

Effects of Variable Viscosity and Thermal Conductivity on MHD Convective Heat Transfer of Immiscible fluids with Heat Source

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Abstract

The magneto-fluid dynamics convective transfer of heat in two immiscible fluids with vertical channel are presented due to the effect of variable viscosity, thermal conductivity and heat source on these fluids. The heat transfer of these problems were transforming into dimensionless form by the differential equations which have been governing the flow. These governing boundary value problems so obtained were solved numerically by using Runge-Kutta 6th order method. The effects of these pertaining parameters on velocity and temperature fields are studied and their results have been presented graphically. The skin-friction and Nusselt number values have been computed and presented in a tabular form.

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I. INTRODUCTION

The two-phase fluid flow phenomena shows importance in pipe flows, fluidized beds, sedimentation process, gas purifications, transport processes and shock waves. Ramana Murthy and Srinivas [12] were analyzed in the study of heat transfer through flow of two immiscible combine stress fluid under an imposed crossways magnetic field with the laws of thermodynamics. Srinivas et al. [17] investigated the effect of radioactive heat transfer on entropy making of two immiscible fluid types which lies between two horizontal parallel plates. Mehdi-Nejad et al. [7] intended heat transfer across an interface formed between the two immiscible fluids and also Srinivas et al. [16] during his study found that effects of the heat transfer in immiscible micropolar and viscous fluids in a upright channel.

Fluids with variable viscosity and thermal conductivity have ample range of applications in many Engineering projects. For instance, these operational situations lubricants can be examined to high temperatures, pressure, shear rate etc, Myers et al. [10] concluded that these effects will influence the fluid properties. An analytical study over fluid flows and heat transfer under stretched sheet through thermal conductivity variables is reported with Subhas Abel[18]. The effect of changeable viscosity and thermal conductivities were studied by Hazarika

and Santana Hazarik[4], over stretched surface through injection with heat radiation. Surajit Dutta [19] emphasized the effects of the same on the flow in continuously moving surface by means of application in transverse magnetic field. Salawu and Dada[13] were investigated the effect of variable viscosity and thermal conductivity resting on radioactive heat transfer through inclined magnetic field along with dissipation in Darcy medium. In view of Anjali Devi and Prakash in [1], presented the problem of variable viscosity and thermal conductivity which cover the result on slander stretch sheet. Dulal Pal and Hiranmoy Mondal [3] enclosed to analyze the influence into variable viscosity and thermal conductivity going on non free convective flow of stretch sheet. Lai with Kulacki[6], Mohamed E.Ali[8], Hossain et al. [5] comprise on study which effect the same lying on flow and heat transfer and found that here be a significant outcome on the fluid flow.

Siddheshwar and Mahabaleswar[15] considered on a visco-elastic fluid in the occurrence of heat basis and also Mukhopadhyay along with Layek[9] were presented free convection flow in accordance by means of the heat transfer of fluids for changeable viscosity over a porous stretch upright surface in existence of thermal emission. An investigator Oluwole Daniel Makinde[11] abstract the effect of radiation, viscosity of variable, and suction and insertion. The result of heat basis might play a significant position in various heat

transfer analysis. In the study of fluid flow according to Sankar in [14], heat transfer in annular enclosed space with heat source. Thus above listed authors have been observing that the significant variation of momentum and temperature are taking place in effect of parameters viscosity variable and thermal conductivity.

Recently, remarkable attention has been shown on the study of convection in immiscible fluid flow suitable fast growth in fluid mechanics research and its significance here various branches of Science. An effect of variables viscosity of fluid and thermal conductivity on heat transfer through the immiscible fluid flow with heat source has not been studied as per the best of my knowledge and available literature. In view of all the above versions, the main inspiration of the present paper has been to study the effect of variables fluid viscosity and thermal conductivity on magneto hydro dynamics (MHD) flow and heat transfer of fluid through vertical channel by heat source.

II. MATHEMATICAL FORMATION

In this paper the formation of the present problem was considered as two infinite parallel plates which have been positioned at $Y = -d_1$ and $Y = d_2$ along with Y -direction firstly as shown in Fig. 1 and both plates were placed in isothermal position with different temperatures t_1 and t_2 respectively. So in this the distance represents left region and another distance represents right region. Here the left region has to be filled with the fluid have density, viscosity and again in the right region it has to be filled with another fluid have density, viscosity. Because of buoyant force, fluid moves in the channel.

Hence to build up the governing equations for this present problem has been considered as follows under the assumption of:

1. The flow has to be assumed in one-dimensional, steady, laminar, immiscible and incompressible.
2. The transport property of left region fluid and right region fluids is understood as constant.
3. Because of buoyant force, fluid moves in the channel
4. The fluid flow has to be completely developed.
5. The flow, temperature and species concentration has to be continuous at the interface.
6. Each of the walls are isothermal and having constant species concentration and, .
7. The flow has to follow Boussinesq approximation

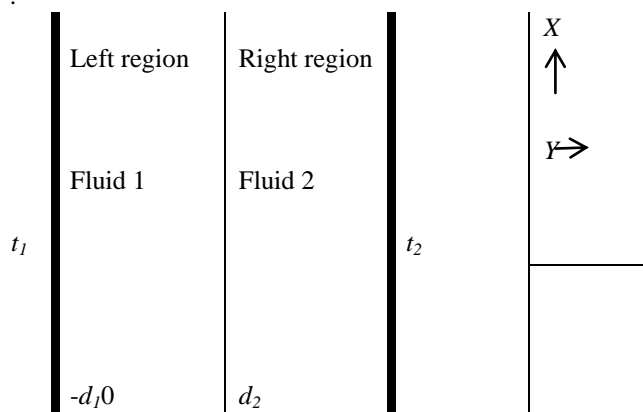


Fig. (1): Geometry of the Problem

III. GOVERNING EQUATIONS

Left region :

$$\frac{dU_1}{dY} = 0 \quad [\text{Continuity}] \quad (1)$$

$$\rho_1 = \rho_0[1 - \beta_{1T}(t_1 - t_0)] \quad (2)$$

[State]

$$\frac{1}{\rho_1} \left[\frac{\partial \mu_1}{\partial Y} \frac{\partial U_1}{\partial Y} + \mu_1 \frac{\partial^2 U_1}{\partial Y^2} \right] + g\beta_{1T}(t_1 - t_0) - \frac{\sigma \beta_0^2 U_1}{\rho} = 0 \quad (3)$$

[Momentum]

$$\frac{1}{\rho C_p} \left[\frac{\partial}{\partial Y} k \frac{\partial t_1}{\partial Y} + Q(t_1 - t_0) + \mu_1 \left(\frac{\partial U_1}{\partial Y} \right)^2 \right] = 0 \quad (4)$$

[Energy]

Right region :

$$\frac{dU_2}{dY} = 0 \quad [\text{Continuity}] \quad (5)$$

$$\rho_2 = \rho_0[1 - \beta_{2T}(t_2 - t_0)] \quad [\text{State}] \quad (6)$$

$$\frac{1}{\rho_2} \left[\frac{\partial \mu_2}{\partial Y} \frac{\partial U_2}{\partial Y} + \mu_2 \frac{\partial^2 U_2}{\partial Y^2} \right] + g\beta_{2T}(t_2 - t_0) - \frac{\sigma \beta_0^2 U_2}{\rho} = 0$$

(7)

$$[\text{Momentum}] \quad \frac{1}{\rho_2 C_p} \left[\frac{\partial}{\partial Y} K_2 \frac{\partial t_2}{\partial Y} + Q(t_2 - t_0) + \mu_2 \left(\frac{\partial U_2}{\partial Y} \right)^2 \right] = 0$$

(8)

[Energy]

To get the numerical solution for the generated system of equations (1) to (8), the initial and boundary conditions defined by Arimen (2) as considered:

$$U_1 = 0 \text{ at } Y = -d_1, \quad U_2 = 0 \text{ at } Y = d_2, \quad U_1(0) = U_2(0),$$

$$t = t_1 \text{ at } Y = -d_1, \quad t = t_2 \text{ at } Y = d_2, \quad t_1(0) = t_2(0),$$

$$\frac{dU_1(0)}{dY} = 0, \quad \frac{dU_2(0)}{dY} = 0, \quad \frac{dt_1(0)}{dY} = 0, \quad \frac{dt_2(0)}{dY} = 0.$$

The following variables are to be used in forming the system of Equations from (1) to (8) in to dimensionless form:

$$y = \frac{Y}{d_1} \text{ (Left region)}, \quad u_1 = \frac{U_1}{U_0}, \quad \theta_1 = \frac{t_1 - t_0}{\Delta t}, \quad M = \frac{\sigma B_0^2 d_1^2}{\mu_\infty}$$

$$\text{(Magnetic field parameter)} \quad Gr = \frac{g \beta_{1T} \Delta t h_1^3}{\nu_1^2} \text{ (Grashof number),}$$

$$\nu_1 = \frac{\mu_\infty}{\rho_1}, \quad R = \frac{U_0 d_1}{\nu_1} \text{ (Reynolds number), } Pr = \frac{\nu_1}{\alpha_0},$$

$$Ec = \frac{U_0^2}{C_{p1} \Delta t} \text{ (Eckert number)}, \quad y = \frac{Y}{d_2} \text{ (Right region),}$$

$$u_2 = \frac{U_2}{U_0}, \quad \theta_2 = \frac{t_2 - t_0}{\Delta t}, \quad d = \frac{d_1}{d_2}, \quad b = \frac{\beta_{1T}}{\beta_{2T}}, \quad \rho = \frac{\rho_1}{\rho_2}, \quad (\rho_2 > \rho_1),$$

$$Cp = \frac{C_{p1}}{C_{p2}}, \quad \nu_2 = \frac{\mu_\infty}{\rho_2}, \quad \alpha_2 = \frac{k_2}{\rho_2 C_{p2}}, \quad U_0 \text{ is characteristic}$$

velocity of fluid.

As the crux of this work it has to study the effect of variables viscosity and thermal conductivity, which are to be considered in the function of temperature as

$$\mu_1 = -\frac{\mu_\infty \theta_r}{\theta_1 - \theta_r} \quad \text{and} \quad \mu_2 = -\frac{\mu_\infty \theta_r}{\theta_2 - \theta_r}$$

For left region and right region respectively where θ_r is variable viscosity parameter of left region and right region respectively. These variable thermal conductivities for left and right regions are taken as $\alpha_1 = \alpha_0(1 + \beta\theta_1)$ and $\alpha_2 = \alpha_0(1 + \beta\theta_2)$ where β is variable conductivity parameter of region 1 and region 2 respectively.

Hence the governing equations will become in tonon dimensional form as:

Left region:

$$\frac{\theta_r}{(\theta_1 - \theta_r)^2} \frac{\partial \theta_1}{\partial y} \frac{\partial u_1}{\partial y} - \frac{\theta_r}{(\theta_1 - \theta_r)} \frac{\partial^2 u_1}{\partial y^2} + \frac{G_r}{R} \theta_1 - \text{Mu}_1 = 0 \quad (9)$$

$$\beta \left(\frac{\partial \theta_1}{\partial y} \right)^2 + (1 + \beta\theta_1) \frac{\partial^2 \theta_1}{\partial y^2} + Q_h \theta_1 - \text{Pr Ec} \left(\frac{\theta_r}{\theta_1 - \theta_r} \right) \left(\frac{\partial u_1}{\partial y} \right)^2 = 0 \quad (10)$$

Right region:

$$\frac{\theta_r}{(\theta_2 - \theta_r)^2} \frac{\partial \theta_2}{\partial y} \frac{\partial u_2}{\partial y} - \frac{\theta_r}{(\theta_2 - \theta_r)} \frac{\partial^2 u_2}{\partial y^2} + \frac{1}{\rho b h^2} \frac{G_r}{R} \theta_2 - \frac{\text{Mu}_2}{h^2} = 0 \quad (11)$$

$$\beta \left(\frac{\partial \theta_2}{\partial y} \right)^2 + (1 + \beta\theta_2) \frac{\partial^2 \theta_2}{\partial y^2} + Q_h \theta_2 - \left(\frac{\theta_r}{\theta_2 - \theta_r} \right) a \text{Pr Ec} \left(\frac{\partial u_2}{\partial y} \right)^2 = 0 \quad (12)$$

The dimensionless boundary and interface conditions thus formed are:

$$u_1(-1) = 0, u_2(1) = 0, u_1(0) = u_2(0), u_1'(0) = 0, \theta_1'(0) = 0 \\ \theta_1(-1) = 1, \theta_2(1) = 0, \theta_1(0) = \theta_2(0). \quad (13)$$

IV. SOLUTIONS OF THE PROBLEM

These governing non linear differential equations from (9) to (12) of momentum and energy have to be solved subjected to their boundary conditions (13). Those obtained equations represent a system of non-linear coupled boundary value problem was solved numerically by using Runge- Kutta 6th order method with the help of software Mathematica 10.4. The Nusselt Number and shearing stress on both walls were calculated in use of following expression:

$$Nu_1 = \left[\frac{\partial \theta_1}{\partial y} \right]_{y=-1}, \quad Nu_2 = \left[\frac{\partial \theta_2}{\partial y} \right]_{y=1}, \quad St_1 = \left[\frac{\partial u_1}{\partial y} \right]_{y=-1}, \\ St_2 = \left[\frac{\partial u_2}{\partial y} \right]_{y=1}.$$

V. RESULT AND DISCUSSION

Thus numerical solution of the set of equations has been analyzed for various values of governing parameters and their result were presented graphically. Grashof number (Gr), Reynolds number (R), Magnetic field parameter (M), Eckert number (Ec), variable viscosity parameter (θ_r) thermal conductivity variable parameter (β) and heat source parameter (Q_h) were fixed when $Gr=3, R=3, M=3, Ec=0.001, \theta_r = -0.8, \beta$

$= 0.6, Q_h = 0.2$ for every profile except in case of unstable parameter.

Thus curves in Fig. 2-7 show the result of different parameters over velocity. Through Grashof number heat transfer indicate the relative effect of the thermal buoyancy force toward the viscous hydrodynamic force in the boundary layer. From Fig. 2, it has to be observed that the thermal buoyancy control the viscous force significantly. The influence with Reynolds number over velocity has been shown in Fig. 3, it was observed to be inertial force dominates the viscous force. Fig. 4 represent the result of Magnetic field parameter and it shows that momentum have been decreasing due to the Lorentz forces which have the capacity to act against the flow. In effect of variable viscosity have been studied and depict in Fig. 5, it shows that as decreases the velocity in both the regions decreases.

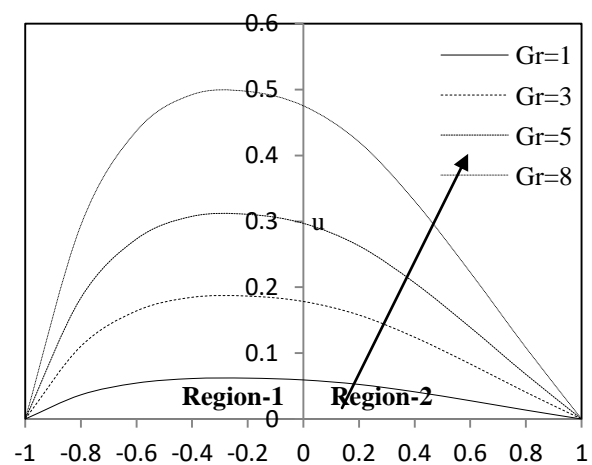
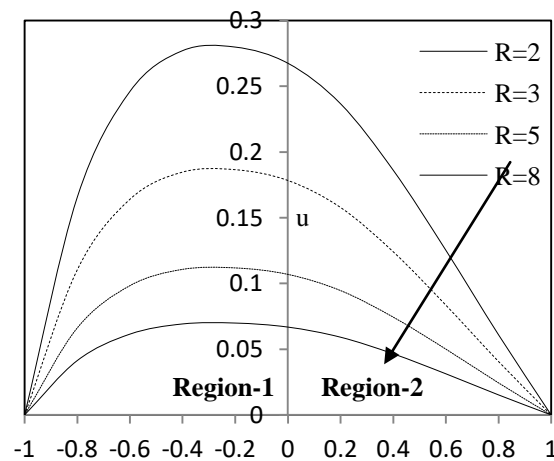


Fig.(2): Velocity Profile for different Gr



Velocity Profile for different R

Fig.(3):

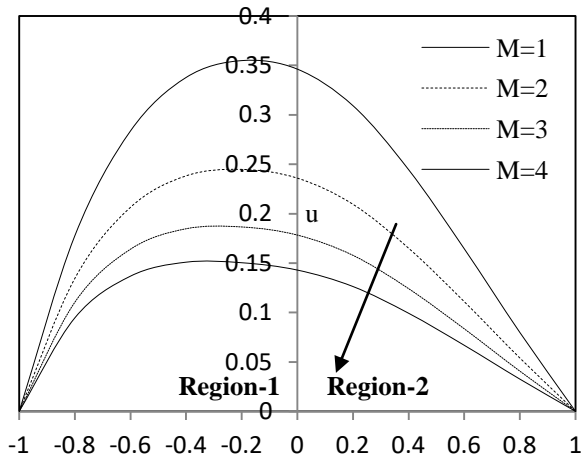


Fig.(4): Velocity Profile for different M

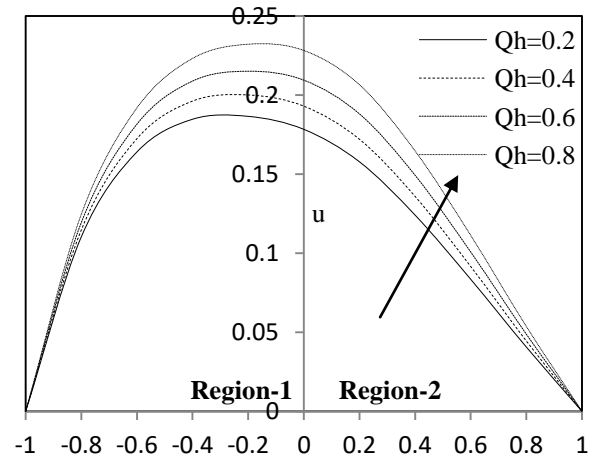


Fig.(7): Velocity Profile for different Q_h

Due to the result of thermal conductivity parameter over velocity has depicted in the Fig. 6, it shows that the boost in non dimensional parameter will result to enhance the momentum for fluids in both regions of the channel due to lower viscosity. The heat source parameter effect is presented in Fig. 7, from this it clear that the more heat source the more momentum.

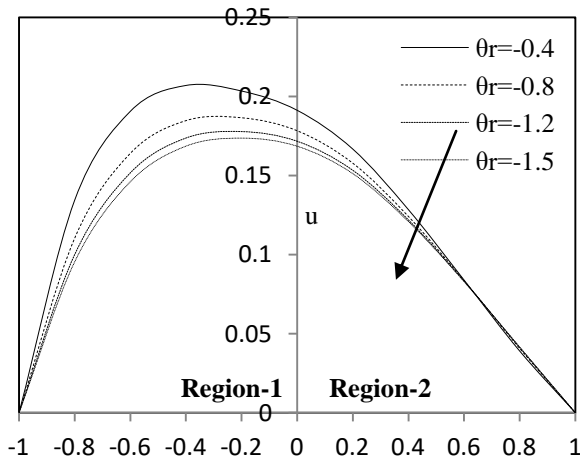


Fig.(5): Velocity Profile for different θ_r

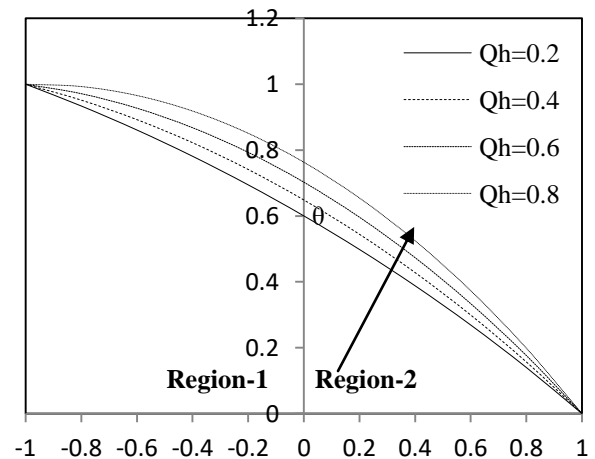


Fig.(8): Temperature Profile for different Q_h

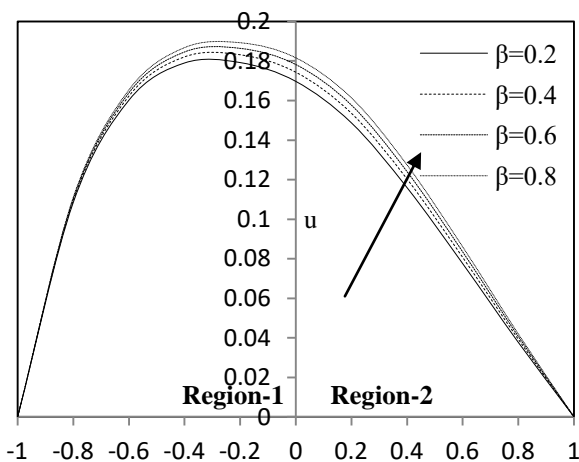


Fig.(6): Velocity Profile for different β

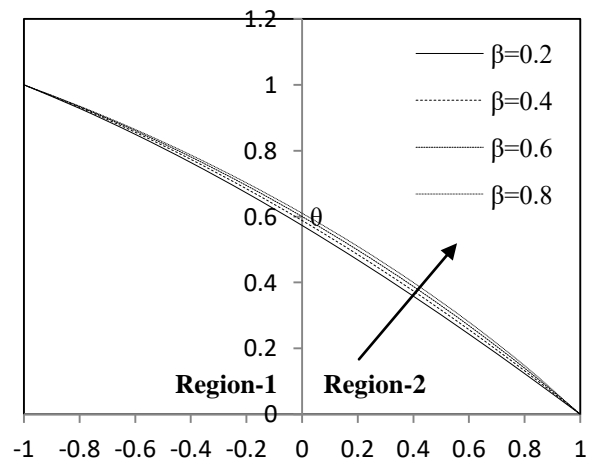


Fig.(9): Temperature Profile for different β

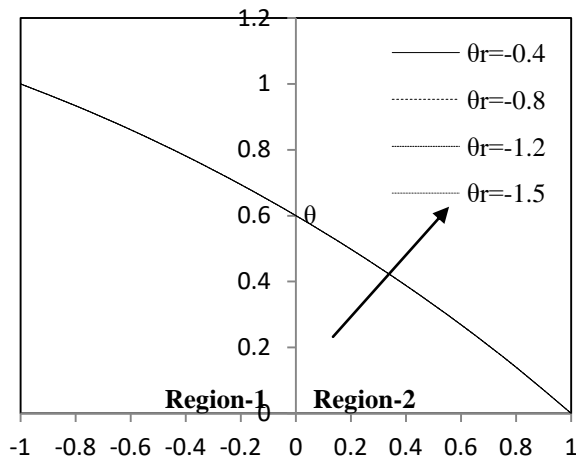


Fig. (10): Temperature Profile for different θ_r

Table 1 show the Shear stress value and Nusselt Number with the effect of all governing parameter. From this table, it was observed that the total Shear stress increase with increase in buoyancy on both the borders (Boundaries of the channel) and . The enhancement for inertial force along with Lorentz force reduces the shearing rate on both walls. Interestingly the increase of viscosity reduces the absolute shearing lying on the left boundary and enhances on the the right boundary. Thus heat source enhancement reduces the absolute heat transfer rate on the hot wall and enhance on the cold wall. So, viscous dissipation reduces the heat transfer rate on both walls. Now the enhancement of thermal conductivity reduces the rate of heat transfer on hot wall which enhances the cold wall.

Table-I Shear Stress and Nusselt Number Values

Gr	R	M	Qh	Ec	β	θ_r	St-I	St-II	Nu-I	Nu-II
1	3	3	0.2	0.001	0.6	-0.8	0.250687	-0.06332	-0.314678	-0.730409
3	3	3	0.2	0.001	0.6	-0.8	0.75206	-0.18995	-0.314675	-0.730401
5	3	3	0.2	0.001	0.6	-0.8	1.25343	-0.31657	-0.314669	-0.730387
8	3	3	0.2	0.001	0.6	-0.8	2.00549	-0.5065	-0.314654	-0.730352
3	2	3	0.2	0.001	0.6	-0.8	1.12809	-0.28492	-0.314671	-0.730391
3	3	3	0.2	0.001	0.6	-0.8	0.75206	-0.18995	-0.314675	-0.730401
3	5	3	0.2	0.001	0.6	-0.8	0.451236	-0.11397	-0.314677	-0.730407
3	8	3	0.2	0.001	0.6	-0.8	0.282022	-0.07123	-0.314678	-0.730408
3	3	1	0.2	0.001	0.6	-0.8	1.10897	-0.37208	-0.314666	-0.730381
3	3	2	0.2	0.001	0.6	-0.8	0.881052	-0.25175	-0.314673	-0.730396
3	3	3	0.2	0.001	0.6	-0.8	0.75206	-0.18995	-0.314675	-0.730401
3	3	5	0.2	0.001	0.6	-0.8	0.666985	-0.15253	-0.314676	-0.730404
3	3	3	0.2	0.001	0.6	-0.8	0.75206	-0.18995	-0.314675	-0.730401
3	3	3	0.4	0.001	0.6	-0.8	0.771769	-0.20607	-0.214998	-0.823603
3	3	3	0.6	0.001	0.6	-0.8	0.79325	-0.22398	-0.105835	-0.932018
3	3	3	0.8	0.001	0.6	-0.8	0.8167	-0.24388	0.0144609	-1.05854
3	3	3	0.2	0.0001	0.6	-0.8	0.75206	-0.18995	-0.314678	-0.730409
3	3	3	0.2	0.001	0.6	-0.8	0.75206	-0.18995	-0.314675	-0.730401
3	3	3	0.2	0.003	0.6	-0.8	0.752059	-0.18994	-0.314668	-0.730385
3	3	3	0.2	0.005	0.6	-0.8	0.752059	-0.18994	-0.314661	-0.730369
3	3	3	0.2	0.001	0.2	-0.8	0.743103	-0.17719	-0.338921	-0.626155
3	3	3	0.2	0.001	0.4	-0.8	0.748101	-0.18397	-0.324945	-0.678441
3	3	3	0.2	0.001	0.6	-0.8	0.75206	-0.18995	-0.314675	-0.730401
3	3	3	0.2	0.001	0.8	-0.8	0.75527	-0.19525	-0.306827	-0.782102
3	3	3	0.2	0.001	0.6	-0.4	0.971967	-0.16954	-0.314676	-0.730402
3	3	3	0.2	0.001	0.6	-0.8	0.75206	-0.18995	-0.314675	-0.730401
3	3	3	0.2	0.001	0.6	-1.2	0.665389	-0.19789	-0.314675	-0.730402
3	3	3	0.2	0.001	0.6	-1.5	0.628013	-0.20117	-0.314674	-0.730402

VI. CONCLUSIONS

- The velocity and temperature are more in region 1 than in region 2 due to smaller density.
- Deviation of viscosity and thermal conductivity enhance the flow significantly.
- The thermal conductivity variation is could not enhance the temperature notably.
- Shear rate was more on the hot plate than on cold plate and also the absolute rate of heat transfer was more on cold plate than on hot plate.
- The variable viscosity and thermal conductivity significantly enhance the rate of shear stress and rate of heat transfer.
- The increase in thermal conductivity and viscosity of the fluid heat transfer is more significant even in immiscible flow.

REFERENCES

- Anjali Devi, S.P. Prakash , M , Temperature dependent viscosity and thermal conductivity effects on hydro magnetic flow over a slandering stretching sheet, Journal of the Nigerian Mathematical Society , Vol. 34, No. 3, 2015, pp.318–330.
- Arimen, T. Truk, M.A and Sylvester, Micro continuum fluid mechanics: A review, International Journal of Engineering Science, Vol. 11, No. 8, 1973, pp.905–930.
- Dulal Pal , Hiranmoy Mondal, Influence of Variable Viscosity on Hydromagnetic Non- Darcy Convective-Radiative Heat Transfer Over a Stretching Sheet with Non-Uniform Heat Source/Sink, International Journal for Computational Methods in Engineering Science and Mechanics, Vol. 15, 2014 No. 6, pp.490–498.
- Hazarika, G.C. and Santana Hazarika, Effects of Variable Viscosity and ThermalConductivity on Magneto

- hydrodynamics mixed Convective Flow over a Stretching Surface with Radiation, International Journal of Scientific Research Engineering and Technology , Vol. 4, No. 9, 2015pp.799–815.
5. Hossain, M.A. Kabir, S. and Rees, D.A. S, O, Natural convection of fluid with variable viscosity from a heated vertical wavy surface, Zeitschrift für angewandte Mathematik und Physik , Vol. 53, No. 1, 2002 pp.48–57.
 6. Lai, F. C. and Kulacki, F. A, The effect of variable viscosity on convective heat transfer along a vertical surface in a saturated porous medium, International Journal of Heat and Mass Transfer, Vol. 15, No. 6, 1990pp.1021–1038.
 7. Mehdi-Nejad, V. Mostaghimi, J. and Chandra, S. Modelling heat transfer in two-fluid interfacial flows, International journal for numerical methods in engineering, Vol. 61, No. 7, 2004 , pp.1028–1048.
 8. Mohamed Ali, E., The effect of variable viscosity on mixed convection heat transfer along a vertical moving surface, International Journal of Thermal Sciences, Vol. 45, No. 1, 2006, pp.60–69.
 9. Mukhopadhyay, S. and Layek, G.C., Effects of thermal radiation and variable fluid viscosity on free convective flow and heat transfer past a porous stretching surface, International Journal of Heat and Mass Transfer, Vol. 51, No. 9-10, 2008 pp.2167–2178.
 10. Myers, T.G. Charpin, J.P.F Tshehla, M.S , The flow of a variable viscosity fluid between parallel plates with shear heating', Applied Mathematical Modelling , Vol. 30, No. 9, .2006 pp.799–815.
 11. Oluwale Daniel Makinde, Effect of variable viscosity on thermal boundary layer over a permeable flat plate with radiation and a convective surface boundary condition, Journal of Mechanical Science and Technology, Vol. 26, No. 5, 2012, pp.1615–1622.
 12. Ramana Murthy, J.V. and Srinivas, First and Second Law Analysis for the MHD Flow of Two Immiscible Couple Stress Fluids between Two Parallel Plates, Heat Transfer – Asian Research, Vol. 44, No. 5, 2015, pp.468–487.
 13. Salawu, S.O. Dada, M.S., Radiative heat transfer of variable viscosity and thermal conductivity effects on inclined magnetic field with dissipation in a non-Darcy medium, Journal of the Nigerian Mathematical Society , Vol. 35, No. 1, 2016, pp.93–106.
 14. Sankar, M. Younghae Do Soorok Ryu and Bongsoo Jang , O. , Cooling of heat sources by natural convection heat transfer in a vertical annulus, Heat Transfer, Part A: Applications, Vol. 68, No. 8, 2015, pp.847–869.
 15. Siddheshwar, P.G. and Mahabaleswar, U.S., Effects of radiation and heat source on MHD flow of a viscoelastic liquid and heat transfer over a stretching sheet, International Journal of Non-Linear Mechanics , Vol. 40, No. 6, 2005, pp.807–820.
 16. Srinivas, G. Suresh Babu, B. and Reddy, B. R. K. Finite element analysis of free convection flow with MHD micropolar and viscous fluids in a vertical channel with dissipative effects, Journal of Naval Architecture and Marine Engineering, Vol. 8, No. 1, 2011, pp.59–69.
 17. Srinivas, J. Ramana Murthy, J.V and Anwar Bég, O., Entropy generation analysis of radiative heat transfer effects on channel flow of two immiscible couple stress fluids, The Brazilian Society of Mechanical Sciences and Engineering, Vol. 39, No. 6, 2017, pp.2191–2202.
 18. Subhas Abel, M. and N. Maheesha, Heat transfer in MHD viscoelastic fluid flow over a stretching sheet with variable thermal conductivity, non-uniform heat source and radiation, Applied Mathematical Modelling , Vol. 32, No. 10, 2008, pp.1965–1983.
 19. Surajit Dutta, Heat transfer in MHD viscoelastic fluid flow over a stretching sheet with variable thermal conductivity, non-uniform heat source and radiation, International Journal of Computer Applications, Vol. 170, No. 9, 2015, pp.0975–8887.