i, j$$

where $c_{ij}, a_i \in R$, $\forall i,j$, ($i = 1,2 \cdots n$, $j = 1,2 \cdots, r$), $\tilde{\gamma}_j$ are FNs, $\tilde{b}_j$ are FNRVs, x_{ij} are the decision variables.

Model-III:

$$\text{Min}Z = \sum_{i=1}^{n} \sum_{j=1}^{r} c_{ij}x_{ij} \quad (9)$$

Subject to

$$\tilde{P}(\sum_{j=1}^{r} x_{ij} \le \tilde{a}_i) \ge \tilde{\beta}_i \quad (10)$$

$$\tilde{P}(\sum_{i=1}^{n} x_{ij} \ge \tilde{b}_j) \ge \tilde{\gamma}_j \quad (11)$$

$$x_{ij} \ge 0, \forall i, j$$

where $c_{ij} \in R$, $\forall i,j$, $(i = 1,2 \cdots, n,\ j = 1,2 \cdots, r)$, $\tilde{\beta}_i$ and $\tilde{\gamma}_j$ are FNs, $\tilde{a}_i$ and $\tilde{b}_j$ are independent FNRVs x_{ij} are the decision variables.

2.2 Solution Methodology

The solution methodology for the proposed model is discussed in the form of theorems. As fuzziness and randomness are considered in constraints, these transformed equivalent constraints are presented in Theorem 2.1, 2.2 and 2.3 as follows. For proof of thorems referAcharya et.al(2014 b)

Theorem 2.1 *If* $\tilde{a}_i$, $i = 1,2, \ldots, n$ *are independent FNRVs then*

$$\tilde{P}(\sum_{j=1}^{r} x_{ij} \le \tilde{a}_i) \ge \tilde{\beta}_i \quad (13)$$

is equivalent to

$$\sum_{j=1}^{r} x_{ij} \le k_{\beta_i^*}\sigma_{a_{i_*}} + \mu_{a_{i_*}} \quad (14)$$

$\tilde{\mu}_{a_i}$ and $\tilde{\sigma}_{a_i}^2$ are mean and variance of $\tilde{a}_i$, which follow Fuzzy Normal Distribution (FND).

Now using theorem (2.1), the deterministic equivalent of the FST problem (1)-(4) is expressed as:

$\boldsymbol{FST1}$: $\text{Min}Z = \sum_{i=1}^{n} \sum_{j=1}^{r} c_{ij}x_{ij}$ (15)

Subject to.

$$\sum_{j=1}^{r} x_{ij} \le k_{\beta_i^*}\sigma_{a_{i_*}} + \mu_{a_{i_*}} \quad (16)$$

$$\sum_{i=1}^{n} x_{ij} \ge b_j \quad (17)$$

$$x_{ij} \ge 0, \forall i, j \quad (8)$$

$\sum_{i=1}^{n} (k_{\beta_i^*}\sigma_{a_{i_*}} + \mu_{a_{i_*}}) \ge \sum_{j=1}^{r} b_j (feasibility\ condition)$

where $\tilde{\beta}_i$, $i = 1,2 \cdots, n$ are FNs and $c_{ij}, b_j \in R$, $\forall i, j$. $\tilde{a}_i$ are FRVs distributed normally. $\tilde{\mu}_{a_i}$ and $\tilde{\sigma}_{a_i}^2$ are mean and variance of $\tilde{a}_i$, which follow FND.

Theorem 2.2 *If* $\tilde{b}_j$, $j = 1,2, \ldots, r$ *are independent FNRVs then* (12)

$$\tilde{P}(\sum_{i=1}^{n} x_{ij} \ge \tilde{b}_j) \ge \tilde{\gamma}_j \quad (19)$$

is equivalent to

$$\sum_{i=1}^{n} x_{ij} \ge (-k_{\gamma_j^*})\sigma_{b_j}^* + \mu_{b_j}^* \quad (20)$$

$\tilde{\mu}_{b_j}$ and $\tilde{\sigma}_{b_j}^2$ are mean and variance of $\tilde{b}_j$, which follow FND.

Now using theorem(2.2), the deterministic equivalent of the FST problem (5)-(8) is expressed as:

$\boldsymbol{FST2}$: $\text{Min}Z = \sum_{i=1}^{n} \sum_{j=1}^{r} c_{ij}x_{ij}$ (21)

Subject to

$$\sum_{j=1}^{r} x_{ij} \le a_i \quad (22)$$

$$\sum_{i=1}^{n} x_{ij} \ge (-k_{\gamma_j^*})\sigma_{b_j}^* + \mu_{b_j}^* \quad (23)$$

$$x_{ij} \ge 0, \forall i, j$$

$\sum_{i=1}^{n} a_i \ge \sum_{j=1}^{r} (-k_{\gamma_j^*})\sigma_{b_j}^* + \mu_{b_j}^* (feasibility\ condition)$ (25)

where $\tilde{\gamma}_j$, $j = 1,2 \cdots, r$ are FNs and $c_{ij}, a_i \in R$, $\forall i, j$. $\tilde{b}_j$ are FRVs distributed normally. $\tilde{\mu}_{b_j}$ and $\tilde{\sigma}_{b_j}^2$ are mean and variance of $\tilde{b}_j$, which follow FND.

Theorem 2.3 *If* $\tilde{a}_i$ *and* $\tilde{b}_j$ *are independent FNRVs then*

$$\tilde{P}(\sum_{j=1}^{n} x_{ij} \le \tilde{a}_i) \ge \tilde{\beta}_i \quad (26)$$

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$$\tilde{P}(\sum_{i=1}^{m} x_{ij} \geq \tilde{b}_j) \geq \tilde{\gamma}_j \quad (27)$$

is equivalent to

$$\sum_{j=1}^{r} x_{ij} \leq k_{\beta_i^*}\sigma_{a_{i_*}} + \mu_{a_{i_*}} \quad (28)$$

$$\sum_{i=1}^{m} x_{ij} \geq (-k_{\gamma_j^*})\sigma_{b_j}^* + \mu_{b_j}^* \quad (29)$$

where $\tilde{\mu}_{a_i}$ and $\tilde{\sigma}_{a_i}^2$ are mean and variance of $\tilde{a}_i$, $\tilde{\mu}_{b_j}$ and $\tilde{\sigma}_{b_j}^2$ are mean and variance of $\tilde{b}_j$, which follow FND.

Now using theorem(2.3), The deterministic equivalent of the (FST) problem (9)-(12) is expressed as:

$$\boldsymbol{FST3}: \text{Min}Z = \sum_{i=1}^{n} \sum_{j=1}^{r} c_{ij}x_{ij} \quad (30)$$

Subject to

$$\sum_{j=1}^{n} x_{ij} \leq k_{\beta_i^*}\sigma_{a_{i_*}} + \mu_{a_{i_*}} \quad (31)$$

$$\sum_{i=1}^{m} x_{ij} \geq (-k_{\gamma_j^*})\sigma_{b_j}^* + \mu_{b_j}^* \quad (32)$$

$$x_{ij} \geq 0, \forall i, j \quad (33)$$

$$\sum_{i=1}^{n} k_{\beta_i^*}\sigma_{a_{i_*}} + \mu_{a_{i_*}} \geq \sum_{j=1}^{r} (-k_{\gamma_j^*})\sigma_{b_j}^* + \mu_{b_j}^* (feasibility\ condition) \quad (34)$$

where $\tilde{\beta}_i$ and $\tilde{\gamma}_j$ are FNs. $\tilde{a}_i$ are FRVs distributed normally. $\tilde{\mu}_{a_i}$ and $\tilde{\sigma}_{a_i}^2$ are mean and variance of $\tilde{a}_i$, which follow FND. $\tilde{b}_j$ are FRVs distributed normally. $\tilde{\mu}_{b_j}$ and $\tilde{\sigma}_{b_j}^2$ are mean and variance of $\tilde{b}_j$, which follow FND. $c_{ij} \in R$, $\forall i, j$

2.3 Numerical Example

$$\textbf{Min}\boldsymbol{Z} = \boldsymbol{8x_{11} + 7x_{12} + x_{13} + x_{21} + 10x_{22} + x_{23} + 7x_{31} + 15x_{32} + 2x_{33}}$$

Subject to

$$\tilde{P}(\sum_{j=1}^{3} x_{1j} \leq \tilde{a}_1) \geq \tilde{\beta}_1 \quad (35)$$

$$\tilde{P}(\sum_{j=1}^{2} x_{2j} \leq \tilde{a}_2) \geq \tilde{\beta}_2 \quad (36)$$

$$\tilde{P}(\sum_{j=1}^{3} x_{3j} \leq \tilde{a}_3) \geq \tilde{\beta}_3 \quad (37)$$

$$\tilde{P}(\sum_{i=1}^{3} x_{i1} \geq \tilde{b}_1) \geq \tilde{\gamma}_1 \quad (38)$$

$$\tilde{P}(\sum_{i=1}^{3} x_{i2} \geq \tilde{b}_2) \geq \tilde{\gamma}_2 \quad (39)$$

$$\tilde{P}(\sum_{i=1}^{3} x_{i3} \geq \tilde{b}_3) \geq \tilde{\gamma}_3 \quad (40)$$

where $\tilde{\beta}_i$ and $\tilde{\gamma}_j$ are FNs. $\tilde{a}_i$ and $\tilde{b}_j$ are normally distributed FRVs with mean $\tilde{\mu}_{a_i}$ and $\tilde{\mu}_{b_j}$, variance $\tilde{\sigma}_{a_i}^2$ and $\tilde{\sigma}_{b_j}^2$ as triangular FNs.

$\tilde{\beta}_1 = \widetilde{0.6} = \langle 0.5/0.6/0.7\rangle$, $\tilde{\beta}_2 = \widetilde{0.2} = \langle 0.1/0.2/0.3\rangle$, $\tilde{\beta}_3 = \widetilde{0.5} = \langle 0.4/0.5/0.6\rangle$,
$\tilde{\gamma}_1 = \widetilde{0.7} = \langle 0.6/0.7/0.8\rangle$, $\tilde{\gamma}_2 = \widetilde{0.8} = \langle 0.7/0.8/0.9\rangle$, $\tilde{\gamma}_3 = \widetilde{0.4} = \langle 0.3/0.4/0.5\rangle$,
$\tilde{\mu}_{a_1} = \langle 2/3/4\rangle$, $\tilde{\mu}_{a_2} = \langle 6/7/8\rangle$, $\tilde{\mu}_{a_3} = \langle 4/5/6\rangle$,
$\tilde{\mu}_{b_1} = \langle 5/6/7\rangle$, $\tilde{\mu}_{b_2} = \langle 3/4/5\rangle$, $\tilde{\mu}_{b_3} = \langle 1/2/3\rangle$,
$\tilde{\sigma}_{a_1}^2 = \langle 2/4/6\rangle$, $\tilde{\sigma}_{a_2}^2 = \langle 2/5/8\rangle$, $\tilde{\sigma}_{a_3}^2 = \langle 3/5/7\rangle$,
$\tilde{\sigma}_{b_1}^2 = \langle 1/3/5\rangle$, $\tilde{\sigma}_{b_2}^2 = \langle 4/6/8\rangle$, $\tilde{\sigma}_{b_3}^2 = \langle 5/7/9\rangle$

Solution:

Using theorem(2.3) and α-cuts of the given FNs and for $\alpha = 0.7$, the deterministic equivalent of the given model becomes

$$\text{Min}Z = 8x_{11} + 7x_{12} + x_{13} + x_{21} + 10x_{22} + x_{23} + 7x_{31} + 15x_{32} + 2x_{33} \quad (41)$$

Subject to

$$\sum_{j=1}^{3} x_{1j} \leq 2.087822248 \quad (42)$$

$$\sum_{j=1}^{3} x_{2j} \leq 7.590932096 \quad (43)$$

$$\sum_{j=1}^{3} x_{3j} \leq 4.542678673 \quad (44)$$

$$\sum_{i=1}^{3} x_{i1} \geq 7.463085723 \quad (45)$$

$$\sum_{i=1}^{3} x_{i2} \geq 6.750870376 \quad (46)$$

$$\sum_{i=1}^{3} x_{i3} \geq 1.814801484 \quad (47)$$

$$x_{ij} \geq 0,$$

An optimal solution is obtained for α=0.7 as follows:

$$(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}) = (0, 2.087822, 0, 2.935361, 4.655571, 0, 4.527725, 0.7476918E-02, 1.814801)$$

and $Z = 99.54166$

3 Conclusion

The objective of our paper is to develop the solution of (FST) problem. The parameters supply and demand are independent normal FRV. A numerical example is discussed to explain the methodology.

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