May – June 2020
ISSN: 0193-4120 Page No. 57 - 60

A Comprehensive Mathematical Study on Elastic Strain Energy and Complementary Strain Energy

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Article Info
Volume 83
Page Number: 57 - 60
Publication Issue:
May - June 2020

Article History
Article Received*: 11August 2019*
Revised*: 18November 2019*
Accepted*: 23January 2020*
Publication*: 07May2020*

Abstract:
To study the behavior of structures we are using Load v/s displacement characteristic values of particular structures in analysis. The displacement of loaded structures can be evaluated by using different methods and one among that is strain energy method. It is an internal energy developed against the deformation due to applied loads. The strain energy is divided in to elastic strain energy and complementary strain energy. Here an attempt is made to evaluate both the energies by simple graphical and mathematical models.

***Keywords** - Strain energy; Negative energy; Complementary strain energy; Total strain energy; Energy models.*

I. INTRODUCTION

Structures are designed for forces like tension, compression, flexure, shear, torsion and bond. Usually the flexural property study indicative for all the above said force parameters. But a special attention is given on torsional property which is predominant requirement in some important structures. Usually the study on torsion in strength of materials deals with circular shafts subjected to torsion. In R.C.C the torsional study is designed by empirical formula. Now here an attempt is made to design for torsional force for rectangular and square sections. This article mainly concentrates on Torsional strain energy.

II. Literature Review

Wang C.K [1]: Explains the energy methods used to determine the deflections, of determinate and indeterminate structures. It explains that energy methods are superior to geometrical methods. Energy methods are successfully applied to matrix methods and Finite element methods easily and more elegantly.

Chandrupatla R[2:] Develops that torsional forces can be evaluated for circular shafts by using FEM, considering triangular elements. The method followed is Galerkin Approach. Here a fundamental approach of FEM method is introduced by using three nodal elements with Torsional force.

Krishnamoorthy C. S [3] : Gives a method for calculation of torsion for non circular sections with torsional stiffness. The article treats the element by using FEM approach. A program for analysis of non circular element subjected to torsion is developed by using Finite element approach.

Gere M[4]: Here the research gap is identified between torsional strain energy and torsional strength is treated. It also dealt with elastic strain energy and complementary strain energy. From this it is decided to study experimentally both.

Bhavikatti S.S [5:] Reported that a formula for rectangular member is proposed for torsional study, for J the polar moment of inertia. A practical approach to study the torsional property of RCC members are developed in this article.

Shah V .L[6] : A design procedure for torsional force is developed by using limit state method. The method is developed by using I. S.-456 2000 codal provisions.

Reddy C.S[7] : Reports an analysis procedure for strain energy and complementary strain energy. It also develops a general procedure for calculation of total strain energy in the structures.

Gambhir M.L[8] : Treats the strain energy as a entity which gives a clear relation between the internal work done and external work. The treatment follows with energy principles.

Muthu K.U.[9]: Explains basics of energy theorems and its applications to evaluate the fundamental relationships with energy theorems.

Ramamrutham.S[10]: It develops concepts and relationships with fundamental energy theorems. It treats the basic structural forms and develops the energy relationships.

III. Theory

We know that deflections can be evaluated by using geometric method. This method is direct. The basic equations of equilibrium, compatibility, boundary conditions and the constitutive laws were used to generate the governing differential equations. These equations were then solved by analytic (double integration), graphic or combined method (Moment Area). Deflection can also be calculated by energy method where internal energy is equated to external work. In energy methods there are two general principles i) Virtual work (virtual displacements) and ii) Complementary virtual work (virtual forces) play a significant part.

Conservation of Energy, Work and Strain: Energy may be created or destroyed, it is a closed system. Mathematically work energy balance can be written as,

$$W = E_s + E_L \qquad \text{------ } 1$$

W= Work Performed

E_s = Energy stored in the body

E_L = Energy loss.

When the structure is loaded with a load the deformation starts in the structure and the deformation is resisted by the internal energy called strain energy. If the load is static then there is no kinetic energy in the structure. When there is no energy loss is the structure the above equation 1 - reduces to

$$W = E_s \qquad \text{--------- } 2$$

The term Es is called elastic strain energy in the structure represented by U. Hence in the conservative structural system with no energy loss,

$$W = U \qquad \text{--------- } 3$$

Work and Complimentary work: On structure the load is applied gradually and hence the graph is not linear that is it is not parallel to X-axis; it takes a curved path or non linear path as shown in figure 1 below.

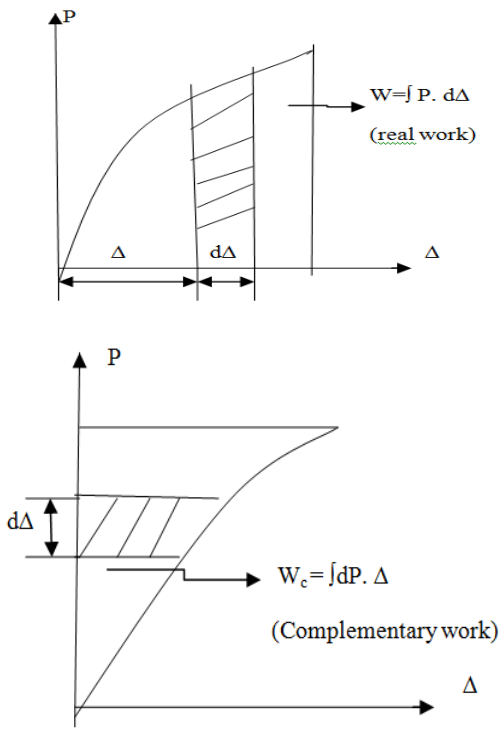

Fig 1. Load Deflection Curve

May – June 2020
ISSN: 0193-4120 Page No. 57 - 60

By definition work is force x displacement.

i.e $W = \int P \,.\, d\Delta$ --------4

If force P is in three dimensions Px , Py , Pz The total work done will be

$W = \int Px \,.\, d\,\Delta\, x + \int Py \,.\, d\Delta y + \int Pz \,.\, d\Delta z$ --------5

This work is real work and graphically it is below the load deflection curve.

The work belonging to linear elastic analysis load deflection curve should be linear.

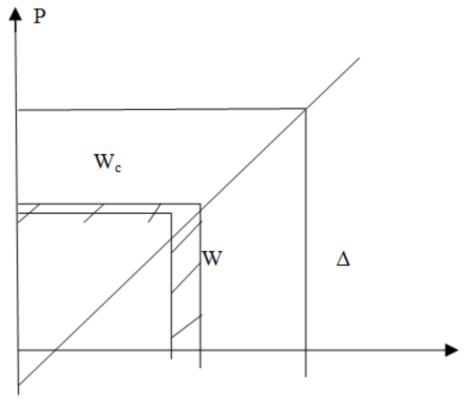

Fig 2. Linear Load Deflection Curve.

From Fig 2

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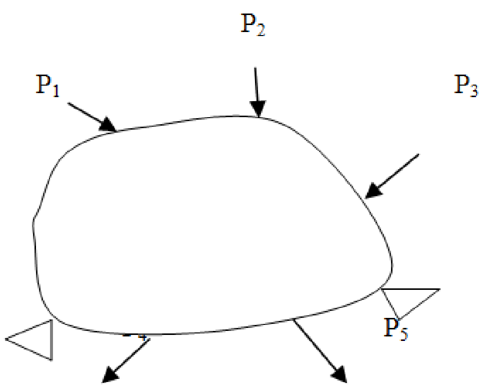
$W = W_c = 0.5 \times P \times \Delta$

W = Area below the curve.

W_c =Area above the curve.

Derivation of Torsional Strain Energy

Summation of external work done on the system is equal to the internal work done.



$W_e + W_i = 0$

Deformations are opposite to internal forces, internal work done can be related to the strain energy of the system U

$W_i = -U$

Relation between external loads and displacements is given by

$U = W_e = \sum_{j=1}^{m} (We)j = \sum_{j=1}^{m} \int P_j \, d\delta_j$

From Strength of materials, in case of a circular section subjected to torsion T, shear stress at a distance r from the centre of the cross section can be given by

$T = T\, r / J$

$U = \int_v T^2 \; dv / (2 \times G)$

$U = \int_L T^2 \; dx / (2\, G\, J)$

For square section $J = 0.1406\, a^4$

Where a is the side of a square. For the members with rectangular section or the built up sections with rectangular components the above equation is used with different meaning for the term J. J is called torsional factor.

IV.Conclusions

Torsional Strain energy is a quantity which gives an idea about torsional strength which can be required for accurate calculation in case of canopy beams, circular water tanks ring beams and shell structures in which twisting moment is predominant. Here in this article an attempt is made to calculate elastic strain energy and complimentary strain energy in which complimentary strain energy is having much more importance is structural analysis. It is proposed to study experimentally the both energies and the details are established.

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