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Optimal Promotion Policies In Hierarchical Grade Structures

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Abstract
Recruitment and Promotion of personnel are effective control mechanisms in human resource management. Optimal policies for promotion are therefore essential in human resource management. Optimal promotion policy models are proposed when a) the impact of recruitment cost is considered under the assumption that the cost of promotion and recruitment are functions of the grade size and b) the impact of attrition cost is considered together with a certain probability of a performance-based promotion, where there are two types of promotions namely, performance-based and career advancement promotions in the grade.

Keywords: *Optimal policy for promotion, Grade structured manpower, Human Resource planning*

1. Introduction

Medium and large organisations typically adopt a hierarchical grade structure for effective and efficient management and operational control. The grade sizes generally depend on the nature of the industry. Grade sizes are determined by recruitment, promotion and attrition. In human resource management, the recruitment into a grade and promotion from one grade to another are better controlled than attrition from a grade. Hence, optimal promotion policies are necessary for human resource management.

Young & Vassiliou [6] have proposed a non-linear manpower model on the promotion of staff considering an expanding non-homogeneous Markov chain model. The application of this model to a larger organisation that is subjected to reorganisation has been proposed by Vassiliou [3]. Grinold & Marshall [1] have proposed a faculty promotion policy using a linear programming approach wherein the inflows to the manpower system are the decision variables and the discounted cost is the objective function. Kalamatianou [2] has proposed a model in which promotion probabilities are functions of seniority in the organisation.

Setlhare [4] has proposed an optimal promotion policy for continuous time manpower model when the cost of promotion from a grade at time t is a function of the number of persons in that grade. In this paper, the impact of recruitment cost into a grade is considered while proposing an optimal promotion policy model wherein both the recruitment cost and promotion cost are functions of the number of persons in that grade. Subramanian [5] has discussed an optimal promotion policy when two types of promotion namely; regular or performance-based promotion and automatic or career-advancement promotion are considered. In the second model of this paper, in proposing an optimal promotion policy, the impact of attrition cost is considered together with a certain probability of passing a screening test for performance-based promotion, when there are two types of promotion namely performance-based promotion and career-advancement promotion in the grade.

2. Model Description and Analysis

2.1. Model I

A continuous time, manpower model is assumed, for which an optimal promotion policy is derived. It is assumed that the manpower system has k grades. The cost of promotion and recruitment are assumed to be functions of the number of persons in a grade, the demand for promotion from the grade, the market cost for the person to be recruited and the current cost of the person considered for promotion. In this paper,

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it is assumed that the cost of promoting a person from grade i to i+1 (i = 1, 2, 3…k-1) is a function of the number of persons present in grade i at time t. It is further assumed that the cost of recruitment of a person to grade i at time t is also a function of the number of persons present in grade i at time t. Notation:

Let $S_i(t)$ denote the number of personnel in grade *i* at time *t*, i= 1, 2, …k.

Let $F[S_i(t)]$ denote the rate of promotion from grade (i-1) to grade i in time (t, t+dt) given that there are $S_i(t)$ personnel in grade i, i= 2, 3 …k.

Let $P[S_i(t)]$ denote the rate of promotion from grade i to grade (i+1) in time (t, t+dt) given that there are $S_i(t)$ personnel in grade i, i= 1, 2, 3 …k-1.

Let $r[S_i(t)]$ denote the rate of recruitment to grade i in time (t, t+dt) given that there are $S_i(t)$ personnel in grade i, i= 1, 2, 3 …k.

Let $C[S_i(t)]$ denote the cost of promoting a person from grade i to grade (i+1) i= 1, 2, 3 …k-1 as well as the cost of promoting *a person from grade (i-1) to grade i, i= 2, 3 …k at in time (t, t+dt) when there are $S_i(t)$ personnel in grade i, i= 2, 3 …k.*

Let $d[S_i(t)]$ denote the cost of recruiting a person to grade i at time (t, t+dt) given that there are $S_i(t)$ personnel in grade i at time t, i = 1, 2, 3 …k.

Optimal policy - model 1

The change in the size of grade i at time t is obtained as:

$$\frac{dS_i(t)}{dt} = F[S_i(t)] - P[S_i(t)] + r[S_i(t)] \qquad (1)$$

The total cost in grade i in time $(t, t+dt)$ is given by:

$$C(t, t+dt) = C[S_i(t)]S_i(t)[F[S_i(t)] + P[S_i(t)]] + d[S_i(t)][S_i(t)][r[S_i(t)]] \qquad (2)$$

The optimal cost is given by the solution to the Euler-Lagrange equation:

$$\frac{\partial f}{\partial S_i} - \frac{d}{dt}\left(\frac{\partial f}{\partial S_i'}\right) = 0 \qquad (3)$$

where,

$$f = S_i(t)[C[S_i(t)][2F\left[S_i(t)\right] - S_i'(t) + r[S_i(t)]\right] + d[S_i(t)][r[S_i(t)]]] \qquad (4)$$

Hence,

$$\{C(S_i)[2F(S_i) - S_i'(t) + r(S_i)] + d(S_i)r(S_i)\}$$
$$+ S_i(t)\frac{d}{dS_i}\left[C[S_i(t)][2F[S_i(t)] + r[S_i(t)] - S_i'(t)] + d[S_i(t)]r[S_i(t)]\right]$$
$$+ \{S_i'(t)C(S_i) + S_i(t)C'(S_i)S_i'(t)\} = 0$$

This implies that

$$\left[C[S_i(t)][2F[S_i(t)] + r[S_i(t)]] + d[S_i(t)]r[S_i(t)]\right] = \frac{k}{S_i(t)} \qquad (5)$$

When $r[S_i(t)] = 0$ and $d[S_i(t)] = 0$, the result derived by Setlhare (2007) $F[S_i(t)] = \frac{k}{S_i(t)C[S_i(t)]}$ is obtained.

In model 1, two cases are illustrated with different assumptions for $C(S_i), d(S_i)$ and $r(S_i)$.

In case 1, $C(S_i) = d(S_i) = constant$ and $r(S_i)$ is a linear function of the grade size S_i. This is a general case where the cost of promotion is equal to the cost of recruitment and both are independent of the size of the grade; the recruitment rate depends on the size of the grade and is a linear function of the grade size.

In case 2, $C(S_i), d(S_i)$ and $r(S_i)$ are linear functions of the grade size S_i. Such a scenario could manifest in situations when the cost of recruitment increases for additional recruitment of rare skills.

2.1.1. Result 1:

Let $C(S_i)$ and $d(S_i)$ be constants while $r(S_i)$ be a linear function of S_i. Let $C(S_i) = d(S_i) =$ d , $r(S_i) = \gamma S_i$ where d and γ are constants.

Then $F(S_i) = \frac{k - 2d\gamma S_i^2}{2dS_i}$

Substituting in equations (1) and (2) the total cost C is obtained.

An illustration of case i is shown in Figure 1.

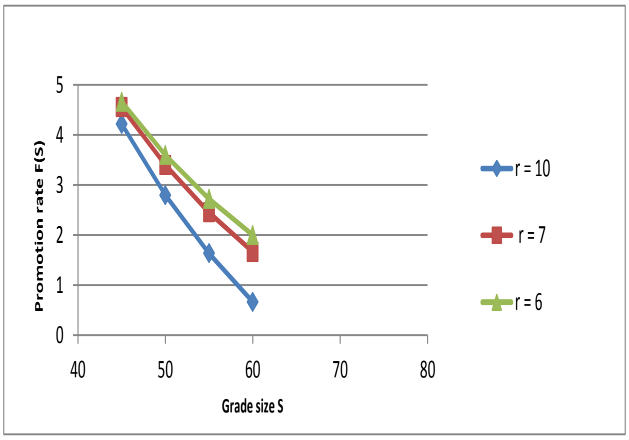

Figure 1: Optimal promotion policy for case i

When the grade size increases, the promotion rate F(S) decreases. The promotion rate increases when the rate of recruitment decreases

2.1.2. Result 2:

Let $C(S_i)$, $d(S_i)$ and $r(S_i)$ be linear functions of S_i.

$C(S_i) = cS_i$
$d(S_i) = dS_i$
$r(S_i) = \gamma S_i$ where c, d and γ are constants

Then $F(S_i) = \frac{k - \gamma S_i^3(c+d)}{2cS_i^2}$

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Substituting in equations (1) and (2) the total cost C is obtained.
An illustration of case ii is shown in Figure 1.

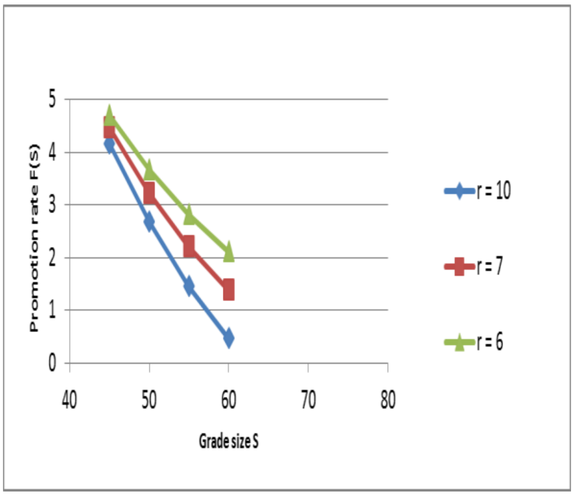

Figure 2: Optimal promotion policy for case ii

When the grade size increases, the promotion rate $F(S)$ decreases. The promotion rate increases when the rate of recruitment decreases.

A comparison of the optimal promotion rate $F(S)$ in case ii when $C(S) = D(S)$, with the optimal promotion rate $F(S)$ in case i is shown in Figure 3

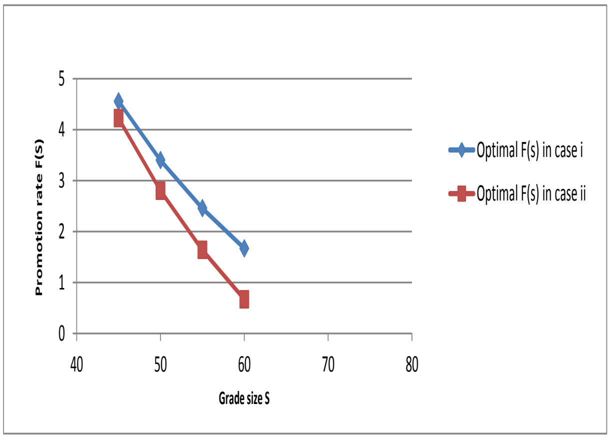

Figure 3: Comparison of case i and case ii

The optimal promotion rate computed in case i is higher than that computed in case ii. This reflects the impact of $C(S)$ and $D(S)$ as linear functions of the grade size.

2.2 Model II

A human resource system with k grades is considered. For any grade i, i = 1, 2… (k-1), let $(0, T_i)$ be the time interval for which promotions from the ith to the (i+1)th grade are considered. Two types of promotion are considered namely:

I Regular promotion (Performance-based): Screening tests are conducted as and when a vacancy arises and only one person (with the maximum score) is promoted to grade (i+1). Let p be the probability that a person is promoted to grade (i+1) after the screening test.

II Career advancement promotion (Time-bound): At the end of $(0, T_i)$, the eligible personnel in grade i are promoted to grade (i+1).

It is assumed that vacancies in grade (i+1) arise as a renewal process. It is also assumed that:

1. The vacancies that arise in grade (i+1) gives rise to a demand for a regular promotion (filled by conducting screening tests) from the ith grade.
2. The screening test is conducted only at the instant when the vacancy arises.
3. Let k be the number of vacancies for regular promotions during $(0, T_i)$.
4. Each regular promotion is for one person only. The cost of one regular promotion from grade i to grade (i+1) is C_{1i}.
5. The eligible personnel in grade i at the end of $(0, T_i)$ are promoted to grade (i+1). The cost of career advancement (time-bound) promotion from grade i to grade (i+1) is C_{2i}.
6. Any vacancy that is filled up in $(0, T_i)$ will not be considered for promotion at the end of $(0, T_i)$.
7. $F(T)$ and $f(T)$ are the c.d.f and p.d.f of the inter-arrival time of vacancies.
8. $F^{(r)}(T)$ and $f^{(r)}(T)$ are the r-fold convolutions of $F(T)$ and $f(T)$.
9. The rate of attrition in grade i is η_i. The cost of attrition from grade i is C_{3i}.
10. The size of the ith grade at the beginning of $(0, T_i)$ is N_i.

Optimal policy

Let C_{T_i} denote the expected total cost of promotion and attrition. Using renewal theory argument,

$$\begin{aligned} C_{T_i} = {} & C_{1i} \sum_{k=0}^{N_i} kp\left[F^{(k)}(T_i) - F^{(k+1)}(T_i)\right] \\ & + C_{2i} \sum_{k=0}^{N_i} [N_i - kp - \eta_i k(1-p)]\left[F^{(k)}(T_i) - F^{(k+1)}(T_i)\right] \\ & + C_{3i} \sum_{k=0}^{N_i} [\eta_i k(1-p)]\left[F^{(k)}(T_i) - F^{(k+1)}(T_i)\right] \end{aligned} \quad (6)$$

$$\begin{aligned} \frac{d(C_{T_i})}{dT_i} = {} & C_{1i} \sum_{k=0}^{N_i} kp\left[f^{(k)}(T_i) - f^{(k+1)}(T_i)\right] \\ & + C_{2i} \sum_{k=0}^{N_i} [N_i - kp - \eta_i k(1-p)]\left[f^{(k)}(T_i) - f^{(k+1)}(T_i)\right] \\ & + C_{3i} \sum_{k=0}^{N_i} [\eta_i k(1-p)]\left[f^{(k)}(T_i) - f^{(k+1)}(T_i)\right] \end{aligned} \quad (7)$$

For an optimal C_{T_i}, $\frac{d(C_{T_i})}{dT_i} = 0$

Hence,

$$\frac{p(C_{1i}-C_{2i})+\eta_i(1-p)(C_{3i}-C_{2i})}{C_{2i}} = \frac{N_i f^{(N_i+1)}(T_i)}{\sum_{k=0}^{N_i} f^{(k)}(T_i) - N_i f^{(N_i+1)}(T_i)} \quad (8)$$

When $\eta_i = 0$ and $p = 1$, the result derived by Subramanian [5]

$$\frac{(C_{1i}-C_{2i})}{C_{2i}} = \frac{N_i f^{(N_i+1)}(T_i)}{\sum_{k=0}^{N_i} f^{(k)}(T_i) - N_i f^{(N_i+1)}(T_i)}$$ is obtained.

Special Case

If the vacancies arrive as a Poisson process with parameter λ > 0, then the inter-arrival time follows an exponential distribution, $f(t) = \lambda e^{-\lambda t}$ $\lambda > 0$, $t > 0$.

Let $f^{(k)}(t)$ denote the k-fold convolution of $f(t)$. The inter-arrival time upto the first *k* arrival is:

$$f^{(k)}(t) = \lambda \frac{(\lambda t)^{k-1}}{(k-1)!} e^{-\lambda t}, \quad \lambda > 0, \quad t > 0$$

Equation (8) now becomes,

$$\frac{p(C_{1i}-C_{2i}) + \eta_i(1-p)(C_{3i}-C_{2i})}{C_{2i}} = \frac{N_i \frac{(\lambda T_i)^{N_i}}{(N_i)!}}{\sum_{k=1}^{N_i} \frac{(\lambda T_i)^{k-1}}{(k-1)!} - N_i \frac{(\lambda T_i)^{N_i}}{(N_i)!}} \quad (9)$$

Solving the above equation, the optimal time T_i^* is obtained.

2.2.1. Result

The optimal cost is obtained from equation (6). The optimal cost is given by:

$$C_{T_i}^* = N_i C_{2i}\left(\sum_{k=0}^{N_i} \frac{(\lambda T_i^*)^k}{k!} e^{-(\lambda T_i^*)}\right) + \left(pC_{1i} - (p+\eta_i-\eta_i p)C_{2i} + \eta_i(1-p)C_{3i}\right)N_i e^{-(\lambda T_i^*)} \frac{(\lambda T_i^*)^{N_i}}{N_i!} + \left(pC_{1i} - (p+\eta_i-\eta_i p)C_{2i} + \eta_i(\lambda T_i^*)C_{3i}(1-p)\right)\left(\sum_{k=1}^{N_i-1} \frac{(\lambda T_i^*)^{k-1}}{(k-1)!} e^{-(\lambda T_i^*)}\right) \quad (10)$$

The behaviour of optimal time and cost for various grade sizes N are illustrated by Figure 4 and Figure 5 respectively. The values of the parameters considered in the illustration are:

$C_1 = 25,\ C_2 = 20,\ C_3 = 30,\ p = 0.1,\ \eta = 0.0005\ and\ \lambda = 0.5.$

The parameter values are approximate values considered for illustrative purposes.

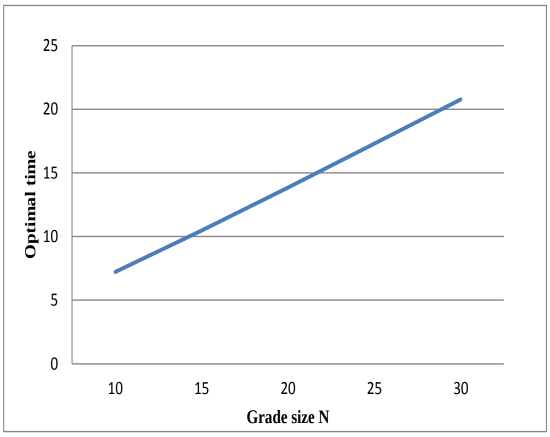

Figure 4: Optimal time for promotion

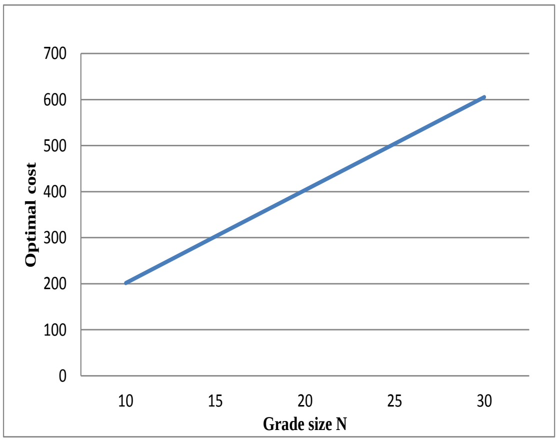

Figure 5: Optimal cost of promotion

The optimal time for promotion increases with the grade size for the given values of $C_{1i}, C_{2i}, C_{3i}, p, \eta$ and λ. The corresponding optimal cost of promotion increases with the grade size.

3. Conclusion

In this paper, two models for optimal promotion policy in manpower systems are proposed. The cost of recruitment into the grade where the cost of promotion and the cost of recruitment are functions of the grade size has been considered in the first model. In the second model, two types of promotion namely: performance-based promotion and career advancement promotion, the impact of attrition cost and a certain probability of passing a screening test for performance-based promotion are considered in proposing an optimal policy.

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