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A New Algorithm for Solving Shortest Path Problems using Octagonal Neutrosophic Numbers

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Abstract:

Shortest path problem, is one of the best decision making problem that can be applied in many fields of science, management and technology. The shortest route algorithm tries to find the shortest route between two nodes in a graph. Neutrosophic theory is very useful to find the better solution for real world problems when incomplete and inconsistent data are given. Octagonal neutrosophic number plays an important role for solving many decision making problems involving incompleteness and indeterminacy. This paper comprises a proposed neutrosophic network method that has been made to find the shortest path between the nodes using octagonal neutrosophic numbers.

Keywords: *Shortest path problem, Floyd's Algorithm, Single valued octagonal neutrosophic number.*

I. INTRODUCTION

This Neutrosophic logic has been generated by Florentine Smaradache [5,6] which is based on non-standard analysis. L AZadeh [9] was created the abstract idea of fuzzy set in 1965. In Neutrosophic set theory, the truth-membership, indeterminacy-membership and falsity-membership functions are independent. In 1998, Smarandache [7,8] introduced Neutrosophic sets as an extensional case of classical sets and intuitionistic fuzzy sets. Neutrosophic logic is used to characterize the logical statement in 3D space, with each dimension representing the membership functions which are within the real standard or non-standard unit interval $]0^-,1^+[$. The octagonal neutrosophic numbers are proposed by K. Selvakumari and S.Lavanya [4] in 2018.

Shortest path problem [2] is a common method used to find shortest path between two vertices, so that the sum of the weights of their corresponding edges can be minimized. K T Atanassov[1] is well known for introducing the concept of generalised sets and intuitionistic fuzzy sets, which are extensions of the concepts of petri sets and fuzzy sets respectively. Dijkstra’s algorithm as well as Floyd’s algorithm[2,3] can be applied to solve both cyclic and acyclic networks. Here we are applying Floyd’s algorithm to find the shortest path between the nodes, where the distance between the given nodes are denoted by Octagonal Neutrosophic numbers.

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II. PRELIMINARIES

2.1 Definition

A fuzzy neutrosophic set A on the universe of a discourse X is defined as A= {x, T_A (x), I_A (x), F_A (x):x∈X} where T_A,I_A,F_A: X→ [0,1] and 0≤ T_A (x)+ I_A (x)+ F_A (x) ≤ 3 where T_A (x) is truth-membership function, I_A (x) is in deterministic function and F_A (x) is non-deterministic function.

2.2 Definition

Let a = {(a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8) ;(T_a, I_a , F_a)} be an octagonal neutrosophic number which is a special neutrosophic set on the set of real numbers, whose truth-membership, indeterminacy-membership and falsity-membership functions are respectively defined by:

$$T_a(x)=\begin{cases} 0 & for\ x < a_1 \\ k\left(\frac{x-a_1}{a_2-a_1}\right) & for\ a_1 \le x \le a_2 \\ k & for\ a_2 \le x \le a_3 \\ k+(T_a-k)\left(\frac{x-a_3}{a_4-a_3}\right) & for\ a_3 \le x \le a_4 \\ T_a & for\ a_4 \le x \le a_5 \\ k+(T_a-k)\left(\frac{a_6-x}{a_6-a_5}\right) & for\ a_5 \le x \le a_6 \\ k & for\ a_6 \le x \le a_7 \\ k\left(\frac{a_8-x}{a_8-a_7}\right) & for\ a_7 \le x \le a_8 \\ 0 & for\ x > a_8 \end{cases}$$

$$I_a(x)=\begin{cases} 1 & for\ x < a_1 \\ 1+(1-k)\left(\frac{a_1-x}{a_2-a_1}\right) & for\ a_1 \le x \le a_2 \\ k & for\ a_2 \le x \le a_3 \\ k+(k-I_a)\left(\frac{a_3-x}{a_4-a_3}\right) & for\ a_3 \le x \le a_4 \\ I_a & for\ a_4 \le x \le a_5 \\ k+(k-I_a)\left(\frac{x-a_5}{a_6-a_5}\right) & for\ a_5 \le x \le a_6 \\ k & for\ a_6 \le x \le a_7 \\ k+(1-k)\left(\frac{x-a_7}{a_8-a_7}\right) & for\ a_7 \le x \le a_8 \\ 1 & for\ x > a_8 \end{cases}$$

$$F_a(x)=\begin{cases} 1 & for\ x < a_1 \\ 1+(1-k)\left(\frac{a_1-x}{a_2-a_1}\right) & for\ a_1 \le x \le a_2 \\ k & for\ a_2 \le x \le a_3 \\ k+(k-F_a)\left(\frac{a_3-x}{a_4-a_3}\right) & for\ a_3 \le x \le a_4 \\ F_a & for\ a_4 \le x \le a_5 \\ F_a+(k-F_a)\left(\frac{x-a_5}{a_6-a_5}\right) & for\ a_5 \le x \le a_6 \\ k & for\ a_6 \le x \le a_7 \\ k+(1-k)\left(\frac{x-a_7}{a_8-a_7}\right) & for\ a_7 \le x \le a_8 \\ 1 & for\ x > a_8 \end{cases}$$

Where 0<k<1

2.3 Definition

The octagonal neutrosophic number

A = {(a_1, a_2,a_3, a_4,a_5,a_6 ,a_7, a_8);(T_A, I_A , F_A)} is defuzzified by a score function defined by

S(A) = $\frac{1}{2}$h [(a_8-a_1) + (a_7-a_2) + (a_6-a_3) + (a_5-a_4)] [($\frac{2+T_A-I_A-F_A}{3}$)] , into a singleton crisp value where h= 0.5.

III. NETWORK TERMINOLOGY

Consider a directed network G*= (V*,E*) consisting of a finite set of nodes η

V*={1,2,3,4,….. η} and a set of m directed edges

E* ⊂U*xV*.

Each edge is denoted by an ordered pair (a, b) where a, b ∈ V' and a ≠b and $Đ_{ab}$ denote a single valued Octagonal Neutrosophic number assigned to the edge (a,b) corresponding to edge length which need to transverse (a,b) from a to b. In optimization problems distance or length can be capacity, cost, time, etc.

Neutrosphic distance along the path is denoted by N(P)

Thus N(P) =$\sum_{(a,b)\in P}$ $Đ_{ab}$

IV. NEUTROSOPHIC OCTAGONAL FLOYD'S ALGORITHM

While Compared to Dijkstra's algorithm, Floyd's algorithm is considered as more general since it determines the shortest route between any two nodes in the given network. Here the given routes are undirected. In this section the distance or weights are represented by single valued octagonal neutrosophic numbers. The algorithm comprises an 'm' node network as an mxm square matrix with 'm' rows and 'm' columns. The distance from α to β is denoted by $đ_{\alpha\beta}$ that is the entry (α, β) of the matrix:

(i) The distance is finite if α is directly linked to β.

(ii) Otherwise it is infinite.

If three nodes α. β, γ are given and if we want to find the shortest distance from α to γ. Here α is directly connected to γ by the distance $đ_{\alpha\beta}$ but while we are determining, it is shorter to reach γ from α passing through β, if $đ_{\alpha\beta}$ + $đ_{\beta\gamma}$< $đ_{\alpha\gamma}$.

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In this situation we can replace the direct route from $\boldsymbol{\alpha} \rightarrow \boldsymbol{\gamma}$ with the indirect route

$\boldsymbol{\alpha} \rightarrow \boldsymbol{\beta} \rightarrow \gamma$. Then the triple operation network can be applied systematically to the given network.

General procedure

Step 1: Define the starting distance matrix $Đ_0$ and node sequence matrix $\dot{S}_0$.

Diagonal cells are marked with (-) symbol since loops are not allowed.

Define the matrices as:

Table 1: General distance matrix $Đ_0$

$Đ_0$	1	2	3	….	β	……	m
1	-	d_{12}	d_{13}	…..	$d_{1\beta}$	……	d_{1m}
2	d_{21}	-	d_{23}	…..	$d_{2\beta}$	……	d_{2m}
3	d_{31}	d_{32}	-	…..	$d_{3\beta}$	…..	d_{3m}
….	….	…..	….	….	…..	…..	….
α	$d_{\alpha 1}$	$d_{\alpha 2}$	$d_{\alpha 3}$	…..	-	…...	$d_{\alpha m}$
….	….	….	….	….	….	….	….
m	d_{m1}	d_{m2}	d_{m3}	…..	$d_{m\beta}$	…..	-

Table 2: Node sequence matrix $\dot{S}_0$

$\dot{S}_0$	1	2	3	…..	β	….	m
1	-	2	3	…..	β	….	m
2	1	-	3	…..	β	….	m
3	1	2	-	…..	β	….	m
:	:	:	:	-	:	….	:
α	1	2	3	…..	-	….	m
:	:	:	:	…..	:	-	:
m	1	2	3	…..	β	….	-

General step ꝁ:

Define the pivot row and pivot column as row ꝁ and column ꝁ respectively and apply the triple operation to each element $đ_{\alpha\beta}$ in $D_{ꝁ-1}$ for all α and β. If the condition

$đ_{\alpha\gamma}$ + $đ_{\gamma\beta}$ < $đ_{\alpha\beta}$ (α≠γ, β≠γ, α≠β) is satisfied then

(a) $đ_{\gamma\beta}$ can be created by replacing $đ_{\alpha\beta}$ in $D_{ꝁ-1}$ with $đ_{\alpha\beta}$ ≠.$đ_{\gamma\beta}$

(b) $S_{ꝁ}$ can be created by replacing $S_{\alpha\beta}$ in $S_{ꝁ-1}$ with ꝁ

(c) Set ꝁ = ꝁ+1 and repeat step ꝁ.

After these m steps, we can find the shortest path between α and β using the following rules from the matrices given by $Đ_m$ and $\dot{S}_m$.

(i) From $đ_{\alpha\gamma}$ and $đ_{\gamma\beta}$ we get the shortest distance between nodes α and β.

(ii) From $\dot{S}_n$ the intermediate node ꝁ= $S_{\alpha\beta}$ can be determine, that results the route $\boldsymbol{\alpha} \rightarrow \boldsymbol{\beta} \rightarrow \gamma$. If $S_{\alpha\gamma}$ = γ and $S_{\gamma\beta}$ = β, then stop the process otherwise repeat

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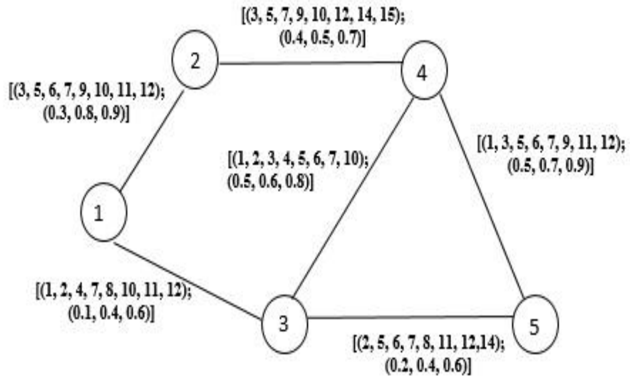
the process between nodes α and γ and between nodes γ and β

V. ILLUSTRATION& RESULTS

❖ Find the shortest route between every pair of nodes. The distance is represented by single valued Octagonal Neutrosophic number.



Step1: First convert the single valued octagonal Neutrosophic number into singleton crisp value, using the score function

$$S(A) = \frac{1}{2}h\,[(a_8-a_1)+(a_7-a_2)+(a_6-a_3)+(a_5-a_4)]\,[(\frac{2+T_A-I_A-F_A}{3})],\ 0<h<1,\ h=0.5.$$

$S[(3,5,6,7,9,10,11,12),(0.3,0.8,0.9)]=(\frac{0.5}{2})(21)(\frac{0.6}{3})=1.05$

$S[(1,2,4,7,8,10,11,12),(0.1,0.4,0.6)]=(\frac{0.5}{2})(27)(\frac{1.1}{3})=2.48$

$S[(3,5,7,9,10,12,14,15),(0.4,0.5,0.7)]=(\frac{0.5}{2})(27)(\frac{1.2}{3})=2.7$

$S[(2,5,6,7,8,11,12,14),(0.2,0.4,0.6)]=(\frac{0.5}{2})(25)(\frac{1.2}{3})=2.5$

$S[(1,3,5,6,7,9,11,12),(0.5,0.7,0.9)]=(\frac{0.5}{2})(23)(\frac{0.9}{3})=1.73$

$S[(1,2,3,4,5,6,7,10),(0.5,0.6,0.8)]=(\frac{0.5}{2})(18)(\frac{1.1}{3})=1.65$

Using corresponding score function, construct matrices $Đ_0$ and $\dot{S}_0$

Distance matrix $Đ_0$

$Đ_0$	1	2	3	4	5
1	-	1.05	2.48	∞	∞
2	1.05	-	∞	2.7	∞
3	2.48	∞	-	1.65	2.5
4	∞	2.7	1.65	-	1.73
5	∞	∞	2.5	1.73	-

Node sequence matrix $\dot{S}_0$

$\dot{S}_0$	1	2	3	4	5
1	-	2	3	4	5
2	1	-	3	4	5
3	1	2	-	4	5
4	1	2	3	-	5
5	1	2	3	4	-

Step 2:ƙ=1

The pivot row and pivot column are shown in the shaded 1st row and 1st column respectively.

$đ_{23}$replaced by $đ_{21} + đ_{13}$ = 2.48+1.05=3.53 and set s_{23} =1

$đ_{32}$ replaced by $đ_{31}+đ_{12}$=2.48+1.05=3.53 and set s_{32} =1

The new matrices obtained are shown in next table.

Distance matrix $Đ_1$

$Đ_1$	1	2	3	4	5
1	-	1.05	2.48	∞	∞
2	1.05	-	**3.53**	2.7	∞
3	2.48	**3.53**	-	1.65	2.5
4	∞	2.7	1.65	-	1.73
5	∞	∞	2.5	1.73	-

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Node sequence matrix $\dot{S}_1$

$\dot{S}_1$	1	2	3	4	5
1	-	2	3	4	5
2	1	-	1	4	5
3	1	1	-	4	5
4	1	2	3	-	5
5	1	2	3	4	-

Step 3: ƙ=2

Here, the shaded second row and second column show the pivot row and pivot column

Respectively.

$Đ_2$ and $\dot{S}_2$ are obtained by the following operation:

$đ_{14}$ replaced by $đ_{12}+đ_{24}$= 1.05+2.7 = 3.75 and set s_{14}= 2.

$đ_{41}$ replaced by $đ_{42}+đ_{21}$=2.7+1.05=3.75 and set s_{41}=2.

The improved matrices are shown in next tables

Distance matrix $Đ_2$

$Đ_2$	1	2	3	4	5
1	-	1.05	2.48	**3.75**	∞
2	1.05	-	3.53	2.7	∞
3	2.48	3.53	-	1.65	2.5
4	**3.75**	2.7	1.65	-	1.73
5	∞	∞	2.5	1.73	-

Node sequence matrix $\dot{S}_2$

$\dot{S}_2$	1	2	3	4	5
1	-	2	3	**2**	5
2	1	-	1	4	5
3	1	1	-	4	5
4	**2**	2	3	-	5
5	1	2	3	4	-

Step 4: ƙ=3

Here select 3rd row and 3rd column as pivot row and pivot column. The cells $đ_{15}$, $đ_{25}$, $đ_{51}$, $đ_{52}$ can be replaced by the operations given below:

$đ_{15}$replacedby $đ_{13}$+ $đ_{35}$ =2.48+2.5=4.98 and set s_{15} =3.

$đ_{25}$replacedby $đ_{23}$+ $đ_{35}$ =3.53+2.5=6.03 and set s_{25} =3.

$đ_{51}$replaced by $đ_{53}$+ $đ_{31}$ =2.5+2.48=4.98 and set s_{51} =3.

$đ_{52}$replaced by $đ_{53}$+ $đ_{32}$ 2.5+3.53=6.03 and set s_{52} =3.

The new matrices obtained are shown below:

Distance matrix $Đ_3$

$Đ_3$	1	2	3	4	5
1	-	1.05	2.48	3.75	**4.98**
2	1.05	-	3.53	2.7	**6.03**
3	2.48	3.53	-	1.65	2.5
4	3.75	2.7	1.65	-	1.73
5	**4.98**	**6.03**	2.5	1.73	-

Node sequence matrix $\dot{S}_3$

$\dot{S}_3$	1	2	3	4	5
1	-	2	3	2	**3**
2	1	-	1	4	**3**
3	1	1	-	4	5
4	2	2	3	-	5
5	**3**	**3**	3	4	-

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Step 5: ƙ=4

The pivot row and pivot column shown by the shaded 4th row and 4th column respectively. $đ_{25}$ and $đ_{52}$ can be improved by the operations which are given below:

$đ_{25}$can be replaced by $đ_{24}$ + $đ_{45}$ = 2.7+1.73=4.43 and set s_{25} =4.

$đ_{52}$can be replaced by $đ_{54}$ + $đ_{42}$ = 1.73+2.7=4.43 and set s_{52} =4.

The new matrices obtained are shown below:

Distance matrix $Đ_4$

$Đ_4$	1	2	3	4	5
1	-	1.05	2.48	3.75	4.98
2	1.05	-	3.53	2.7	**4.43**
3	2.48	3.53	-	1.65	2.5
4	3.75	2.7	1.65	-	1.73
5	4.98	**4.43**	2.5	1.73	-

Node sequence matrix $\dot{S}_4$

$\dot{S}_4$	1	2	3	4	5
1	-	2	3	**2**	**3**
2	1	-	**1**	4	**4**
3	1	**1**	-	4	5
4	**2**	2	3	-	5
5	**3**	**4**	3	4	-

Step 6:

The final matrices comprise the necessary information to find the shortest path between every two nodes.

Distance matrix $Đ_4$

$Đ_4$	1	2	3	4	5
1	-	1.05	2.48	**3.75**	**4.98**
2	1.05	-	**3.53**	2.7	**4.43**
3	2.48	**3.53**	-	1.65	2.5
4	**3.75**	2.7	1.65	-	1.73
5	**4.98**	**4.43**	2.5	1.73	-

Node sequence matrix $\dot{S}_4$

$\dot{S}_4$	1	2	3	4	5
1	-	2	3	**2**	**3**
2	1	-	**1**	4	**4**
3	1	**1**	-	4	5
4	**2**	2	3	-	5
5	**3**	**4**	3	4	-

To determine the shortest route check whether $\boldsymbol{S_{\alpha\beta} = \beta}$ **or not.** Otherwise α and β are linked through at least one of the other intermediate vertex.

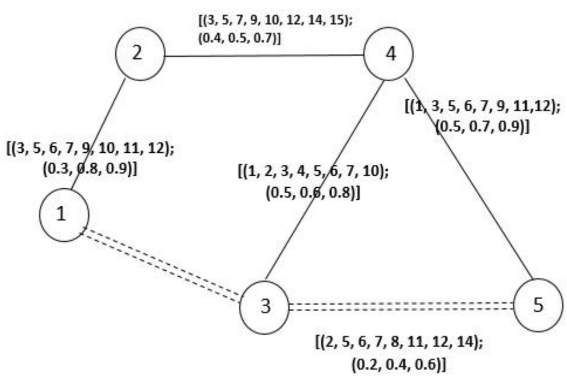
This result table is given below:

Result Table

Source Vertex(α)	Destination Vertex (β)	Distance $d_{\alpha\beta}$	Shortest Path
1	2	1.05	1→2
1	3	2.48	1→3
1	4	3.75	1→2→4
1	5	4.98	1→3→5
2	3	3.53	2→1→3
2	4	2.7	2→4
2	5	4.43	2→4→5
3	4	1.65	3→ 4
3	5	2.5	3→ 5
4	5	1.73	4→5

Node 1 leads to node 5 through the shortest route 1→**3→5** and the **value of the route = 4.98.**

This gives the end of the process.



VI. CONCLUSION

A New algorithm for solving shortest path problem on a network with single valued octagonal neutrosophic edge lengths are developed in this paper and the given edge weights are uncertain in this situation. The octagonal neutrosophic numbers are defuzzified, using score functions into a single crisp value. This algorithm helps us to make best decisions in choosing the best of all possible alternative paths. It is considered as one of the most fundamental and combinational problem which can apply in various field of science, management and technology. This algorithm is a remarkably successful method for solving the uncertainty. We can do further research in the application of these algorithms.

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