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Estimation of Inflection Point of Interpolated FFF Beam Data Profile from a Medical Linear Accelerator using Five Point Central Difference Method

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Abstract

Background: Treating Malignant and Benign tumours with High energy Photon beam radiotherapy from Linear accelerator (Linac) is a common practice in Medicine. To deliver uniform doses throughout the field of irradiation, Flattening filters are used in the Linacs conventionally. But Modern radiotherapy techniques uses the same high energy photon beams in Flattening Filter Free (FFF) mode with very high dose rate delivery to make high dose gradient and faster dose delivery. Hence the analysis of these FFF beam profiles are extremely important from quality assurance perspective before delivering on the patients.

Aim: To determine the inflection points and Penumbra for 6MV FFF and 10MV FFF beam profiles through software coding and compare with the existing conventional graphical method.

Method & Materials: In plane and cross plane beam profiles of a 20x20sqcm field is taken as standard for analysing and comparing the symmetry and penumbra of 6MV FFF and 10MV FFF beams. Conventionally the tangents are drawn at the beam edges of the profiles and midpoint of tangents are fixed as Inflection points. In this study, by taking the second derivative and five point central deviation formula the inflection points are found and the coding to determine the infection points, thereby penumbra are written in python3. To see the consistency of data comparable with conventional method, 12 different profiles taken during different part of a year is studied and presented.

Results: Infection points and penumbra of 6MV FFF and 10 MV FFF beams determined by python programming is having less deviation compared with Conventional method. The values obtained are well within the tolerance limits set by national standards. The maximum variation of penumbra values for 6mv FFF beam in plane is 2.3 mm and cross plane is 2.6mm from conventional method. The same variation for 10FFF beam is 2mm for in plane and 2.5mm for cross plane. Inflection points variation also within 3% of manual methods for both the energies.

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1. Introduction

Radiation Dosimetry is the determination of the absorbed dose in the disease affected tissue from exposure to ionizing radiation. It weaken or breaks up DNA cells and either damage the cells enough to kill them or causing them to mutate. Unflattened or FFF beam (Flattening filter free beam) is nowadays commonly used in Stereotactic Radiosurgery / Stereotactic Radiotherapy which result in more focused high energy beam so that the success rate of treating the patients become high. The FFF beam has the natural S shaped sigmoidal curve and the inflection point is the point on the curve where the curvature is more sharper than other part of the curve(where the curve bends up or bends down). By analyzing the inflection point, various beam characteristics can be calculated. In this paper, the point of inflection is estimated using manual method, MS-Excel, InterReg 2014 and the five point central difference method. The point of inflection plays vital role in the dosimetric analysis of FFF beam..

The accurate calculation of how much dose is to be applied to the inflicted region of the disease affected tissue in radiotherapy is very much crucial to the success of trapy in the treatment. A difference of few percent of dose may lead to the failure of treatment due to under / over dosage of radiation. In the treatment of cancer tissues, high energy beams are used conventionally. Modern SRS/SRT uses unflattened beam of 6MV and 10MV photon beams. Removing the filter from the beam path, FFF beams results in the increased dose and reduces the head leakage. Also it changes the beam quality of the FFF beam and the natural shape of the beam profile into S shaped sigmoidal curves which result in increased success rate in the treatment of radiotherapy. This Sigmoidal curves is widely studied in various disciplines like data science, artificial neural network, machine and deep learning. Sigmoidal curves have applications in first and second order derivatives. Minima, Maxima and concave up / concave down nature of the sigmoidal curve is studied using the first and second order derivatives. Inflection point is the prime concept in the study of sigmoidal curves. An inflection point is the point on the curve where the curve bends up or bends down and $\frac{d2}{dx2}(f(x))=0$.

The important property of the inflection point is that it lies in the region where the gradient value of the curve is high. Also this property which is the desirable characteristics of the beam curve is used to find the inflection point of the curve. The various way of discovering the point of inflection such as the manual method, MS-Excel, InterReg 2014 and the modified five point central difference formula is analyzed using the 6 MV FFF inline and cross line beam profile data. Many medical physicists follow manual method in determining the inflection point which is laborious and more prone to error which result in the wrong determination of the inflection point and may lead to over / under dose. Determining the beam profile characteristics of various FFF beams such as maximum dose depth d_{max}, percent dose at the depth of 10cm d_{10}, PDD and TPR , field size and penumbra and RDV (Reference Dose Value) were studied by pichandi[1]. Inflection point method of studying beam profile characteristics (dosimetric analysis) of unflattened beam by finding the inflection point of the ulflattened beam using the Cauchy principle of first and second order derivatives is used by Shende in [2].

Point of Inflection

Points on the path of the curve where concavity changes are called inflection points in which second derivative of the curve function is absolute zero. Concavity is determined by the checking the whether the second order derivative changes it sign from positive to negative or vice versa and at the inflection point where the direction of bending of the curve changes from bends up or bends down. IPs lie between the between where the curvature bends up or bends down.

Let function $f:[m,n] \rightarrow R$, $f \in C^{(r)}$, r>=2 which is convex for $x \in[m,q]$, and concave for $x \in[q,n]$, q is the unique point of inflection of the function f in [m,n] and x be an arbitrary point $x \in[m,n]$.

The point of inflection must satisfy the following condition

Concave Up

The function is concave up at $x=x_0$ in which the sign of the first order derivative of the function change from positive (+ve) to negative (-ve) and the function has its local maximum value

$\frac{d}{dx}(f(x)$ is +ve (positive) and $\frac{d2}{dx2}(f(x))>0$

a. Concave Down

The function is concave down at $x=x_0$ in which the sign of the first order derivative of the function change from negative (-ve) to positive (+ve) and the function has its local maximum value

$\frac{d}{dx}(f(x)$ is –ve (negative) and $\frac{d2}{dx2}(f(x))<0$

Inflection point

At the point of inflection where

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$\frac{d2}{dx2}(f(x))$ =0 is the constraint that must be satisfied at x, whereas $\frac{d2}{dx2}(f(x))$ =0 and $\frac{d3}{dx3}(f(x)) \neq 0$ is the sufficient constraint.

2. Fff Beam Profile Characteristics

Photon FFF beam

The modern recent apparatus of linear accelerators such as Elekta HD, Varian and True beam are operating Unflattened beam. The flattening filter actually hinders the path of the photon beam and hence the it is removed and Unflattened beam is used. This removal changes the natural shape of the curve into S shaped sigmoidal curves. More sophisticated treatment planning techniques such SRT/ SRS does not require homogeneous dose distribution. So removal of flattening filter increases the dose rate much higher when compared to FF beams and the reduces exposure time of the photon beams which result in increased success of the treatment. Baic[5] compared the conventional FF and modern FFF beams based on their dosimetric parameters and presented the clinical applications of FFF beams. The various beam profile parameters were studied by different authors. The important parameters of unflattened beams are beam symmetry, Field size , beam stability and penumbra which result in the increased dose rate , dose per pulse, diminished energy variation across the beam and less leakage.

Beam profile parameters of FFF photon beams

Beam profile can be measured for inline plane and cross line plane. In In plane, the beam head is along patient and the measurement are taken whereas the cross plane the beam head is cross to the patient. The S shaped sigmoidal curve of the beam reveals that FFF beam has high intensity at the center of the beam and it falls on either side of the symmetric edge. This is due to the removing the filter which obstruct the beam and 50% intensity level occurs at the path of the curve where gradient is high (which is bending up / bending down) of beam profile. But in general, 50% intensity level occurs at the central part of the beam curve in the FF beam.

Beam Symmetry

Symmetry of the S Shaped sigmoidal curves plays vital role in the left and right part of the beam curve.

Field Size

Field size is the length between the left and right point of inflection on the symmetric S Shaped sigmoidal beam curve.

Stability of the beam

The distance from the central part of the beam curve at 90%, 75% and 60% is used to measure the stability of the FFF beams.

Beam penumbra

RDV is the The dose value at point of inflection. Let Points X_a and X_b which lies at a distance of 1.6 and 0.4 times of RDV . The distance between X_a and X_b on both the side of the symmetric S shaped sigmoidal curve is called as beam penumbra.

Quality Index

QI is measured to certify that quality of the photon beam energy has not changed significantly and is used as quality indicator. It is dependent upon the beam which increases linearly . The other parameters such as PDD at 10cm and depth of dose maximum are also used as quality indicators.

Estimation of Inflection Point

To calculate the point of inflection of the S shaped sigmoidal curve, there exists numerous method in the literature proposing theory about determination and estimation of the inflection point of the continuous curve. The inflection point is used to study the various properties of the curve such as the size of field and penumbra of the photon beam for the analysis of the dosimetric properties of the beam. AERB of India which constituted a Task Group has pointed out the manual graphical method of calculating the point of inflection and further analysis of FFF beam. Manually IPs are defined as the midpoint on either side of the region of the S shaped sigmoidal curve the dose rate is maximum (where the curvature sharply bends) of the FFF beam curve. By identifying the region where the dose value is high, let (A) be point where curvature starts its sharp bending and let (B) be the end point of the symmetric S shaped sigmoidal curve. Let 'h' be the length between the points A and B. The point which lies at h/2 is called as the inflection point. Field size is calculated as the length between point of inflection at both the sides of S shaped sigmoidal curve. The distance measure between X_a and X_b on either side of the beam profile is referred as penumbra.

3. Estimation of Inflection Point

Mathematical method definition of First and Second Order derivatives using central difference method The application of first and second order derivatives in determining the inflection point in which second order derivative becomes absolute zero is used in

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determining the point of inflection. Here 5 point Cauchy derivative formula is used to compute the first and second order derivatives.

According to the Cauchy theorem, derivative of a continuous function is as follows as in Eq.(1). If continuous function f(a) defined on [a, a+h], then

$$\text{min of first order derivative} \leq \frac{f(a+h)-f(a)}{h} \leq \text{max of first order derivative}$$

in [x, x+h] Eq. (1)

The first order derivative of f(a) is defined using 5 point central difference formula as in Eq.(2)

$$f'(a) = \lim_{h \to 0} \frac{f(a-2h)-8f(a-h)+8f(a+h)-f(a+2h)}{12h}$$

Eq.(2)

From first order derivative using 5 point central difference formula, 5 point second order derivative formula is derived by differentiating the first order formula. And f''(x) is as follows as in Eq.(3)

$$f''(a) = \frac{d}{dx}\left[f'(a)\right] = \frac{d}{dx} \lim_{h \to 0} \frac{f(a-2h)-8f(a-h)+8f(a+h)-f(a+2h)}{h}$$

$$= \lim_{h \to 0} \frac{f'(a-2h)-8f'(a-h)+8f'(a+h)-f'(a+2h)}{h}$$

$$f''(a) = \lim_{h \to 0} \frac{f(a-2h)+16f(a-h)-30f(a)+16f(a+h)-f(a+2h)}{12h^3}$$

Eq.(3)

Different method of estimating the inflection point of the beam

In literature there exist different method of estimating the inflection point of a continuous curve. After locating the inflection point on the beam curve, field size and penumbra can also be determined using the inflection point.

Manual Method

The inflection point which lies at path of the sigmoidal curve where the gradient is high (sharply bending side of the curve) of the FFF beam. In Fig 1, 'h' is the length of the curve where the gradient is high . The half point of h is considered as the inflection point. The manual method of determining the inflection point is time consuming and laborious for medical physicists and may lead to error when inflection point is wrongly identified.

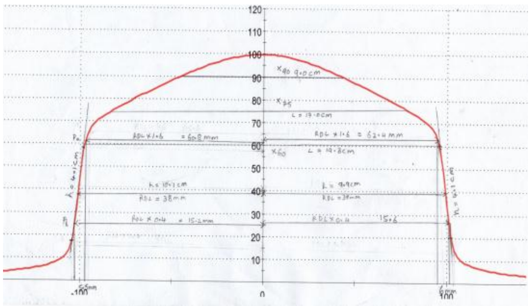

Figure 1: Manual method of finding inflection point

MS-Excel

MS-Excel method of determining inflection point can be easy as compared to manual method. But it also cumbersome for medical physicists as huge volume of profile data is to be exported from linacs whenever data is generated monthly. Fig. 2 shows the MS-Excel chart for finding the inflection point.

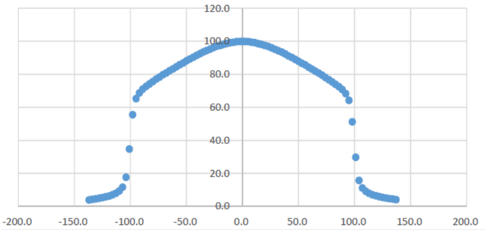

Figure 2: MS-Excel Chart showing inflection point

InterReg 2014

InterReg is an interpolation and Regression software which is used for statistical data analysis. Here the beam profile data can be interpolated using cubic spline interpolation and akima spline interpolation. The interpolated data can be added into the existing data and more accurate estimation of inflection point can be determined.

Python Program using five point central difference method

Determination of an inflection point using the first order Cauchy derivative formula by python code programming was analyzed by the author Ravindra Shende [2]. In this method, the h value is large from the profile data and to determine the exact point of inflection, h value need to be as small as possible. Hence in this modified python programming , the given beam profile data is interpolated using the

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akima spline interpolation and the added interpolated data using the 5 point central difference formula is used to find the first and second order derivatives.

R Software –inflection package

Estimation of the inflection point of a continuous curve using R function is implemented by Demetris T Christopoulos in his research paper [4].

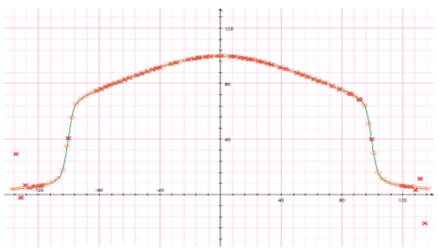

Figure 3: InterReg method of determining the inflection point

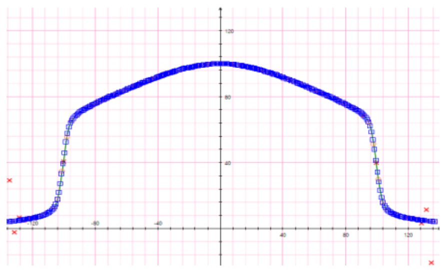

Figure 4: InterReg method (after applying akima spline interpolation) for determining the inflection point

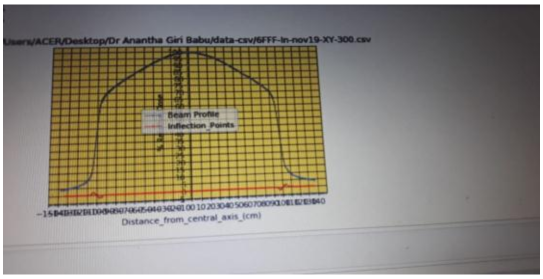

Figure 5: Inflection point using akima spline interpolated beam profile using modified python code using central difference method

4. Implementation

The modified python code using five point central difference method along with akima spline interpolated beam profile data yields better result when compare to the results obtained by the python code programming used by R Shende [2]. In the following table , the comparison of modified python code using five point central difference method and python code programming used by R Shende is compared. Fig 5 and Fig6 shows the result of python code using first order derivatives and akima spline interpolated beam profile using modified python programming using five point central difference method. Table 1 compares the beam profile parameters using the python programming using first order derivatives and akima spline interpolated beam profile by modified python programming using five point central difference method.

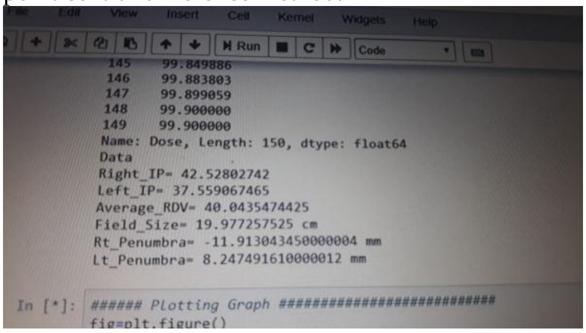

Figure 5: Python code using first order derivatives

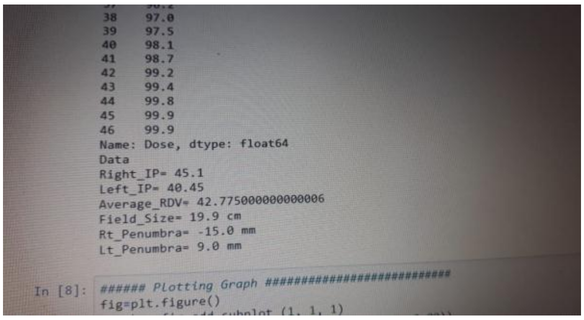

Figure 6: Akima interpolated beam profile using Modified python code using five point central difference method

Table 1: Result of Python Programming Using first principle of derivatives and Akima Spline Interpolated Beam Profile Using Modified five point central difference method

	Python Programming Using first principle of derivatives	**Akima Spline Interpolated Beam Profile Using Modified five point central difference method**
Right IP	**45.1**	**42.52802742**
Left IP	**40.45**	**37.559067465**
Avg RDV	**42.775**	**40.0435474425**
Field Size	**19.9 cm**	**19.977257525cm**
RtPenumbra	**-15.0 mm**	**-11.913043450000004mm**
Lt Penumbra	**9.0 mm**	**8.247491610000012mm**

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5. Conclusion

As the values determined by akima spline interpolated beam profile using modified python coding using five point central difference method is yielding better result than python programming using first derivatives and the conventional manual method, this method can be used for regular use in regular QA protocols, which may save lot of analyzing time of physicists and engineers and may reduce the manual graphical errors. This may help in tuning of beams instantaneously by interpretation of results obtained through code. This method can be further expanded using machine learning for further in determining the inflection point.

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