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Cartesian Product on Vague Bi-ideals and Vague Interior Ideals of a Γ-Semiring

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Abstract:
In this paper, we introduce and study the concept of cartesian product of vague sets of a Γ-semiring R and we characterize vague Γ-semiring, left(resp. right) vague ideal, vague bi-ideal and vague interior ideals of R in terms of cartesian product of vague sets of R.
Keywords: *vague set, left (resp. right) vague ideal, vague bi-ideal, vague interior ideal.*

I. INTRODUCTION

Semiring is an important algebraic tool in many areas of mathematics, for example, coding and language theory, automata theory, combinatorics, functional analysis and graph theory. M.K.Rao[12] introduced the concept of Γ-semiring as a generalization of semiring as well as Γ-ring. The properties of an ideal in semirings and Γ-semirings were somewhat different from the properties of the usual ring ideals. Moreover the concept of Γ-semiring not only generalizes the concept of semiring and Γ-ring but also the concept of ternary semiring.

Zadeh, L.A.[14] introduced the study of fuzzy sets in 1965. Mathematically a fuzzy set on a set X is a mapping μ into [0,1] of real numbers; for p in X, μ(p) is called the membership of p belonging to X. The membership function gives only an approximation for belonging but it does not give any information of not belonging. To counter this problem, Gau, W.L. and Buehrer, D.J.[11] introduced the concept of vague sets. A vague set ψ of a set X is a pair of functions (t_ψ, f_ψ), where tψ and fψ are fuzzy sets on X satisfying $t_\psi(p)+f_\psi(p) \leq 1$, ∀ p ∈ X. A fuzzy set t_ψ of X can be identified with the pair $(t_\psi,1-t_\psi)$. Thus the theory of vague sets is a generalization of fuzzy sets. The concepts of fuzzy Γ-semirings, vague Γ-semirings, vague ideals, vague bi-ideals and vague interior ideals of a Γ-semiring have been introduced and studied by Bhargavi, Y. and Eswarlal, T.[1-9]. Ersoy, B.A., Tepecik, A. and Demir, I.[10] studied cartesian product of fuzzy prime ideals of rings. Samit Kumar Majumder and Sujit Kumar Surdar [13] studied cartesian product of fuzzy prime and fuzzy semiprime ideals of semigroups.

In this paper, we introduce and study the concept of cartesian product of vague sets of a Γ-semiring R and we characterize vague Γ-semiring, left(resp. right) vague ideal, vague bi-ideal and vague interior ideals of R in terms of cartesian product of vague sets of a Γ-semiring.

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II. Preliminaries

In this section we recall some of the fundamental concepts and definitions, which are necessary for this paper.

Definition 2.1: Let R and Γ be two additive commutative semigroups. Then R is called Γ-semiring if there exists a mapping $R \times \Gamma \times R \rightarrow R$ image to be denoted by $a\alpha b$ for a, $b \in R$ and $\alpha \in \Gamma$ satisfying the following conditions.

1. $a\alpha(b + c) = a\alpha b + a\alpha c$
2. $(a + b)\alpha c = a\alpha c + b\alpha c$
3. $a(\alpha + \beta)c = a\alpha c + a\beta c$
4. $a\alpha (b\beta c) = (a\alpha b)\beta c$, $\forall$ a, b, c $\in$ R; $\alpha, \beta \in \Gamma$.

Definition 2.2: A non-empty subset S of a Γ-semiring R is said to be a sub Γ-semiring of R if (S, +) is a sub semigroup of (R, +) and $a\alpha b \in S$, $\forall$ a, b $\in$ S and $\alpha \in \Gamma$.

A non-empty subset S of a Γ-semiring R is said to be a left(resp. right) ideal of R if (S, +) is a sub semigroup of (R, +) and $x\alpha a \in S$ (resp. $a\alpha x \in S$), $\forall$ $a \in S$; $x \in R$; $\alpha \in \Gamma$.

A sub Γ-semiring S of a Γ-semiring R is said to be a bi-ideal of R if $S\Gamma R\Gamma S \subseteq S$.

A sub Γ-semigroup S of R is called an interior ideal of R if $R\Gamma S\Gamma R \subseteq S$.

Definition 2.3: Let X be any non-empty set. A mapping $\mu : X \rightarrow [0,1]$ is called a fuzzy subset.

Definition 2.4: A vague set ψ in the universe of discourse X is a pair (t_ψ, f_ψ), where $t_\psi : X \rightarrow [0, 1]$, $f_\psi : X \rightarrow [0, 1]$ are mappings such that $t_\psi(p) + f_\psi(p) \leq 1$, $\forall$ $p \in X$. The mappings t_ψ and f_ψ are called true membership mapping and false membership mapping respectively.

Definition 2.5: The interval $[t_\psi(p), 1 - f_\psi(p)]$ is called the vague value of p in ψ and it is denoted by $V_\psi(p)$ i.e., $V_\psi(p) = [t_\psi(p), 1 - f_\psi(p)]$.

Definition 2.6: Let R be a Γ-semiring. A vague set $\psi = (t_\psi, f_\psi)$ of R is said to be vague Γ-semiring if the following conditions are true:

For all p, q $\in$ R; $\gamma \in \Gamma$,

$V_\psi (p + q) \geq \min\{ V_\psi(p), V_\psi(q)\}$ and

$V_\psi(p\gamma q) \geq \min\{ V_\psi(p), V_\psi(q)\}$

i.e., (i). $t_\psi(p + q) \geq \min\{t_\psi(p), t_\psi(q)\}$,

$1 - f_\psi(p + q) \geq \min\{1 - f_\psi(p), 1 - f_\psi(q)\}$ and

(ii). $t_\psi(p\gamma q) \geq \min\{t_\psi(p), t_\psi(q)\}$,

$1 - f_\psi(p\gamma q) \geq \min\{1 - f_\psi(p), 1 - f_\psi(q)\}$.

Definition 2.7: A vague set $\psi = (t_\psi, f_\psi)$ of a Γ-semiring R is said to be left (resp. right) vague ideal of R if the following conditions are true:

For all p, q $\in$ R; $\gamma \in \Gamma$,

$V_\psi (p + q) \geq \min\{ V_\psi(p), V_\psi(q)\}$ and

$V_\psi(p\gamma q) \geq V_\psi(q)$ (resp. $V_\psi(p\gamma q) \geq V_\psi(p)$)

i.e., (i). $t_\psi(p + q) \geq \min\{t_\psi(p), t_\psi(q)\}$,

$1 - f_\psi(p + q) \geq \min\{1 - f_\psi(p), 1 - f_\psi(q)\}$ and

(ii). $t_\psi(p\gamma q) \geq t_\psi(q)$ (resp. $t_\psi(p\gamma q) \geq t_\psi(p)$),

$1 - f_\psi(p\gamma q) \geq 1 - f_\psi(q)$ (resp. $1 - f_\psi(p\gamma q) \geq 1 - f_\psi(p)$).

Definition 2.8: A vague Γ-semiring $\psi = (t_\psi, f_\psi)$ of a Γ-semiring R is called a vague bi-ideal of R if for all p, q, r $\in$ R; $\alpha, \beta \in \Gamma$, $V_\psi(p\alpha q\beta r) \geq \min\{V_\psi(p), V_\psi(r)\}$

i.e., $t_\psi(p\alpha q\beta r) \geq \min\{ t_\psi(p), t_\psi(r)\}$ and $1 - f_\psi(p\alpha q\beta r) \geq \min\{1-f_\psi(p), 1-f_\psi(r)\}$.

Definition 2.9: A vague Γ-semiring $\psi = (t_\psi, f_\psi)$ of a Γ-semiring R is called a vague interior ideal of R if for all p, q, r $\in$ R; $\alpha, \beta \in \Gamma$, $V_\psi(p\alpha q\beta r) \geq V_\psi(q)$

i.e., $t_\psi(p\alpha q\beta r) \geq t_\psi(q)$ and $1 - f_\psi(p\alpha q\beta r) \geq 1 - f_\psi($

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III. Cartesian Product on Vague Sets of A Γ-Semiring

In this section, we introduce and study the concept of cartesian product of vague sets of a Γ-semiring R and we characterize vague Γ-semiring, left(resp. right) vague ideal, vague bi-ideal and vague interior ideals in terms of cartesian product of vague sets. Throughout this paper unless otherwise mentioned R is a Γ-semiring.

Definition 3.1: Let ψ and ϕ be vague sets of R. Then the cartesian product of ψ and ϕ is defined by $V_{(\psi \times \phi)}((m, n)) = \min\{ V_{\psi}(m), V_{\phi}(n)\}$, for all $(m, n) \in R \times R$.

Lemma 3.2: If ψ and ϕ are vague sets of R, then $(\psi \times \phi)_{(\alpha, \beta)} = \psi_{(\alpha, \beta)} \times \phi_{(\alpha, \beta)}$, where $\alpha, \beta \in [0, 1]$ with $\alpha < \beta$.

Theorem 3.3: If ψ and ϕ are vague Γ-semiring of R, then $\psi \times \phi$ is a vague Γ-semiring of $R \times R$.

Proof: Suppose ψ and ϕ are vague Γ-semiring of R.

Let $(m, n), (p, q) \in R \times R; \gamma \in \Gamma$

Now,

1.$V_{\psi \times \phi}((m, n) + (p, q)) = V_{\psi \times \phi}((m + p, n + q)) = \min\{V_{\psi}(m + p), V_{\phi}(n + q)\} \geq \min\{\min\{V_{\psi}(m), V_{\psi}(p)\}, \min\{V_{\phi}(n), V_{\phi}(q)\}\} = \min\{\min\{V_{\psi}(m), V_{\phi}(n)\}, \min\{V_{\psi}(p), V_{\phi}(q)\}\} = \min\{V_{\psi \times \phi}((m, n)), V_{\psi \times \phi}((p, q))\}$

2.$V_{\psi \times \phi}((m, n) \gamma (p, q)) = V_{\psi \times \phi}((m\gamma p, n\gamma q)) = \min\{V_{\psi}(m\gamma p), V_{\phi}(n\gamma q)\} \geq \min\{\min\{V_{\psi}(m), V_{\psi}(p)\}, \min\{V_{\phi}(n), V_{\phi}(q)\}\} = \min\{\min\{V_{\psi}(m), V_{\phi}(n)\}, \min\{V_{\psi}(p), V_{\phi}(q)\}\} = \min\{V_{\psi \times \phi}((m, n)), V_{\psi \times \phi}((p, q))\}$

Thus $\psi \times \phi$ is a vague Γ-semiring of $R \times R$.

Theorem 3.4: If ψ and ϕ are left(resp. right) vague ideals of R, then $\psi \times \phi$ is a left(resp. right) vague ideal of $R \times R$.

Proof: Suppose ψ and ϕ are left vague ideals of R.

Let $(m, n), (p, q) \in R \times R; \gamma \in \Gamma$

Now,

1.$V_{\psi \times \phi}((m, n) + (p, q)) = V_{\psi \times \phi}((m + p, n + q)) = \min\{V_{\psi}(m + p), V_{\phi}(n + q)\} \geq \min\{\min\{V_{\psi}(m), V_{\psi}(p)\}, \min\{V_{\phi}(n), V_{\phi}(q)\}\} = \min\{\min\{V_{\psi}(m), V_{\phi}(n)\}, \min\{V_{\psi}(p), V_{\phi}(q)\}\} = \min\{V_{\psi \times \phi}((m, n)), V_{\psi \times \phi}((p, q))\}$

2.$V_{\psi \times \phi}((m, n) \gamma (p, q)) = V_{\psi \times \phi}((m\gamma p, n\gamma q)) = \min\{V_{\psi}(m\gamma p), V_{\phi}(n\gamma q)\} \geq \min\{V_{\psi}(p)\}, V_{\phi}(q)\} = V_{\psi \times \phi}((p, q))$.

Thus $\psi \times \phi$ is a left vague ideal of $R \times R$.

Similarly we can prove $\psi \times \phi$ is a right vague ideal of $R \times R$.

Theorem 3.5: If ψ and ϕ are vague bi-ideals of R, then $\psi \times \phi$ is a vague bi-ideal of $R \times R$.

Proof: Suppose ψ and ϕ are vague bi-ideals of R.

From theorem:3.3, $\psi \times \phi$ is a vague Γ-semiring of $R \times R$.

Let $(m, n), (p, q), (u, v) \in R \times R; \gamma, \eta \in \Gamma$

Now,

$V_{\psi \times \phi}((m, n) \gamma (p, q) \eta (u, v)) = V_{\psi \times \phi}((m\gamma p\eta u, n\gamma q\eta v)) = \min\{V_{\psi}(m\gamma p\eta u), V_{\phi}(n\gamma q\eta v)\} \geq \min\{\min\{V_{\psi}(m), V_{\psi}(u)\}, \min\{V_{\phi}(n), V_{\phi}(v)\}\} = \min\{\min\{V_{\psi}(m), V_{\phi}(n)\}, \min\{V_{\psi}(u), V_{\phi}(v)\}\} = \min\{V_{\psi \times \phi}((m, n)), V_{\psi \times \phi}((u, v))\}$

Thus $\psi \times \phi$ is a vague bi-ideal of $R \times R$.

Theorem 3.6: If ψ and ϕ are vague interior ideals of R, then $\psi \times \phi$ is a vague interior ideal of $R \times R$.

Proof: Suppose ψ and ϕ are vague interior ideals of R.

From theorem:3.3, $\psi \times \phi$ is a vague Γ-semiring of $R \times R$.

Let $(m, n), (p, q), (u, v) \in R \times R; \gamma, \eta \in \Gamma$

Now,

$V_{\psi \times \phi}((m, n) \gamma (p, q) \eta (u, v)) = V_{\psi \times \phi}((m\gamma p\eta u, n\gamma q\eta v)) = \min\{V_{\psi}(m\gamma p\eta u), V_{\phi}(n\gamma q\eta v)\} \geq \min\{V_{\psi}(p), V_{\phi}(q)\} = V_{\psi \times \phi}((p, q))$

Thus $\psi \times \phi$ is a vague interior ideal of $R \times R$.

Theorem 3.7: Let ψ and ϕ be vague Γ-semiring of R, then $\psi \times \phi$ is a vague Γ-semiring of $R \times R$ if and only if $(\psi \times \phi)_{(\alpha, \beta)}$ is a sub Γ-semiring of $R \times R$.

Proof: Suppose $\psi \times \phi$ is a vague Γ-semiring of $R \times R$.

Let $(m, n), (p, q) \in (\psi \times \phi)_{(\alpha, \beta)}; \gamma \in \Gamma$.

$\Rightarrow V_{\psi \times \phi}((m, n)) \geq [\alpha, \beta]$ and $V_{\psi \times \phi}((p, q)) \geq [\alpha, \beta]$.

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Now, $V_{\psi \times \phi}((m, n) + (p, q)) = \min\{V_{\psi \times \phi}((m, n)), V_{\psi \times \phi}((p, q))\} \geq [\alpha, \beta]$.
That implies $(m, n) + (p, q) \in (\psi \times \phi)_{(\alpha, \beta)}$.
Also, $V_{\psi \times \phi}((m, n)\ \gamma\ (p, q)) = \min\{V_{\psi \times \phi}((m, n)), V_{\psi \times \phi}((p, q))\} \geq [\alpha, \beta]$.
That implies $(m, n)\ \gamma\ (p, q) \in (\psi \times \phi)_{(\alpha, \beta)}$.
Thus $(\psi \times \phi)_{(\alpha, \beta)}$ is a sub Γ-semiring of R × R.
Conversely suppose that $(\psi \times \phi)_{(\alpha, \beta)}$ is a sub Γ-semiring of R × R.
Let $(m, n), (p, q) \in R \times R$; $\gamma \in \Gamma$.
Let $V_{\psi \times \phi}((m, n)) = [\alpha_1, \beta_1]$ and $V_{\psi \times \phi}((p, q)) \geq [\alpha_2, \beta_2]$ with $[\alpha_2, \beta_2] \leq [\alpha_1, \beta_1]$.
Put $[\alpha, \beta] = \min\{[\alpha_1, \beta_1], [\alpha_2, \beta_2]\}$.
Then $(m, n), (p, q) \in (\psi \times \phi)_{(\alpha, \beta)}$.
So, $(m, n) + (p, q), (m, n)\ \gamma\ (p, q) \in (\psi \times \phi)_{(\alpha, \beta)}$.
That implies $V_{\psi \times \phi}((m, n) + (p, q)) \geq [\alpha, \beta] = \min\{V_{\psi \times \phi}((m, n)), V_{\psi \times \phi}((p, q))\}$ and
$V_{\psi \times \phi}((m, n)\ \gamma\ (p, q)) \geq [\alpha, \beta] = \min\{V_{\psi \times \phi}((m, n)), V_{\psi \times \phi}((p, q))\}$.
Thus $\psi \times \phi$ is a vague Γ-semiring of R × R.

Theorem 3.8: Let ψ and ϕ be two left(resp. right) vague ideals of R, then $\psi \times \phi$ is a left(resp. right) vague ideal of R × R if and only if $(\psi \times \phi)_{(\alpha, \beta)}$ is a left (resp. right) ideal of R × R.

Proof: Suppose $\psi \times \phi$ is a left vague ideal of R × R.
Let $(m, n), (p, q) \in (\psi \times \phi)_{(\alpha, \beta)}$; $(u, v) \in R \times R$; $\gamma \in \Gamma$.
$\Rightarrow V_{\psi \times \phi}((m, n)) \geq [\alpha, \beta]$ and $V_{\psi \times \phi}((p, q)) \geq [\alpha, \beta]$.
Now, $V_{\psi \times \phi}((m, n) + (p, q)) = \min\{V_{A \times B}((m, n)), V_{\psi \times \phi}((p, q))\} \geq [\alpha, \beta]$.
That implies $(m, n) + (p, q) \in (\psi \times \phi)_{(\alpha, \beta)}$.
Also, $V_{\psi \times \phi}((u, v)\ \gamma\ (m, n)) = V_{\psi \times \phi}((u\gamma m, v\gamma n)) = \min\{V_\psi(u\gamma m), V_\phi(v\gamma n)\} \geq \min\{V_\psi(m)\}, V_\phi(n)\} = V_{\psi \times \phi}((m, n)) \geq [\alpha, \beta]$.
That implies $(u, v)\ \gamma\ (m, n) \in (\psi \times \phi)_{(\alpha, \beta)}$.
Thus $(\psi \times \phi)_{(\alpha, \beta)}$ is a left ideal of R × R.
Conversely suppose that $(\psi \times \phi)_{(\alpha, \beta)}$ is a left ideal of R × R.
Let $(m, n), (p, q) \in R \times R$; $\gamma \in \Gamma$.
Let $V_{\psi \times \phi}((m, n)) = [\alpha_1, \beta_1]$ and $V_{\psi \times \phi}((p, q)) \geq [\alpha_2, \beta_2]$ with $[\alpha_2, \beta_2] \leq [\alpha_1, \beta_1]$.
Put $[\alpha, \beta] = \min\{[\alpha_1, \beta_1], [\alpha_2, \beta_2]\}$.
Then $(m, n), (p, q) \in (\psi \times \phi)_{(\alpha, \beta)}$.
So, $(m, n) + (p, q), (m, n)\ \gamma\ (p, q), (p, q)\ \gamma\ (m, n) \in (\psi \times \phi)_{(\alpha, \beta)}$.
That implies $V_{\psi \times \phi}((m, n) + (p, q)) \geq [\alpha, \beta] = \min\{V_{\psi \times \phi}((m, n)), V_{\psi \times \phi}((p, q))\}$ and
$V_{\psi \times \phi}((m, n)\ \gamma\ (p, q)) \geq [\alpha, \beta] = V_{\psi \times \phi}((p, q))$.
Thus $\psi \times \phi$ is a left vague ideal of R × R.
Similarly we can prove for right ideals also.

Theorem 3.9: Let ψ and ϕ be vague bi-ideals of R, then $\psi \times \phi$ is a vague bi-ideal of R × R if and only if $(\psi \times \phi)_{(\alpha, \beta)}$ is a bi-ideal of R × R.

Proof: Suppose $\psi \times \phi$ is a vague bi-ideal of R × R.
From theorem:3.7, $(\psi \times \phi)_{(\alpha, \beta)}$ is a sub Γ-semiring of R × R.
Let $(m, n), (p, q) \in (\psi \times \phi)_{(\alpha, \beta)}$; $(u, v) \in R \times R$; $\gamma, \eta \in \Gamma$.
$\Rightarrow V_{\psi \times \phi}((m, n)) \geq [\alpha, \beta]$ and $V_{\psi \times \phi}((p, q)) \geq [\alpha, \beta]$.
Now, $V_{\psi \times \phi}((m, n)\ \gamma\ (u, v)\ \eta\ (p, q)) = \min\{V_{\psi \times \phi}((m, n)), V_{\psi \times \phi}((p, q))\} \geq [\alpha, \beta]$.
That implies $(m, n)\ \gamma\ (u, v)\ \eta\ (p, q) \in (\psi \times \phi)_{(\alpha, \beta)}$.
Thus $(\psi \times \phi)_{(\alpha, \beta)}$ is a bi-ideal of R × R.
Conversely suppose that $(\psi \times \phi)_{(\alpha, \beta)}$ is a bi-ideal of R × R.
From theorem:3.7, $\psi \times \phi$ is a vague Γ-semiring of R × R.
Let $(m, n), (p, q), (u, v) \in R \times R$; $\gamma, \eta \in \Gamma$.
Let $V_{\psi \times \phi}((m, n)) = [\alpha_1, \beta_1]$ and $V_{\psi \times \phi}((p, q)) \geq [\alpha_2, \beta_2]$ with $[\alpha_2, \beta_2] \leq [\alpha_1, \beta_1]$.
Put $[\alpha, \beta] = \min\{[\alpha_1, \beta_1], [\alpha_2, \beta_2]\}$.
Then $(m, n), (p, q) \in (\psi \times \phi)_{(\alpha, \beta)}$.
So, $(m, n)\ \gamma\ (u, v)\ \eta\ (p, q) \in (\psi \times \phi)_{(\alpha, \beta)}$.
That implies $V_{\psi \times \phi}((m, n)\ \gamma\ (u, v)\ \eta\ (p, q)) \geq [\alpha, \beta] = \min\{V_{\psi \times \phi}((m, n)), V_{\psi \times \phi}((p, q))\}$.
Thus $\psi \times \phi$ is a vague bi- ideal of R × R.

Theorem 3.10: Let ψ and ϕ be vague interior ideals of R, then $\psi \times \phi$ is a vague interior ideal of R × R if and only if $(\psi \times \phi)_{(\alpha, \beta)}$ is a interior ideal of R × R.

Proof: Suppose $\psi \times \phi$ is a vague interior ideal of R × R.
From theorem:3.7, $(\psi \times \phi)_{(\alpha, \beta)}$ is a sub Γ-semiring of R × R.
Let $(m, n), (p, q) \in R \times R$; $(u, v) \in (\psi \times \phi)_{(\alpha, \beta)}$; $\gamma, \eta \in \Gamma$.
$\Rightarrow V_{\psi \times \phi}((u, v)) \geq [\alpha, \beta]$.
Now, $V_{\psi \times \phi}((m, n)\ \gamma\ (u, v)\ \eta\ (p, q)) \geq V_{\psi \times \phi}((u, v)) \geq [\alpha, \beta]$.
That implies $(m, n)\ \gamma\ (u, v)\ \eta\ (p, q) \in (\psi \times \phi)_{(\alpha, \beta)}$.

Thus $(\psi \times \phi)_{(\alpha, \beta)}$ is a interior ideal of $R \times R$.
Conversely suppose that $(\psi \times \phi)_{(\alpha, \beta)}$ is a interior ideal of $R \times R$.
From theorem:3.7, $\psi \times \phi$ is a vague Γ-semiring of $R \times R$.
Let $(m, n), (p, q), (u, v) \in R \times R$; $\gamma, \eta \in \Gamma$.
Let $V_{\psi \times \phi}((u, v)) = [\alpha, \beta]$.
Then $(u, v) \in (\psi \times \phi)_{(\alpha, \beta)}$.
So, $(m, n)\, \gamma\, (u, v)\, \eta\, (p, q) \in (\psi \times \phi)_{(\alpha, \beta)}$.
That implies $V_{\psi \times \phi}((m, n)\, \gamma\, (u, v)\, \eta\, (p, q)) \geq [\alpha, \beta] = V_{\psi \times \phi}((u, v))$.
Thus $\psi \times \phi$ is a vague interior ideal of $R \times R$

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