

Radio Analytic Mean Labeling of Some Standard Graphs

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Article Info

Volume 83

Page Number: 14579 - 14584

Publication Issue:

March - April 2020

Article History

Article Received: 24 July 2019

Revised: 12 September 2019

Accepted: 15 February 2020

Publication: 22 April 2020

Abstract:

A Radio Analytic mean labelling of a connected graph G is a one to one map f from the vertex $v(G)$ to set Z such that for any two distinct vertices of u and v of G . $d(u,v) + \left\lfloor \frac{|f(u)^2 - f(v)^2|}{2} \right\rfloor \geq 1 + \dim G$. The Radio Analytic mean number n , is the maximum number assigned to any vertex of G . The Radio mean number of G , $(rmn(G))$ is the minimum value of taken all over Radio Analytic mean labeling of G . In this paper we find the Radio Analytic mean number of path graph, cycle graph, star graph, Bi-star graph, complete Bipartite graph.

Keywords: Radio Analytic mean labeling, Radio Analytic mean graph, path graph, cycle graph, star graph, Bi-star graph, complete Bipartite graph;

I. INTRODUCTION

Throughout this paper we consider finite, simple, undirected and connected graphs. $V(G)$ and $E(G)$ respectively denote the vertex set and edge set of G . Radio labeling or multilevel distance labeling is motivated by the channel assignment problem for radio transmitters. First defined the concept of Radio Labeling of Graph G . [1]. Motivated by problems we consider the Radio k Labeling of Cartesian product of two Graphs [2]. In various Authors are discussed in Radio multilevel distance Labeling for path and cycles, Radio number of trees, Radio number of square of paths in [3][4][5]. Radio Labeling of Graph is motivated by restrictions in assigning channel task for Radio Transmitters [8]. Ponraj introduced the notion of radio mean labeling of graphs and investigated radio mean number of some graphs [10]. T.Tharmaraj and P.B. Saraija

are introduced the concept of Analytic Mean labeling in [12]. A graph $G(v,E)$ is said to be an Analytic mean graph. If we can assign the vertices in V with distinct elements from $0,1,2,\dots,p-1$ in such a way that when each edge $e=uv$ is labeled $f(uv) = \frac{|f(u)^2 - f(v)^2|}{2}$. Poomalai et.al [11] introduced Radio analytic mean labeling of some graphs. In this paper we find the Radio Analytic mean number of path graph, cycle graph, star graph, Bi-star graph, complete Bipartite graph.

II. PRELIMINARIES

Definition 1.1: A Radio Analytic mean labeling of a connected graph G is a one to one map f from the vertex set $v(G)$ to set Z such that for any two distinct vertices of u and v of G . Such that for each edge uv . $d(u,v) + \left\lfloor \frac{|f(u)^2 - f(v)^2|}{2} \right\rfloor \geq 1 + \dim G$. The Radio Analytic mean number n , is the maximum number assigned to any vertex of G . The Radio mean number of G , $(rmn(G))$ is the minimum value of taken all over Radio Analytic mean labeling of G .

Definition 1.2 : Let v_1, v_2, \dots, v_n be the alternating vertices are adjacent is called the path graph. It has n vertices and $n-1$ edges. It is denoted by P_n

Definition 1.3 : Let G be the graph C_n . we draw a path P_n . The connecting the first and last vertices by an edge we form the cycle C_n . The cycle graph is admitted Radio Analytic mean condition .

Definition 1.4 : Let V be the centre vertex. Let V_1, V_2, \dots, V_n be the end vertices of the star $k_{1,n}$. we assign the label 1 to the centre vertices. Remaining vertices $2 \leq i \leq n$.

Definition 1.5: The n -bistar graph $B_{n,n}$ is graph obtained from two copies of $k_{1,n}$ by joining the vertices of maximum degree by an edge

Definition 1.6: A complete bipartite graph is a graph whose vertex set $V(G)$ can be separated into two subsets of v_1 and v_2 so that each edge of G has one end in v_1 and other end in v_2 is called a bipartition of G .

Definition 1.7: The distance $d(u,v)$ from a vertex u to a vertex v in a connected graph G is the minimum of the lengths of the u - v paths in G

Definition 1.8: The diameter $\text{dim}(G)$ of G is the greatest eccentricity among the vertices of G

Main Results:

Theorem 2.1: The Radio Analytic mean number of path graph P_n is $2n-3$ for $n \geq 5$

Proof: Let the vertex set and edge set of path graph $V(P_n) = v_i : 1 \leq i \leq n$ and

$E(P_n) = \{v_i v_{i+1} : 1 \leq i \leq n-1\}$. Now $V(P_n) = n$ and $E(G) = n-1$.

Let assign label of vertices as defined $f: V(P_n) \rightarrow \{1, 2, \dots, 2n-3\}$. The diameter of the path graph is $n-1$

Now we verify the Radio analytic mean condition for the path graph . Then condition is

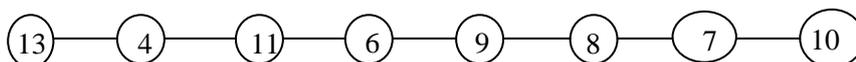
$$d(u,v) + \left\lceil \frac{|f(u)^2 - f(v)^2|}{2} \right\rceil \geq 1 + \text{diam}G \text{ for every pair of vertices the } P_n.$$

No of vertices	V_1	V_2	V_3	V_4	V_5
2	1	2	-	-	-
3	1	2	3	-	-
4	1	3	4	2	-

When n is odd

$$f(v_1) = 2n-3, f(v_{2i}) = 2i+2, 1 \leq i \leq \frac{n-1}{2}, f(v_{2i+1}) = 2n-3-2i, 1 \leq i \leq \frac{n-1}{2}$$

Example: Radio Analytic mean labeling of path P_8 . i.e $\text{ramn}(P_8) = 13$



Radio Analytic mean labeling of path P_7 . i.e $\text{ramn}(P_7) = 11$

when n is even

$$f(v_1) = 2n-3, f(v_{2i}) = 2i+2, 1 \leq i \leq \frac{n}{2}, f(v_{2i+1}) = 2n-3-2i, 1 \leq i \leq \frac{n}{2}-1$$

Now we check the Radio Analytic mean condition

When n is odd

Case(i): consider the pair (v_1, v_{2i}) , $1 \leq i \leq \frac{n-1}{2}$

$$d(v_1, v_{2i}) + \left\lceil \frac{|(2n-3)^2 - (2i+2)^2|}{2} \right\rceil \geq 1 + \text{diam}G = 1 + n-1$$

$$d(v_1, v_{2i}) + \left\lceil \frac{|(2n-3)^2 - (2i+2)^2|}{2} \right\rceil \geq n$$

Case(ii) : consider the pair (v_1, v_{2i+1}) , $1 \leq i \leq \frac{n-1}{2}$

$$d(v_1, v_{2i+1}) + \left\lceil \frac{|(2n-3)^2 - (2i+2)^2|}{2} \right\rceil \geq n$$

Case(iii): consider the pair (v_{2i}, v_{2j+1}) , $1 \leq i, j \leq \frac{n-1}{2}$

$$d(v_{2i}, v_{2j+1}) + \left\lceil \frac{|(2i+2)^2 - (2n-3-2j)^2|}{2} \right\rceil \geq n$$

When n is even

Case (i): consider the pair (v_1, v_{2i}) , $1 \leq i \leq \frac{n}{2}$

$$d(v_1, v_{2i}) + \left\lceil \frac{|(2n-3)^2 - (2i+2)^2|}{2} \right\rceil \geq n$$

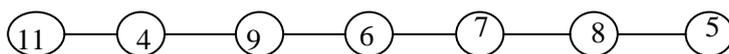
Case(ii): consider the pair (v_1, v_{2i+1}) , $1 \leq i \leq \frac{n}{2}-1$

$$d(v_1, v_{2i+1}) + \left\lceil \frac{|(2n-3)^2 - (2n-3-2i)^2|}{2} \right\rceil \geq n$$

Case (iii): consider the pair (v_{2i}, v_{2j+1}) , $1 \leq i \leq \frac{n}{2}, 1 \leq j \leq \frac{n}{2}-1$

$$d(v_{2i}, v_{2j+1}) + \left\lceil \frac{|(2n+2)^2 - (2n-3-2j)^2|}{2} \right\rceil \geq n$$

The Radio Analytic mean condition is satisfied for all pairs of vertices. Hence f is Radio Analytic mean labeling of P_n .Therefore radio analytic mean number of f is $2n-3$. Radio analytic mean labeling number of $P_n = 2n-3$. i.e $\text{ramn}(P_n) = 2n-3$



Theorem 2.2: The Radio Analytic mean number of cycle graph C_n is $2n$, $n \geq 5$

Let G be the graph C_n . we draw a path P_n . The connecting the first and last vertices by an edge we form the cycle C_n . The cycle graph is admitted Radio Analytic mean condition

Define the vertex label $f: V(G) \rightarrow \{1, 2, 3 \dots 2n\}$ as follows .

When n is odd

$$f(v_1) = 2n, \quad f(v_{2i}) = n+i, \quad 1 \leq i \leq \frac{n-1}{2}, \quad f(v_{2i+1}) = n-i, \quad 1 \leq i \leq \frac{n-1}{2},$$

when n is even

$$f(v_1) = 2n, \quad f(v_{2i}) = n+i, \quad 1 \leq i \leq \frac{n}{2}, \quad f(v_{2i+1}) = n-i, \quad 1 \leq i \leq \frac{n}{2} - 1$$

Now we check the Radio Analytic mean condition

When n is odd

The diameter of circle of n vertices is $\frac{n-1}{2}$

Case(i) : consider the pair (v_1, v_{2i}) , $1 \leq i \leq \frac{n-1}{2}$

$$d((v_1, v_{2i})) + \left\lceil \frac{|(2n)^2 - (n+i)^2|}{2} \right\rceil \geq 1 + \text{diam}G = 1 + \frac{n-1}{2}$$

$$d((v_1, v_{2i})) + \left\lceil \frac{|3n^2 - 2ni - i^2|}{2} \right\rceil \geq \frac{n+1}{2}$$

Case(ii): consider the pair (v_1, v_{2j+1}) , $1 \leq i, j \leq \frac{n-1}{2}$

$$d(v_1, v_{2j+1}) + \left\lceil \frac{|(2n)^2 - (n-j)^2|}{2} \right\rceil \geq 1 + \text{diam}G = 1 + \frac{n-1}{2}$$

$$d(v_1, v_{2j+1}) + \left\lceil \frac{|3n^2 + 2nj - j^2|}{2} \right\rceil \geq \frac{n+1}{2}$$

case(iii): consider the pair (v_{2i}, v_{2j+1}) , $i \leq i, j \leq \frac{n-1}{2}$

$$d(v_{2i}, v_{2j+1}) + \left\lceil \frac{|(n+i)^2 - (n-j)^2|}{2} \right\rceil \geq \frac{n-1}{2}$$

$$d(v_{2i}, v_{2j+1}) + \left\lceil \frac{|2n(i+j) + i^2 - j^2|}{2} \right\rceil \geq \frac{n-1}{2}$$

case(iv): consider the pair (v_{2i}, v_{2j}) , $i \neq j$, $1 \leq i, j \leq \frac{n-1}{2}$

$$d(v_{2i}, v_{2j}) + \left\lceil \frac{|(n+i)^2 - (n+j)^2|}{2} \right\rceil \geq \frac{n-1}{2}$$

$$d(v_{2i}, v_{2j}) + \left\lceil \frac{|2n(i-j) + i^2 - j^2|}{2} \right\rceil \geq \frac{n-1}{2}$$

case(v): consider the pair (v_{2i+1}, v_{2j+1}) , $i \neq j$, $1 \leq i, j \leq \frac{n-1}{2}$

$$d(v_{2i+1}, v_{2j+1}) + \left\lceil \frac{|(n-i)^2 - (n-j)^2|}{2} \right\rceil \geq \frac{n-1}{2}$$

$$d(v_{2i+1}, v_{2j+1}) + \left\lceil \frac{|2n(j-i) + i^2 - j^2|}{2} \right\rceil \geq \frac{n-1}{2}$$

When n is even

The diameter of circle of n vertices is $\frac{n}{2}$

Case(i): consider the pair (v_1, v_{2i}) , $1 \leq i \leq \frac{n}{2}$

$$d(v_1, v_{2i}) + \left\lceil \frac{|(2n)^2 - (n+i)^2|}{2} \right\rceil \geq 1 + \text{diam}G = 1 + \frac{n}{2}$$

$$d(v_1, v_{2i}) + \left\lceil \frac{|3n^2 - 2ni - i^2|}{2} \right\rceil \geq \frac{n+2}{2}$$

Case (ii): consider the pair (v_1, v_{2j+1}) , $1 \leq j \leq \frac{n}{2} - 1$

$$d(v_1, v_{2j+1}) + \left\lceil \frac{|(2n)^2 - (n-j)^2|}{2} \right\rceil \geq 1 + \text{diam}G = 1 + \frac{n}{2}$$

$$d(v_1, v_{2j+1}) + \left\lceil \frac{|3n^2 + 2nj - j^2|}{2} \right\rceil \geq \frac{n+2}{2}$$

case(iii): consider the pair (v_{2i}, v_{2j+1}) , $1 \leq i \leq \frac{n}{2}$, $1 \leq j \leq \frac{n}{2} - 1$

$$d(v_{2i}, v_{2j+1}) + \left\lceil \frac{|(n+i)^2 - (n-j)^2|}{2} \right\rceil \geq \frac{n+2}{2}$$

$$d(v_{2i}, v_{2j+1}) + \left\lceil \frac{|2n(i+j) + i^2 - j^2|}{2} \right\rceil \geq \frac{n+2}{2}$$

case(iv): consider the pair (v_{2i}, v_{2j}) , $i \neq j$, $1 \leq i, j \leq \frac{n}{2}$

$$d(v_{2i}, v_{2j}) + \left\lceil \frac{|(n+i)^2 - (n+j)^2|}{2} \right\rceil \geq \frac{n+2}{2}$$

$$d(v_{2i}, v_{2j}) + \left\lceil \frac{|2n(i-j) + i^2 - j^2|}{2} \right\rceil \geq \frac{n+2}{2}$$

case(v): consider the pair (v_{2i+1}, v_{2j+1}) , $i \neq j$, $1 \leq i, j \leq \frac{n}{2} - 1$

$$d(v_{2i+1}, v_{2j+1}) + \left\lceil \frac{|(n-i)^2 - (n-j)^2|}{2} \right\rceil \geq \frac{n+2}{2}$$

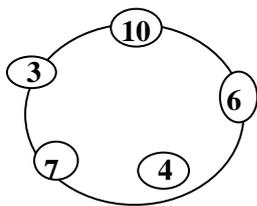
$$d(v_{2i+1}, v_{2j+1}) + \left\lceil \frac{|2n(j-i) + i^2 - j^2|}{2} \right\rceil \geq \frac{n+2}{2}$$

The Radio Analytic mean condition is satisfied for all pairs of vertices. Hence f is Radio Analytic mean labeling of C_n . Therefore radio analytic mean number

of f is $2n$. Radio analytic mean labeling number of $c_n = 2n$. i.e $\text{ramn}(c_n) = 2n$.

Example 2:

$c_n = n$ is odd



$\text{ramn}(c_5)$

10
 $\text{ramn}(c_6) = 12$

Theorem 2.3: The Radio Analytic mean number of star graph $(k_{1,n})$ is $n+1$ for $n \geq 5$

Proof: Let G be a graph of $k_{1,n}$. Let v and v_i be the centre vertex and pendent vertices of G respectively. Then $V(G) = n+1$, $E(G) = n$. The diameter of the star graph is 2.

Define the vertex label $f: V(G) \rightarrow \{1, 2, 3 \dots n+1\}$.
Now $f(v) = 1$, $f(v_i) = 1+i$, $1 \leq i \leq n$

Now we verify the Radio Analytic mean condition for the star graph.

$d(u,v) + \left\lceil \frac{|f(u)^2 - f(v)^2|}{2} \right\rceil \geq 1 + \text{diam}G$ for every pair of vertices $(k_{1,n})$

Case(i): verify the pair (v, v_i) for $1 \leq i \leq n$

$$d(v, v_i) + \left\lceil \frac{|f(v)^2 - f(v_i)^2|}{2} \right\rceil \geq 1 + \text{diam}G = 1 + 2 = 3$$

$$d(v, v_i) + \left\lceil \frac{|1 - (i+1)^2|}{2} \right\rceil \geq 3$$

Example:

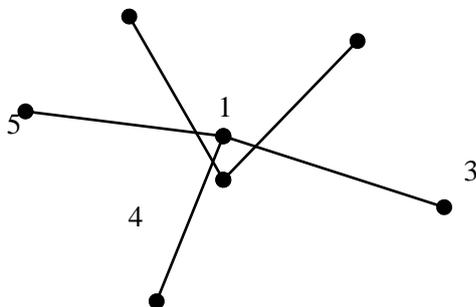
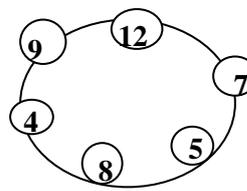


Figure 2: The Radio Analytic mean number of $(k_{1,n})$ is $n+1$ for $n \geq 5$

Theorem 2.4: The Radio Analytic mean number of Bistar graph is $(B_{n,n})$ is $2n+2$, $n \geq 4$.

Proof: Let the vertex set and edge set of Bistar graph $V(B_{n,n}) = \{u_i, v_i : 1 \leq i \leq n\}$ and

$c_n = n$ is even



$$d(v, v_i) + \left\lceil \frac{|-i^2 - 2i|}{2} \right\rceil \geq 3 = d(v, v_i) + \left\lceil \frac{|i^2 + 2i|}{2} \right\rceil \geq 3$$

Case(ii): verify the pair (v_i, v_j) $i \neq j$, $1 \leq i, j \leq n$

$$d(v_i, v_j) + \left\lceil \frac{|f(v_i)^2 - f(v_j)^2|}{2} \right\rceil \geq 1 + \text{diam}G = 1 + 2 = 3$$

$$d(v_i, v_j) + \left\lceil \frac{|(i+1)^2 - (j+1)^2|}{2} \right\rceil \geq 3$$

$$d(v_i, v_j) + \left\lceil \frac{|i^2 - j^2 + 2(i-j)|}{2} \right\rceil \geq 3$$

The Radio Analytic mean condition is satisfied for all pairs of vertices. Hence f is Radio Analytic mean labeling of $(k_{1,n})$. Therefore Radio analytic mean number of f is $n+1$. Radio analytic mean labeling number of $k_{1,n}$. i.e $\text{ramn}(G) = n+1$

6

2

$f(v) = n+2,$
 $f(v_i) = (n+2) + i, 1 \leq i \leq n.$

We have to prove that all the pairs of **Bistar graph** satisfied Radio Analytic mean condition.

$d(u,v) + \left\lceil \frac{|f(u)^2 - f(v)^2|}{2} \right\rceil \geq 1 + \text{diam}G$

case(i): verify the pair (u,v), $1 \leq i \leq n$

$d(u,v) + \left\lceil \frac{|f(u)^2 - f(v)^2|}{2} \right\rceil \geq 1 + \text{diam}G$

$1 + \left\lceil \frac{|(n+1)^2 - (n+2)^2|}{2} \right\rceil \geq 4$

Case(ii): verify the pair (u,u_i) for $1 \leq i \leq n$

$d(u, u_i) + \left\lceil \frac{|f(u)^2 - f(u_i)^2|}{2} \right\rceil \geq 1 + \text{diam}G$

$1 + \left\lceil \frac{|(n+1)^2 - (i)^2|}{2} \right\rceil \geq 4$

Case(iii): verify the pair (u_i, u_j) $i \neq j$ for $1 \leq i, j \leq n$

$d(u_i, u_j) + \left\lceil \frac{|f(u_i)^2 - f(u_j)^2|}{2} \right\rceil \geq 1 + \text{diam}G$

Example:

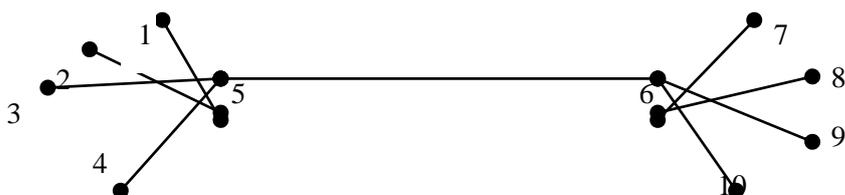


Figure 3
Radio Analytic mean number of $(B_{n,n})$ is $2n+2, n \geq 4.$

Theorem 2.5: The Radio Analytic mean number of complete Bipartite graph $K_{x,y}$ is $3+n, n \geq 4$

Proof: Let the vertex set and edge set of complete Bipartite graph be $V(K_{x,y}) = \{V_i, W_i \mid 1 \leq i \leq x, \text{ and } 1 \leq i \leq y\}$

Now $V(G) = x+y$ & $E(G) = xy$. We define a bijective map $f: V(K_{x,y}) \rightarrow \{1, 2, 3, \dots, 3+n\}$

Let $f(V_i) = i, 1 \leq i \leq x$

$f(w_i) = n+i, 1 \leq i \leq y.$

The diameter of the complete Bipartite graph is 2.

Now we check the Radio Analytic mean condition.

$d(u,v) + \left\lceil \frac{|f(u)^2 - f(v)^2|}{2} \right\rceil \geq 1 + \text{diam}G$

Case(i): verify the pair $(v_i, w_i), 1 \leq i \leq x$ and $1 \leq i \leq y$

$d(v_i, w_i) + \left\lceil \frac{|f(v_i)^2 - f(w_i)^2|}{2} \right\rceil \geq 1 + \text{diam}G = 1+2=3$

$2 + \left\lceil \frac{|(i+1)^2 - (i+1)^2|}{2} \right\rceil \geq 4$

Case(iv): verify the pair $(u, v_i), 1 \leq i \leq n$

$d(u, v_i) + \left\lceil \frac{|f(u)^2 - f(v_i)^2|}{2} \right\rceil \geq 1 + \text{diam}G$

$2 + \left\lceil \frac{|(n+1)^2 - (n+2+i)^2|}{2} \right\rceil \geq 4$

Case(v): verify the pair (u_i, v_i) for $1 \leq i \leq n$

$d(u_i, v_i) + \left\lceil \frac{|f(u_i)^2 - f(v_i)^2|}{2} \right\rceil \geq 1 + \text{diam}G$

$3 + \left\lceil \frac{|(i)^2 - (n+2+i)^2|}{2} \right\rceil \geq 4$

Case(vi): verify the pair (v, v_i) for $1 \leq i \leq n$

$d(v, v_i) + \left\lceil \frac{|f(v)^2 - f(v_i)^2|}{2} \right\rceil \geq 1 + \text{diam}G$

$3 + \left\lceil \frac{|(n+2)^2 - (n+2+i)^2|}{2} \right\rceil \geq 4$

Case(vii): verify the pair (v_i, v_j) for $1 \leq i \leq n$

$d(v_i, v_j) + \left\lceil \frac{|f(v_i)^2 - f(v_j)^2|}{2} \right\rceil \geq 1 + \text{diam}G$

$2 + \left\lceil \frac{|(n+2+i)^2 - (n+2+j)^2|}{2} \right\rceil \geq 4$

Hence these cases are establishes all the pairs satisfied the radio analytic mean condition

The Ra
 $1 + \left\lceil \frac{|(i)^2 - (n+i)^2|}{2} \right\rceil \geq 3$

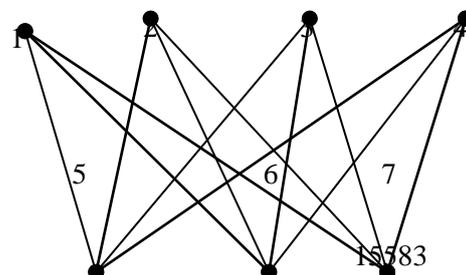
Case(ii): verify the pair (v_1, v_{2i}) for $1 \leq i \leq x$

$d(v_1, v_{2i}) + \left\lceil \frac{|f(v_1)^2 - f(v_{2i})^2|}{2} \right\rceil \geq 3$

$2 + \left\lceil \frac{|(i)^2 - (i+1)^2|}{2} \right\rceil \geq 3$

Consequently this process remaining pairs satisfied The Radio Analytic mean condition.

Example:



The Radio Analytic mean number of $(k_{x,y})$ is $3+n, n \geq 4$.

Conclusion: In this paper the concept of Radio Analytic mean condition behavior of path graph, cycle graph, star graph, Bistar graph, Bipartite graph are admitted RAMN. In future this work we can extended to various graphs.

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