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Fixed Point Theorems taking Concept of Fuzzy Sets

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Abstract:

The target of this manuscript is to establish some fixed point results using generalized CLR property under integral type contractive condition in fuzzy- metric space.

Keywords: *Fuzzy- metric space, mapping for weakly -compatible, CLR property.*

INTRODUCTION

The fuzzy metric spaces [f-m space] was introduced by [7] using the basic concepts given by [13], and [6] modified this concept introduced by [7]. [1] gave the idea of property E.A. for a pair of self mappings which contains the class of non-compatible mappings. The concept of common limit range property [CLR} is proved by [12] as modification of E.A. property , this modification not need the condition of closedness. [5] specified the joint common limit in the range of mappings (JCLR) property in f- m space. Some fixed point results established by [10] for new type of common limit in the range property in metric space. Branciari [2] established integral type mappings for complete metric spaces. By the motivation of all above the work is done in this paper in f-m spaces which contains new type of common limit in the range property under integral type contractive condition.

2. Preliminaries

The reader can see the basic definitions , concepts and examples for f- m spaces from the work of [1], [6], [7],[8].[9] [10], [13].Throughout the work in this manuscript $(X, M,*)$ is taken as f-m space

$$\lim_{t\to\infty} M(x, y, t) = 1, \text{ for all } x, y \in X.$$

The definition of C L R$_g$ property can be seen in the work of [12] Popa and Patriciu [10] introduced a new type of common limit range property for self-mappings in metric space as
Motivated form above of [10], (In f-m space) we can have:

Definition Let $(X, M,*)$ $is\ a\ f\ m\ spaces$. A, S and T are mappings from X to X . (A, S) is said to satisfy common limit range property with respect to T (shortly CLR$_{(A,S)T}$ property), if there exists a sequence x_n in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = t$, for some $t \in S(X) \cap T(X)$.

Lemma [9]: If for all $x, y \in X, t > 0$ and $k \in (0, 1)$, $M(x, y, kt) \geq M(x, y, t)$, then $x = y$. For another basic result for common fixed point one can see [10].Now the following results are used for the new theorem

(δ_1) A Lebesque-integrable mapping
$\emptyset : R^+ \to R^+$

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is taken to be nonnegative- summable :

$$\int_0^{\in} \emptyset(t)dt \geq 0, \text{ where } \in > 0.$$

(δ_2) Let (X, M, *), be a f- m spaces and S & T, A& B, mappings from X to X . It is expressed as

$$m(x,y,t) = min\left\{\begin{matrix} M(Sx,Ty,t), M(By,Ty,t), \\ M(Ax,Ty,t), M(By,Sx,t), M(Ax,Sx,t)\end{matrix}\right\},$$

for all $x, y \in X,\ t > 0$.

3. Main result

Theorem 3.1: Let $(X, M, *)$ *be a f m space*. Self-mappings A,B.S and T are defined on X :

(3.1) $x, y \in X,\ t > 0$ and $k \in (0,1)$,

$$\int_0^{M(Ax,By,kt)} \emptyset(t)dt \geq \int_0^{m(x,y,t)} \emptyset(t)dt\,;$$

where $m(x,y,t)$ is defined in (δ_2) and if (A,S) and T enjoys the $\mathrm{CLR}_{(A,S)T}$ property, then $C(A,S) \neq \emptyset$ and $C(B,T) \neq \emptyset$.

Common fixed point will be proved for A, B, S, T if $(A\ \&S)$, $(B\&T)$ are weakly compatible.

Proof: Since T *and* (A,S) enjoys $\mathrm{CLR}_{(A,S)T}$ property, then a sequence x_n in X for

$$lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z, \text{ for some } z \in S(X) \cap T(X).$$

Since $z \in T(X)$, there exists $u \in X$ such that $z = Tu$. Using (3.1), we have

$$\int_0^{\mathrm{M}(\mathrm{A}x_n,\mathrm{Bu,kt})} \emptyset(t)dt \geq \int_0^{m(x_n,u,t)} \emptyset(t)dt\,;$$

where $m(x_n,u,t) = min\left\{\begin{matrix} M(Sx_n,Tu,t), M(Ax_n,Sx_n,t), M(Bu,Tu,t), \\ M(Ax_n,Tu,t), M(Bu,Sx_n,t)\end{matrix}\right\},$

If $n \to \infty$

$$\int_0^{\mathrm{M(z,Bu,kt)}} \emptyset(t)dt \geq \int_0^{m(x_n,u,t)} \emptyset(t)dt\,;$$

where $m(x_n,u,t) = min\left\{\begin{matrix} M(z,z,t), M(z,z,t), M(Bu,z,t), \\ M(z,z,t), M(Bu,z,t)\end{matrix}\right\},$

$$= min\{1,1, M(Bu,z,t),\ 1, M(Bu,z,t)\} = M(Bu,z,t).$$

i.e.

$$\int_0^{\mathrm{M(z,Bu,kt)}} \emptyset(t)dt \geq \int_0^{\mathrm{M(z,Bu,t)}} \emptyset(t)dt\,;$$

then we have,

$$M(z,Bu,kt) \geq M(z,Bu,t).$$

we have, $Bu = z = Tu$. Therefore $C(B,T) \neq \emptyset$.

Since $z \in S(X)$, *For* $v \in X$ there will be $z = Sv$. Using (3.1),

$$\int_0^{\mathrm{M(A}v\mathrm{,Bu,kt)}} \emptyset(t)dt \geq \int_0^{m(v,u,t)} \emptyset(t)dt\,;$$

where $m(v,u,t) = min\left\{\begin{matrix} M(Sv,Tu,t), M(Av,Sv,t), M(Bu,Tu,t), \\ M(Av,Tu,t), M(Bu,Sv,t)\end{matrix}\right\}.$

$$\int_0^{\mathrm{M(A}v\mathrm{,z,kt)}} \emptyset(t)dt \geq \int_0^{m(v,u,t)} \emptyset(t)dt\,;$$

where $$m(v,u,t) = min\begin{Bmatrix} M(z,z,t), M(Av,z,t), M(z,z,t), \\ M(Av,z,t), M(z,z,t) \end{Bmatrix},$$
$$= min\{1, M(Av,z,t), 1, M(Av,z,t), 1\} = M(Av,z,t).$$
i.e.
$$\int_0^{M(Av,z,kt)} \emptyset(t)dt \geq \int_0^{M(Av,z,t)} \emptyset(t)dt\,;$$
then we have, $\mathrm{M}(\mathrm{A}v, z, \mathrm{kt}) \geq \mathrm{M}(\mathrm{A}v, z, \mathrm{t})$.
we have, $Av = z = Sv$. Therefore $C(A,S) \neq \emptyset$.

Hence, $z = Av = Sv = Bu = Tu$ and z is coincidence point of (A,S) & (B,T).

Uniqueness : Let p be another point of coincidence of (A,S), i.e., $p = Aw = Sw$. Using (3.1), we get
$$\int_0^{M(Aw,Bu,kt)} \emptyset(t)dt \geq \int_0^{m(w,u,t)} \emptyset(t)dt\,;$$
where $$m(w,u,t) = min\begin{Bmatrix} M(p,z,t), M(p,p,t), M(z,z,t), \\ M(p,z,t), M(p,p,t) \end{Bmatrix}.$$
$$= min\{M(p,z,t), 1, 1, M(p,z,t), 1\} = M(p,z,t).$$
i.e.
$$\int_0^{M(p,z,kt)} \emptyset(t)dt \geq \int_0^{M(p,z,t)} \emptyset(t)dt\,;$$
$$M(p,z,kt) \geq M(p,z,t).$$

It is clear by the basic results that $p = z$. Thus, z is the unique coincidence point of (A,S). Similarly, using (3.1) it is easy to see that z is the unique coincidence point of (B,T).

Further, by the weakly compatibleness of (A,S) and (B,T) and then by basic result , z will become as unique common fixed point of A, B, S and T.
Special Case: If we take $\emptyset(t) = 1$, for all t $\in R^+$ in Theorem 3.1, then we have following:

Corollary 3.2: Let A, B, S and T be self-mappings of a fuzzy metric space $(X, M, *)$ satisfying:

(3.2) for all $x, y \in X$, $t > 0$ and for a number $k \in (0,1)$,
$$M(Ax, By, kt) \geq m(x,y,t);$$
where $m(x,y,t)$ is defined in (δ_2) and if (A,S) and T enjoys the $\mathrm{CLR}_{(A,S)T}$ property, then $C(A,S) \neq \emptyset$ and $C(B,T) \neq \emptyset$. Moreover, if (A,S) and (B,T) are weakly compatible then unique common fixed point will be obtained by A, B, S and T

Proof: follows form Theorem 3.1 by taking $\emptyset(t) = 1$, for all t $\in R^+$.

Remark 3.3: Note that our result requires neither the completeness of the subspace nor the containment of ranges.

Theorem 3.4: Let $(X, M, *)$ *be a f m space*. Self-mappings A,B.S and T are defined on X :

(3.4) for all $x, y \in X$, $t > 0$ and for a number $k \in (0,1)$,
$$\int_0^{M(Ax,By,kt)} \emptyset(t)dt \geq \int_0^{\gamma(m(x,y,t))} \emptyset(t)dt\,;$$
where $\gamma : [0,1] \to [0,1]$ is a continuous - nondecreasing $\gamma(s) > s$, $s \in (0,1)$ and if T & (A,S) and enjoys the $\mathrm{CLR}_{(A,S)T}$ property, then $C(A,S) \neq \emptyset$ and $C(B,T) \neq \emptyset$.
Unique common fixed point will be obtained in X for A,B,S,T , if (A,S) and (B,T) are weakly compatible

Proof: Since (A,S) and T enjoys $\mathrm{CLR}_{(A,S)T}$ property, then there exists a sequence x_n in X such that

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$$lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z, \text{ for some } z \in S(X) \cap T(X).$$

Since $z \in T(X)$, there exists $u \in X$ such that $z = Tu$. Using (3.4), we have

$$\int_0^{M(Ax_n,Bu,kt)} \emptyset(t)dt \geq \int_0^{\gamma(m(x_n,u,t))} \emptyset(t)dt\,;$$

where $\gamma(m(x_n,u,t)) = \gamma\left(min\left\{\begin{matrix} M(Sx_n,Tu,t), M(Ax_n,Sx_n,t), M(Bu,Tu,t), \\ M(Ax_n,Tu,t), M(Bu,Sx_n,t) \end{matrix}\right\}\right).$

$taking\ n \to \infty$, we get,

$$\int_0^{M(z,Bu,kt)} \emptyset(t)dt \geq \int_0^{\gamma(m(x_n,u,t))} \emptyset(t)dt\,;$$

where $\gamma(m(x_n,u,t)) = \gamma\left(min\left\{\begin{matrix} M(z,z,t), M(z,z,t), M(Bu,z,t), \\ M(z,z,t), M(Bu,z,t) \end{matrix}\right\}\right),$

$= \gamma(min\{1,1,M(Bu,z,t),1,M(Bu,z,t)\}),$

$= \gamma(M(Bu,z,t)) > M(Bu,z,t).$

i.e.

$$\int_0^{M(z,Bu,kt)} \emptyset(t)dt \geq \int_0^{\gamma(m(x_n,u,t))} \emptyset(t)dt > \int_0^{M(z,Bu,t)} \emptyset(t)dt\,;$$

then

$M(z,Bu,kt) \geq M(z,Bu,t)$. we have, $Bu = z = Tu. So\ C(B,T) \neq \emptyset$.

Since $z \in S(X)$, there exists $v \in X$ such that $z = Sv$. Using (3.4), we have

$$\int_0^{M(Av,Bu,kt)} \emptyset(t)dt \geq \int_0^{\gamma(m(v,u,t))} \emptyset(t)dt\,;$$

where $\gamma(m(v,u,t)) = \gamma\left(min\left\{\begin{matrix} M(Sv,Tu,t), M(Av,Sv,t), M(Bu,Tu,t), \\ M(Av,Tu,t), M(Bu,Sv,t) \end{matrix}\right\}\right).$

$$\int_0^{M(Av,z,kt)} \emptyset(t)dt \geq \int_0^{\gamma(m(v,u,t))} \emptyset(t)dt\,;$$

where $\gamma(m(v,u,t)) = \gamma\left(min\left\{\begin{matrix} M(z,z,t), M(Av,z,t), M(z,z,t), \\ M(Av,z,t), M(z,z,t) \end{matrix}\right\}\right),$

$= \gamma(min\{1,M(Av,z,t),1,M(Av,z,t),1\}),$

$= \gamma\big(M(Av,z,t)\big) > M(Av,z,t).$

i.e.

$$\int_0^{M(Av,z,kt)} \emptyset(t)dt \geq \int_0^{\gamma(m(v,u,t))} \emptyset(t)dt > \int_0^{M(Av,z,t)} \emptyset(t)dt\,;$$

$M(Av,z,kt) \geq M(Av,z,t).$

Since, $Av = z = Sv$. Hence $C(A,S) \neq \emptyset$.

So $z = Av = Sv = Bu = Tu$ and z is coincidence point of (A,S) & (B,T).

Uniqueness for z can be proved easily as

Let p be another coincidence point of (A,S), i.e., $p = Aw = Sw$. Using (3.4), we get

$$\int_0^{M(Aw,Bu,kt)} \emptyset(t)dt \geq \int_0^{\gamma(m(w,u,t))} \emptyset(t)dt\,;$$

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where $\gamma\big(m(w,u,t)\big) = \gamma\left(min\left\{\begin{matrix} M(p,z,t), M(p,p,t), M(z,z,t), \\ M(p,z,t), M(p,p,t) \end{matrix}\right\}\right),$

$$= \gamma(min\{M(p,z,t), 1, 1, M(p,z,t), 1\}),$$

$$= \gamma\big(M(p,z,t)\big) > M(p,z,t).$$

i.e.

$$\int_0^{\mathrm{M(p,z,kt)}} \emptyset(t)dt \geq \int_0^{\gamma(m(w,u,t))} \emptyset(t)dt > \int_0^{M(p,z,t)} \emptyset(t)dt\,;$$

then we have,

$$M(p,z,kt) \geq M(p,z,t).$$

It is clear that , $p = z$. This proves the uniqueness of (A,S). Similarly it can be done for (B,T).

Further, if (A,S) and (B,T) are weakly compatible, then is the unique common fixed point of A, B, S and T.

Conclusion

The fixed point theorems in f-m space using generalized CLR_g property under integral type contractive condition for four mappings is established. The results are proved without using continuity of the involved mappings, completeness of the subspace and containment of ranges.

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