# Application of Linear Programming in Nurse Scheduling Problem 

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#### Abstract

: The main object of this paper is to find application of linear programming in Nurse Scheduling Problem and for this to find minimum number of nursing staff for different shifts (Morning and Evening) for a particular hospital to satisfy its daily nursing staff demand and to minimize the total cost of hospital and to optimize the nursing staff for scheduling of their working shifts and day -off.


Keywords: Linear Programming Model, Nurse Scheduling, Cost Minimization, Objective Function, Constraints, Lingo Software.

## INTRODUCTION

Linear programming technique is an important tool for decision making in business policies for optimization. In linear programming there is a linear function called objective function generally denoted by Z which is to be maximized or minimized with given set of condition.In business linear programming can also used to find when to hire more workers, and when to increase demand of available workers. Many organizations can use linear programming method to schedule the jobs provide by organizations that are done in different shifts for optimum output of the organizations. It is used to find minimum number of workers for company. Linear programming is an efficient tool for solving problems related to nurse-scheduling, which contain the working shifts, minimum number of staff as required, satisfaction, caring of patients and reduce the cost of hospital. ManmohanPatidar et al. (2015) applied technique of linear programming for solving nurse scheduling problem in hospital management. Ahmed Ali El Adoly et al. (2018) gave a new formulation and solution for the nurse scheduling problem and many other auther used linear programming for solution of nurse scheduling
problems. The main intension of this paper is to find optimum number of nursing staff for a particular hospital for two shift (Morning and Evening) to reduced the cost of hospital. The mathematical formulation of linear programming problem is Maximize or Minimize

$$
\begin{equation*}
\mathrm{Z}=\mathrm{c}_{1} \mathrm{H}_{1}+\mathrm{c}_{2} \mathrm{H}_{2}+\ldots+\mathrm{c}_{\mathrm{n}} \mathrm{H}_{\mathrm{n}} \text { (objective function) } \tag{i}
\end{equation*}
$$

subject to the constraints

$$
\left.\begin{array}{c}
\mathrm{a}_{11} \mathrm{H}_{1}+\mathrm{a}_{12} \mathrm{H}_{2}+\ldots+\mathrm{a}_{1 \mathrm{n}} \mathrm{H}_{\mathrm{n}}(\leq,=, \geq) \mathrm{b}_{1} \\
\mathrm{a}_{21} \mathrm{H}_{1}+\mathrm{a}_{22} \mathrm{H}_{2}+\ldots+\mathrm{a}_{2 \mathrm{n}} \mathrm{H}_{\mathrm{n}}(\leq,=,, \geq) \mathrm{b}_{2} \\
\ldots \ldots \ldots \ldots \\
\ldots \ldots \ldots \ldots \ldots  \tag{ii}\\
\ldots \ldots \ldots \ldots \ldots \\
\mathrm{a}_{\mathrm{m} 1} \mathrm{H}_{1}+\mathrm{a}_{\mathrm{m} 2} \mathrm{H}_{2}+\ldots+\mathrm{a}_{\mathrm{mn}} \mathrm{H}_{\mathrm{n}}(\leq,=, \geq) \mathrm{b}_{\mathrm{m}}
\end{array}\right\}
$$

and non-negative restrictions

$$
\mathrm{H}_{\mathrm{j}} \geq 0, \quad \mathrm{j}=1,2 \ldots \mathrm{n}
$$

Manuscript Detail.

Where $a_{i j}$ 's, $b_{i}$ 's and $c_{j}$ 's are constants and $H_{j}$ 's are variables.
In the conditions given by (ii) there may be any of the three signs $\leq,=, \geq$.

The standard form of the linear programming problem of $n$ variables and $m$ constraints can be written as follows:

Maximize or Minimize

$$
\mathrm{z}=\mathrm{c}_{1} \mathrm{H}_{1}+\mathrm{c}_{2} \mathrm{H}_{2}+\ldots+\mathrm{c}_{\mathrm{n}} \mathrm{H}_{\mathrm{n}}+0 \cdot \mathrm{R}_{1}+0 \cdot \mathrm{R}_{2}+\ldots+0 \cdot \mathrm{R}_{\mathrm{m}}
$$

(objective function)
subject to the constraints
and non-negative restrictions

$$
H_{j} \geq 0, \quad R_{i} \geq 0, j=1,2 \ldots n, i=1,2 \ldots m
$$

Where $a_{i j}$ 's, $b_{i}$ 's and $c_{j}$ 's are constants and $H_{j}$ 's and $\mathrm{R}_{\mathrm{i}}$ 's are variables.

## ASSUMPTION FOR PROBLEM

(a) It is assumed that the number of nurses needed on a day is fixed.
(b) It is assumed that every nurse works six consecutive days in a week.
(c) It is assumed that all relation are linear.

## DATA PRESENTATION AND ANALYSIS

The number of nurses needed on a particular day for a particular hospital is given as follows-

| Day | Number of Nurses |  |
| :---: | :---: | :---: |
|  | Shift I | Shift II |
| Monday | 50 | 30 |
| Tuesday | 40 | 25 |
| Wednesday | 60 | 40 |

$$
\begin{align*}
& \mathrm{a}_{11} \mathrm{H}_{1}+\mathrm{a}_{12} \mathrm{H}_{2}+\ldots+\mathrm{a}_{1 n} \mathrm{H}_{\mathrm{n}}+\mathrm{R}_{1}=\mathrm{b}_{1} \\
& \mathrm{a}_{21} \mathrm{H}_{1}+\mathrm{a}_{22} \mathrm{H}_{2}+\ldots+\mathrm{a}_{2 \mathrm{n}} \mathrm{H}_{\mathrm{n}}+\mathrm{R}_{2}=\mathrm{b}_{2} \\
& \text {....... ....... } \\
& \text {....... ....... ......... ......... } \\
& \mathrm{a}_{\mathrm{m} 1} \mathrm{H}_{1}+\mathrm{a}_{\mathrm{m} 2} \mathrm{H}_{2}+\ldots+\mathrm{a}_{\mathrm{mn}} \mathrm{H}_{\mathrm{n}}+\mathrm{R}_{\mathrm{m}}=\mathrm{b}_{\mathrm{m}} \tag{iv}
\end{align*}
$$

| Thursday | 50 | 20 |
| :---: | :---: | :---: |
| Friday | 100 | 50 |
| Saturday | 80 | 30 |
| Sunday | 30 | 15 |

## MODEL FORMULATION

Let $\mathrm{H}_{1}$ be the number of nurses starting duty from Monday in shift I (Monday - Saturday)

Let $\mathrm{H}_{3}$ be the number of nurses starting duty from Tuesday in shift I (Tuesday - Sunday)

Let $\mathrm{H}_{2}$ be the number of nurses starting duty from Monday in shift II (Monday - Saturday)

Let $\mathrm{H}_{4}$ be the number of nurses starting duty from Tuesday in shift II (Tuesday - Sunday)

Let $\mathrm{H}_{5}$ be the number of nurses starting duty from Wednesday in shift I (Wednesday - Monday)

Let $\mathrm{H}_{6}$ be the number of nurses starting duty from Wednesday in shift II (Wednesday - Monday)

Let $\mathrm{H}_{7}$ be the number of nurses starting duty from Thursday in shift I (Thursday - Tuesday)

Let $\mathrm{H}_{8}$ be the number of nurses starting duty from Thursday in shift II (Thursday - Tuesday)

Let $\mathrm{H}_{9}$ be the number of nurses starting duty from Friday in shift I (Friday - Wednesday)

Let $\mathrm{H}_{10}$ be the number of nurses starting duty from Friday in shift II (Friday - Wednesday)

Let $\mathrm{H}_{11}$ be the number of nurses starting duty from Saturday in shift I (Saturday - Thursday)

Let $\mathrm{H}_{12}$ be the number of nurses starting duty from Saturday in shift II (Saturday - Thursday)

Let $\mathrm{H}_{13}$ be the number of nurses starting duty from Sunday in shift I (Sunday - Friday)

Let $\mathrm{H}_{14}$ be the number of nurses starting duty from Sunday in shift II (Sunday - Friday)

Let Z is total number of nurses to be minimize.

The mathematical form of above data is

## Minimize

$\mathrm{Z}=\mathrm{H}_{1}+\mathrm{H}_{2}+\mathrm{H}_{3}+\mathrm{H}_{4}+\mathrm{H}_{5}+\mathrm{H}_{6}+\mathrm{H}_{7}+$
$\mathrm{H}_{8}+\mathrm{H}_{9}+\mathrm{H}_{10}+\mathrm{H}_{11}+\mathrm{H}_{12}+\mathrm{H}_{13}+\mathrm{H}_{14}$

Subject to

$$
\begin{aligned}
& H_{1}+H_{5}+H_{7}+H_{9}+H_{11}+H_{13} \geq 50 \\
& H_{2}+H_{6}+H_{8}+H_{10}+H_{12}+H_{14} \geq 30 \\
& H_{1}+H_{3}+H_{7}+H_{9}+H_{11}+H_{13} \geq 40 \\
& H_{2}+H_{4}+H_{8}+H_{10}+H_{12}+H_{14} \geq 25 \\
& H_{1}+H_{3}+H_{5}+H_{9}+H_{11}+H_{13} \geq 60 \\
& H_{2}+H_{4}+H_{6}+H_{10}+H_{12}+H_{14} \geq 40 \\
& H_{1}+H_{3}+H_{5}+H_{7}+H_{11}+H_{13} \geq 50 \\
& H_{2}+H_{4}+H_{6}+H_{8}+H_{12}+H_{14} \geq 20 \\
& H_{1}+H_{3}+H_{5}+H_{7}+H_{9}+H_{13} \geq 100 \\
& H_{2}+H_{4}+H_{6}+H_{8}+H_{10}+H_{14} \geq 50 \\
& H_{1}+H_{3}+H_{5}+H_{7}+H_{9}+H_{11} \geq 80 \\
& H_{2}+H_{4}+H_{6}+H_{8}+H_{10}+H_{12} \geq 30 \\
& H_{3}+H_{5}+H_{7}+H_{9}+H_{11}+H_{13} \geq 30 \\
& H_{4}+H_{6}+H_{8}+H_{10}+H_{12}+H_{14} \geq 15
\end{aligned}
$$

and $\mathrm{H}_{1}, \mathrm{H}_{2}, \mathrm{H}_{3}, \ldots, \mathrm{H}_{14} \geq 0$
using surplus variables the problem converted to

## Minimize

$\mathrm{Z}=\mathrm{H}_{1}+\mathrm{H}_{2}+\mathrm{H}_{3}+\mathrm{H}_{4}+\mathrm{H}_{5}+\mathrm{H}_{6}+\mathrm{H}_{7}+\mathrm{H}_{8}+\mathrm{H}_{9}+\mathrm{H}_{10}+\mathrm{H}_{11}+\mathrm{H}_{12}$
$+\mathrm{H}_{13}+\mathrm{H}_{14}+0 . \mathrm{R}_{1}+0 . \mathrm{R}_{2}+0 . \mathrm{R}_{3}+0 . \mathrm{R}_{4}+0 . \mathrm{R}_{5}+0 . \mathrm{R}_{6}+0 . \mathrm{R}_{7}+0 . \mathrm{R}_{8}+$
$0 . \mathrm{R}_{9}+0 . \mathrm{R}_{10}+0 . \mathrm{R}_{11}+0 . \mathrm{R}_{12}+0 . \mathrm{R}_{13}+0 . \mathrm{R}_{14}$

Subject to

$$
\begin{aligned}
& \mathrm{H}_{1}+\mathrm{H}_{5}+\mathrm{H}_{7}+\mathrm{H}_{9}+\mathrm{H}_{11}+\mathrm{H}_{13}-\mathrm{R}_{1}=50 \\
& \mathrm{H}_{2}+\mathrm{H}_{6}+\mathrm{H}_{8}+\mathrm{H}_{10}+\mathrm{H}_{12}+\mathrm{H}_{14}-\mathrm{R}_{2}=30 \\
& \mathrm{H}_{1}+\mathrm{H}_{3}+\mathrm{H}_{7}+\mathrm{H}_{9}+\mathrm{H}_{11}+\mathrm{H}_{13}-\mathrm{R}_{3}=40 \\
& \mathrm{H}_{2}+\mathrm{H}_{4}+\mathrm{H}_{8}+\mathrm{H}_{10}+\mathrm{H}_{12}+\mathrm{H}_{14}-\mathrm{R}_{4}=25 \\
& \mathrm{H}_{1}+\mathrm{H}_{3}+\mathrm{H}_{5}+\mathrm{H}_{9}+\mathrm{H}_{11}+\mathrm{H}_{13}-\mathrm{R}_{5}=60 \\
& \mathrm{H}_{2}+\mathrm{H}_{4}+\mathrm{H}_{6}+\mathrm{H}_{10}+\mathrm{H}_{12}+\mathrm{H}_{14}-\mathrm{R}_{6}=40 \\
& \mathrm{H}_{1}+\mathrm{H}_{3}+\mathrm{H}_{5}+\mathrm{H}_{7}+\mathrm{H}_{11}+\mathrm{H}_{13}-\mathrm{R}_{7}=50 \\
& \mathrm{H}_{2}+\mathrm{H}_{4}+\mathrm{H}_{6}+\mathrm{H}_{8}+\mathrm{H}_{12}+\mathrm{H}_{14}-\mathrm{R}_{8}=20 \\
& \mathrm{H}_{1}+\mathrm{H}_{3}+\mathrm{H}_{5}+\mathrm{H}_{7}+\mathrm{H}_{9}+\mathrm{H}_{13}-\mathrm{R}_{9}=100 \\
& \mathrm{H}_{2}+\mathrm{H}_{4}+\mathrm{H}_{6}+\mathrm{H}_{8}+\mathrm{H}_{10}+\mathrm{H}_{14}-\mathrm{R}_{10}=50 \\
& \mathrm{H}_{1}+\mathrm{H}_{3}+\mathrm{H}_{5}+\mathrm{H}_{7}+\mathrm{H}_{9}+\mathrm{H}_{11}-\mathrm{R}_{11}=80 \\
& \mathrm{H}_{2}+\mathrm{H}_{4}+\mathrm{H}_{6}+\mathrm{H}_{8}+\mathrm{H}_{10}+\mathrm{H}_{12}-\mathrm{R}_{12}=30 \\
& \mathrm{H}_{3}+\mathrm{H}_{5}+\mathrm{H}_{7}+\mathrm{H}_{9}+\mathrm{H}_{11}+\mathrm{H}_{13}-\mathrm{R}_{13}=30 \\
& \mathrm{H}_{4}+\mathrm{H}_{6}+\mathrm{H}_{8}+\mathrm{H}_{10}+\mathrm{H}_{12}+\mathrm{H}_{14}-\mathrm{R}_{14}=15 \\
& \text { and } \mathrm{H}_{1}, \mathrm{H}_{2}, \mathrm{H}_{3}, \ldots, \mathrm{H}_{14}, \mathrm{R}_{1}, \mathrm{R}_{2}, \ldots, \mathrm{R}_{14} \geq 0
\end{aligned}
$$

The above linear programming model has 14 variables, so using Lingo software for solving above linear programming problem, we get an optimal solution as

$$
\begin{aligned}
& \mathrm{H}_{1}=70, \mathrm{H}_{2}=35, \mathrm{H}_{3}=30, \mathrm{H}_{4}=15, \mathrm{H}_{5}=0, \mathrm{H}_{6}=0, \\
& \mathrm{H}_{7}=0, \mathrm{H}_{8}=0, \mathrm{H}_{9}=0, \mathrm{H}_{10}=0, \mathrm{H}_{11}=0, \mathrm{H}_{12}=0, \\
& \mathrm{H}_{13}=0, \mathrm{H}_{14}=0,
\end{aligned}
$$

and Minimize $\mathrm{Z}=150$

## INTERPRETATION OF RESULT

Based on assumed data the optimum result of the above problem specified that the minimum value of Z is 150 and value of $\mathrm{H}_{1}=70$, $\mathrm{H}_{2}=35, \mathrm{H}_{3}=30, \mathrm{H}_{4}=15$ and value of other ${ }^{2}$ variables zero.

## CONCLUSION

From the detailed conversation as above it is concluded that the minimum number of nurses for a particular hospital is 150 and number of nurses starting duty from Monday in shift I is 70, in shift II is 35 and number of nurses starting duty from Tuesday in shift I is 30 , in shift II is 15 and there is no need of nurses starting duty from Wednesday, Thursday, Friday, Saturday and Sunday.

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