March - April 2020
ISSN: 0193-4120 Page No. 10917 - 10925

On the Psi -Instability of a Non-Linear Difference System

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Abstract: The objective of this work to give sufficient condition for on ψ – Instability of nonlinear difference system

x(m+1) = A (m) x(m) +f (m, x(m)) (1.1)

and the linear difference system of the form

x(m+1) = A (m) x(m) +B (m) x(m) (1.2)

As a perturbed system of x (m+1) =A(m) x (m) (1.3)

In equation (1.1) and (1.2) we consider that *A(m) and B(m)* are n ×n matrix valued mappings on *N* and f is a vector function of order *n* on N

I. Introduction

Difference equations involved in many areas such as mathematical modeling, finite element methods, control theory, numerical methods, and more problems of discrete mathematical structures. The theory of difference equations gives the better explanation to construction of discrete mathematical models, when compared to continuous models.

The stability of solutions of differential system work have been done by Lakshmi kantham and Rama Mohana Rao [4], by Mahfoud [5] , Avramescu [2], by Hara, Yoneyama and Itoh [3] and others. Further, the instability of the solutions of a systems of differential equations was studied by Coppel [6] and Diamandescu, A. was study the Ψ –stability [7] and Ψ–instability of a nonlinear Volterra integro-differential system [8]. Further, Ψ- Stability for nonlinear difference equations was studied by T.S.Rao, G. Suresh Kumar and M.S.N. Murty [10]. Recently the concept of strong stability for difference system was studied by Naga Jyothi G, T S Rao, Suresh Kumar. G and Nageswara Rao.T [11].

The ψ - instability for nonlinear difference system are not yet discussed. With the motivation of the above works, here we obtain sufficient conditions for the instability for the zero solution of nonlinear difference system.

II. Definitions

Let Ψ=be a diagonal matrix of order m defined by Ψ=dig [Ψ1, Ψ_2, ----Ψ_n]

Where $\psi_i :$ N $\to \infty$. i=1. 2, ----n

Then the matrix Ψ (m) is an nonsingular square matrix of order n, for all $m \in N$.

Definition2.1 [1] : If the vector valued function u(m) in R^n satisfies the difference equation (1.1), then u(m) is called a solution of (1.1). It is clear that u(m)=0 (zero vector) is always solution of (1.1) and is called trivial solution or zero solution of (1.1). And also u (m) is trivial solution or zero solution of (1.2).

March - April 2020
ISSN: 0193-4120 Page No. 10917 - 10925

Definition 2.2[1] : Any *n X n* matrix Z (m) whose columns are linear independent solutions of difference equation (1.1) is called a fundamental matrix or principal matrix of (1.1).

Definition 2.3[1] . If for each m_0 in *N* and every ε> 0, there exists δ= δ(ε, m_0) > 0 such that any solution x(m) of (1) which satisfies the inequality $\|\Psi(m_0)x(m_0)\|<\delta$, also exists and satisfies the inequality $\|\Psi(m)x(m)\| < \varepsilon$ for all m $\geq m_0$, then the zero solution of (1.1) is called to be Ψ-stable on *N*. Otherwise it is called Ψ- instable.

III. Main Results

Lemma3.1.

Let Z (m) be an non singular matrix mapping on N and let Q be a projection. If there is mapping $\mu : N \to (0,\infty)$ and a non-negative value L such that

$$\sum_{s=0}^{t} \mu(r)\left|\psi(m)Z(m)QZ^{-1}(r)\psi^{-1}(r)\right| \leq L$$

Then $\forall\ m \geq 0$ & $\sum_{r=0}^{+\infty} \mu(r) = \infty$

there exists a constant M >0 such that

$$\left|\psi(m)Z(m)Q\right| \leq M e^{-L^{-1}\sum_{r=0}^{m}\mu(r)} \quad \forall\ m \geq 0.$$

Consequently $\lim_{m\to\infty}\left|\psi(m)Z(m)Q\right| = 0$.

Proof:

If Q=0, the conditions are trivially true

If $Q \neq 0$

Let s(m)= $\left|\psi(m)Z(m)Q\right|^{-1}$ for $\forall\ m \geq 0$

$$\sum_{s=0}^{t}[\mu(r)s(r)]\psi(m)Z(m)Q =$$

$$\sum_{s=0}^{t}\mu(r)\,|\,\psi(m)Z(m)Q\,|\,Z^{-1}(r)\psi^{-1}(r)\psi(r)Z(r)Qs(r)$$

for $m \geq 0$

$$[\sum_{s=0}^{t}\mu(r)s(r)]\left|\psi(m)Z(m)Q\right| \leq$$

$\sum_{r=0}^{m}\mu(r)\left|\psi(m)Z(m)QZ^{-1}(r)\psi^{-1}(r)\right|$ At the Thus,

$\left|\psi(r)Z(r)Q\right|s(r)$ for $m \geq 0$

scalar mapping $g(m) = \sum_{r=0}^{m}\mu(r)s(r)$ satisfies for

$$g(m)s^{-1}(m) = \sum_{r=0}^{m}\mu(m)\left|\psi(m)Z(m)Q\right|^{-1}\left|\psi(m)Z(m)Q\right|$$

$$= \sum_{r=0}^{m}\mu(r)\left|\psi(m)Z(m)QZ^{-1}(r)\psi^{-1}(r)\right|$$

$$\leq L \quad \forall\ m \geq 0$$

we have $g(m+1) = \mu(m)s(m)$

$$g(m+1) \geq L^{-1}\mu(m)g(m) \quad \forall\ m \geq 0$$

It gives that $g(m) \geq g(m_1)e^{-L^{-1}\sum_{r=m_1}^{m}\mu(m)}$ for all $m \geq m_1 \geq 0$

Hence $\left|\psi(m)Z(m)Q\right| = s^{-1}(m) \leq Lg^{-1}(m)$

$$\leq Lg^{-1}(m_1)e^{-L^{-1}\sum_{s=t_1}^{t}\mu(r)} \quad forall\ m \geq m_1 \geq 0$$

since $\left|\psi(m)Z(m)Q\right|$ is function on $[0, m_1]$

Now we got a non-negative constant M such that

$$\left|\psi(m)Z(m)Q\right| \leq Me^{-L^{-1}\sum_{r=0}^{m}\mu(r)} \quad for\ m \geq 0$$

Consequently $\lim_{m\to\infty}\left|\psi(m)Z(m)Q\right| = 0$

Hence proved.

Lemma 3.2.

If Z (m) be an non singular matrix which is a mapping of m on N and let Q be a projection. If there is a non-negative value L such that

$$\sum_{r=m}^{\infty}\left|\psi(m)Z(m)QZ^{-1}(r)\psi^{-1}(r)\right|\leq L$$
$\forall\ m\geq 0$

Then for any vector $y_0\in R^n \ni Qy_0\neq 0$,
$\lim_{m\to\infty}\sup\left\|\psi(m)Z(m)Qy_0\right\|=+\infty$.

Proof:

Let Q be a projection and consider Z (m) be an non singular matrix which is a mapping of m on N .

Let $s(m)=\left\|\psi(m)Z(m)Qy_0\right\|^{-1}$ for $m\geq 0$

From the identity

$$(\sum_{r=m}^{T}s(r))\psi(m)Z(m)Qy_0 = \sum_{s=t}^{T}\psi(m)Z(m)QZ^{-1}(r)\psi^{-1}(r)\psi(r)Z(r)Qy_0s(r)$$ for

$T\geq m\geq 0$, It gives that

$$(\sum_{r=m}^{T}s(r))\left\|\psi(m)Z(m)Qy_0\right\| \leq \sum_{r=m}^{T}\left|\psi(m)Z(m)QZ^{-1}(r)\psi^{-1}(r)\right| \left\|\psi(r)Z(r)Qy_0\right\|s(r)\ \ \forall\ T\geq m\geq 0$$

Let Q be a projection and consider Z (m) be an non singular matrix which is a mapping of m on N
Let $s(m)=\left\|\psi(m)Z(m)Qy_0\right\|^{-1}$ for $m\geq 0$

$(\sum_{r=m}^{T}s(r))s^{-1}(m)\leq L\ \ \forall\ T\geq m\geq 0$ From that $\sum_{r=m}^{T}s(r)$ exists.

Thus $\lim_{m\to\infty}\inf s(m)=0 \Rightarrow$
$\lim_{m\to\infty}\sup\left\|\psi(m)Z(m)Qy_0\right\|=+\infty$

Hence proved.

IV.Ψ-Instability of Linear Difference System

The main purpose of this section is discuss the ψ – instability of zero solutions of linear equation (1.2) and (1.3).

Theorem 4.1: .The zero solution of (1.3) is ψ – unstable on N iff there is a projection Q such that $|\psi(m)Z(m)Q|$ is unbounded on N, where Z(m) is principal matrix for (1.3)

Proof:

Let Z(m) be a principal matrix for(1.3) .

Prove that the zero solution of (1.3) is

ψ – un stable on N $\Leftrightarrow$

there is a projection Q such that $|\psi(m)Z(m)Q|$ is unbounded on N.

Necessary Condition :

Let the zero solution of (1.3) is

ψ – un stable on N.

Then by known theorem, $|\psi(m)Z(m)Q|$ is unbounded on N.

Sufficient Condition: Suppose $|\psi(m)Z(m)Q|$ is unbounded on N.

March - April 2020
ISSN: 0193-4120 Page No. 10917 - 10925

To show that the zero solution of (1.3) is ψ – unstable on N.

If possible assume that the zero solution of (1.3) is Ψ-stable on N.

Then by definition, for each ε>0 and $m_0 \geq 0$ $\exists$ a $\delta = \delta(\in, m_0)$ >0 such that any solution x(m) of (1.3) which gives $\|\psi(m_0)y(m_0)\| < \delta(\in, m_0) \Rightarrow \|\psi(m)y(m)\| < \in \quad \forall$ $m \geq m_0$.

Let $m_0 \geq 0$ and $y_0 \in R^n$ such that $|\psi(m_0)Z(m_0)Q| \neq 0$ and

$$\|y_0\| < \frac{\rho}{|\psi(m_0)Z(m_0)Q|}$$

$$< \rho \, |\psi(m_0)Z(m_0)Q|^{-1} = \rho_0$$

We have $\|\psi(m_0)Z(m_0)Qy_0\| < \rho$

It follows that $|\psi(m)Z(m)Qy_0| < \in \quad \forall \quad m \geq m_0$ (1)

Thus for $v \in R^n, \|v\| = 1,$ we have $|\psi(m)Z(m)Q\rho_0 v| < \in \quad \forall \quad m \geq m_0$.

Thus $|\psi(m)Z(m)Q| < \in \rho_0^{-1} \quad \forall \quad m \geq m_0$

Which is Contradiction to eq (1.3)

Our supposition zero solution of (1.3) is Ψ-stable on N is wrong

Hence zero solution of (1.3) is Ψ -unstable on N

Theorem 4.2: If there is a projection Q≠0 and non-negative value L such that principal matrix Z(m) for equation (1.3) $\sum_{r=m}^{\infty} |\psi(m)Z(m)QZ^{-1}(r)\psi^{-1}(r)| \leq L \quad \forall$ $m \geq 0$.Then , the zero solution of (1.3) is ψ – unstable on N.

Proof:

Let Q≠0 be a projection and a non-negative constant L and Z(m) be principal matrix of x(m+1)=A(m)x(m)---------(1.3) and Z(m) satisfies the inequality $\sum_{r=m}^{\infty} |\psi(m)Z(m)QZ^{-1}(r)\psi^{-1}(r)| \leq L$ $\forall \quad m \geq 0$.

In country way, we assume that the zero solution of (1.3) is ψ – Stable on N, instead of showing that the zero solution of (1.3) is ψ – unstable on N

Then from the definition of Ψ-stable, for each ε>0 and any m_0>0 $\exists$ a $\rho = \rho(\in, m_0)$ >0 such that any solution y(m) of (1.3) which satisfies the inequality $\|\psi(m_0)y(m_0)\| < \rho(\in, m_0)$ exists and satisfies the equality $\|\psi(m)y(m)\| < \in \quad \forall$ $m \geq m_0$.

Without changing the principality, we may assume Z(0) = I (or we replace Z(m) with $Z(m)Z^{-1}(m)$ and p with $Z(0)Z^{-1}(0)$).

For $m_0 = 0$, we can choose $y(0) \in R^n$ such that (I-Q)y(0)=0 and

$$0 < \|\psi(0)y(0)\| < \rho(\in, 0).$$

Then $\|\psi(m)y(m)\| < \in \quad \forall \quad m \geq 0$ ------------------- (1)

On other hand,

$$\psi(m)y(m) = \psi(m)Z(m)Z^{-1}(0)y(0)$$

$$= \psi(m)y(m)Qy(0)$$

From above lemma,

March - April 2020
ISSN: 0193-4120 Page No. 10917 - 10925

We got that $\lim_{m\to\infty}\sup\|\psi(m)y(m)\| = \infty$.

Which is Contradiction to equation (1)

Our supposition " zero solution of (1.3) is Ψ-stable on N" is wrong.

Hence zero solution of (1.3) is Ψ-unstable on N.

Theorem4.3 If

a) There exists supplementary projections Q_1, Q_2, $Q_2 \neq 0$ and a nonnegative value J such that the principal matrix Z(m) of (1.3) satisfies the condition

$$\sum_{r=0}^{m}\left|\psi(m)Z(m)Q_1Z^{-1}(r)\psi^{-1}(r)\right| +$$
$$\sum_{r=m}^{\infty}\left|\psi(m)Z(m)Q_2Z^{-1}(r)\psi^{-1}(r)\right| \le J \quad \forall\ m \ge 0$$

b) C (m) is a $n\times n$ matrix mapping for $m \ge 0$ and satisfies the following conditions.

$L = \sup_{m\ge 0}\left|\psi(m)C(m)\psi^{-1}(m)\right|$ is very small number such that LJ<1.

$$\lim_{m\to\infty}\left|\psi(m)C(m)\psi^{-1}(m)\right| = 0$$

Then (1.2) is ψ -unstable on N.

Proof:

case (1) Suppose there exists supplementary projections Q_1, Q_2, $Q_2 \neq 0$ and a non negative value J such that the principal matrix Z(m) of the equation x(m+1) = A(m)x(m)-------(1.3) satisfies the condition

$$\sum_{r=0}^{m}\left|\psi(m)Z(m)Q_1Z^{-1}(r)\psi^{-1}(r)\right| +$$
$$\sum_{s=t}^{\infty}\left|\psi(m)Z(m)Q_2Z^{-1}(r)\psi^{-1}(r)\right| \le J$$

$\forall\ m \ge 0$

To show that the linear equation x(m+1)=A(m)x(m)+B(m)x(m)--- (1.2) is Ψ-unstable on N.

We prove this in controversy

If possible assume that equation (1.2) satisfies the definition of Ψ-stability on N.

Then from the Ψ-stable definition, for each ε>0 and $m_0 \ge 0$, $\exists\ a\ \rho = \rho(\in, m_0) > 0$ such that any y(m) of (1.2) which gives $\|\psi(m_0)y(m_0)\| < \rho$ exists and satisfies the equality $\|\psi(m)x(m)\| < \in$ $\forall\ m \ge m_0$.

Without changing the principality, we assume that Y (0) = I

For m_0 we can choose $y(0)\in R^n$ such that $Q_1y(0)=0$ and $0<\|\Psi(0)y(0)\|<\rho$. Then $\|\Psi(m)y(m)\|<\in$ for all $m\ge m_0$ We consider the mapping

$$y(m) = x(m) - \sum_{s=0}^{t} Z(m)Q_1Z^{-1}(r)B(r)x(r)$$
$$+\sum_{s=t}^{\infty} Z(m)Q_2Z^{-1}(r)B(r)x(r)$$

for $m \ge 0$

for $u \ge m \ge 0$, we have

$$\left|\sum_{r=m}^{u} Z(m)Q_2Z^{-1}(r)B(r)x(r)\right| \le \left|\psi^{-1}(m)\right|$$
$$\sum_{r=m}^{u}\left|\psi(m)Z(m)Q_2Z^{-1}(r)\psi^{-1}(r)\right| \qquad m \ge m_0.$$
$$\|\psi(r)B(r)\psi^{-1}(r)\|\|\psi(r)x(r)\|$$

March - April 2020
ISSN: 0193-4120 Page No. 10917 - 10925

$$\leq \left|\psi^{-1}(m)\right| L \in$$
$$\sum_{r=m}^{u}\left|\psi(m)Z(m)Q_2Z^{-1}(r)\psi^{-1}(r)\right|$$

It follows that $\sum_{r=m}^{u} Z(m)Q_2Z^{-1}(r)B(r)x(r)$ is convergent. Thus the mapping $y(m)$ exist for $m \geq 0$.

Clearly, the mapping $y(m)$ is differentiable on N and we have

$$y(m+1) = x(m+1)$$
$$-\sum_{r=0}^{m} Z(m+1)Q_1Q^{-1}(r)B(r)x(r)$$
$$-Z(m)Q_1Z^{-1}(m)B(m)x(m)$$
$$+\sum_{r=m}^{\infty} Z(m+1)Q_2Q^{-1}(m)B(m)x(m)$$
$$-Z(m)Q_2Z^{-1}(m)B(m)x(m)$$

$$= A(m)x(m) + B(m)x(m) -$$
$$\sum_{r=0}^{m} A(m)Z(m)Q_1Z^{-1}(r)B(r)x(r)$$
$$-Z(m)Z^{-1}(m)B(m)x(m)(Q_1+Q_2)$$
$$+\sum_{r=m}^{\infty} A(m)Z(m)Q_2Z^{-1}(r)B(r)x(r)$$

$$= A(m)x(m) + B(m)x(m) - A(m)$$
$$[\sum_{r=0}^{m} Z(m)Q_1Z^{-1}(r)B(r)x(r)]$$
$$-B(m)x(m)(I)$$
$$+A(m)[\sum_{r=m}^{\infty} A(m)Z(m)Q_2Z^{-1}(r)B(r)x(r)]$$
$$= A(m)[x(m) - \sum_{r=0}^{m} Z(m)Q_1Z^{-1}(r)B(r)x(r)$$
$$+\sum_{r=m}^{\infty} Z(m)Q_2Z^{-1}(r)B(r)x(r)]$$

$$= A(m)y(m)$$

Thus the mapping $y(m)$ satisfies the (1.3) on N.

Since,

$$\leq \in + \left\|\psi(r)x(r)\right\|\left|\psi(r)B(r)\psi^{-1}(r)\right|$$
$$[\sum_{r=0}^{m}\left|\psi(m)Z(m)Q_1Z^{-1}(r)\psi^{-1}(r)\right|$$
$$+\sum_{s=t}^{\infty}\left|\psi(m)Y(m)p_2Z^{-1}(r)\psi^{-1}(r)\right|$$

$$\leq \in + \in LJ$$

$$\left\|\psi(m)y(m)\right\| \leq \in (1+LJ) \quad for\ m \geq 0$$

$$\leq \in (1+LJ) \quad \text{for } m \geq 0$$

Now we got that the solution $y(m)$ is ψ -bounded on N.

On the other hand,

$$\begin{aligned} y(m) &= Z(m)Z^{-1}(0)y(0) \\ &= Z(m)[Q_1+Q_2]y(0) \\ &= YZ(m)[Q_1y(0)+Q_2y(0)] \\ &= Z(m)[0+Q_2Z(0)] \\ Y(t) &= Z(t)Z_2y(0)] \end{aligned}$$

If $p_2y(0) \neq 0$ from lemma 2, we got that $\limsup_{t\to\infty}\left\|\psi(t)Y(t)p_2(0)\right\| = \infty$

This is wrong

Thus $Q_2y(0) = 0$ and then $y(m) = 0$ on N.

There after
$$x(t) = \sum_{r=0}^{m} Z(m)Q_1Z^{-1}(r)B(r)x(r) -$$
$$\sum_{r=m}^{\infty} Z(m)Q_2Z^{-1}(r)B(r)x(r) \quad for\ t \geq 0$$

It gives that

March - April 2020
ISSN: 0193-4120 Page No. 10917 - 10925

$$\|\psi(m)x(m)\| = \left\| \begin{array}{l} \psi(m)[\sum_{r=0}^{u} Z(m)p_1 Y^{-1}(r)B(r)x(r) \\ -\sum_{r=u}^{\infty} Y(t)p_2 Y^{-1}(r)B(r)x(r)] \end{array} \right\|$$

$$\leq LJ \sup_{m\geq 0}\|\psi(m)x(m)\| \quad for \ m\geq 0$$

Which is contradiction to $\|\psi(m)x(m)\| < \in$

So, our assumption that (1.2) is Ψ-stable on N" is wrong.

Hence (1.2) is Ψ-unstable on N.

Part (b) can be obtained from (a), in similar way.

Hence proved.

V. Ψ-Instability of the nonlinear difference equations

The object of this section gives the sufficient conditions for instability of the zero solution of the nonlinear difference equation (1.1)

Theorem 5.1: If

(1) There are supplementary projections Q_1, Q_2, Q_2#0 and a non-negative value J>0 such that the principal matrix Z(m) of the equation (1.3) satisfies the condition

$$\sum_{r=0}^{m}\left|\psi(m)Z(m)Q_1Z^{-1}(r)\psi^{-1}(r)\right| + \sum_{r=m}^{\infty}\left|\psi(m)Z(m)Q_2Z^{-1}(r)\psi^{-1}(r)\right| \leq J \quad \forall \ \mathrm{m}\geq 0$$

(2) The mapping f (t, s, x) satisfies the inequality

$$\|\psi(m)f(m,r,x(r)\| \leq f(m,r)\|\psi(r)x(r)\|,$$
$$0\leq r\leq m<\infty,$$

for every mapping $x: N \to R^n$, here

f(m, r) is a nonnegative mapping on D which satisfies $\sum_{r=0}^{m} f(m,r) \leq L \ \forall \ m\geq 0$ where L is positive constant.

(3) LJ<1.

Then, the zero solution of (1.1) is ψ -un stable on N.

Proof:

In country way, we suppose that the trivial solution of (1.1) is ψ -stable on N. Then by definition of ψ -stable for each ∈>0 and any t_0>0, there exists a ρ=ρ(∈,m_0)>0 such that any solution x(t) of (1.1) which satisfies the inequalit $\|\psi(m_0)x(m_0)\| < (\in, m_0)$ exists and satisfies the equality $\|\psi(m)x(m)\| < \in \quad for \ m\geq m_0$. Without changing the principality, we may assume that Z (0) = I.

For $m_0 = 0$, we can choose $x(0)\in R^n$ such that $p_1x(0)=0$ and

$$0\leq \|\psi(0)x(0)\| < \delta(\in,0).$$

Then $\|\psi(m)x(m)\| < \in \ \forall \ m\geq 0$

Consider the mapping

$$y(m) = x(m) - \sum_{s=0}^{t} Z(m)Q_1Z^{-1}(r)\sum_{u=0}^{s} F(r,v,x(v)) + \sum_{s=t}^{\infty} Z(m)Q_2Z^{-1}(m)\sum_{u=0}^{s} F(r,v,x(v)) \quad for \ m\geq 0$$

March - April 2020
ISSN: 0193-4120 Page No. 10917 - 10925

for $u \geq m \geq 0$, *we have*

$$\| \sum_{m}^{v} Z(m)Q_2 Z^{-1}(r) \sum_{0}^{r} f(r,v,x(v)) \|$$

$$\leq \left|\psi^{-1}(m)\right| \sum_{r=m}^{u} \left|\psi(m)Z(m)Q_2 Z^{-1}(r)\psi^{-1}(r)\right|$$

$$\sum_{v=0}^{r} \|\psi(r) f(r,v,x(v))\|$$

$$\leq L \in \left|\psi^{-1}(m)\right| \sum_{s=m}^{u} \left|\psi(m)Z(m)Q_2 Z^{-1}(r)\psi^{-1}(r)\right|$$ It follows that

$$\sum_{r=m}^{u} Z(m)Q_2 Z^{-1}(r) \sum_{u=0}^{r} f(r,v,x(v))$$ is convergent.

Thus the mapping w (m) exists on N

Clearly the mapping w(m) is differentiable on N and we have

$$w(m+1) = A(m)x(m) + \sum_{r=0}^{m} f(m,r,x(r)) -$$

$$A(m)\sum_{s=0}^{t} Y(m)Y^{-1}(m) \sum_{u=0}^{s} f(r,u,x(v)) -$$

$$Y(m)(Q_1+Q_2)Z^{-1}(t)\sum_{v=0}^{m} f(x,v,v,x(v)) +$$

$$A(m)\sum_{s=t}^{\infty} Y(m)Q_2 Y^{-1}(r)\sum_{v=0}^{r} f(r,v,x(r))$$

$$w(m+1) = A(m)w(m)$$

$$w(m+1) = x(m+1) -$$

$$\sum_{r=0}^{m} Y(m+1)Q_1 Z^{-1}(r)\sum_{u=0}^{s} f(r,v,x(v))$$

$$-Z(m)Q_1 Z^{-1}(m)\sum_{v=0}^{m} f(m,v,x(v)) +$$

$$\sum_{r=m}^{\infty} Z(m+1)Q_2 Z^{-1}(r)\sum_{v=0}^{r} f(r,u,x(u)) - z(m)Q_2 Z^{-1}(m)$$

$$\sum_{v=0}^{m} f(m,v,x(v))$$

Thus x (m) satisfies the linear equation (1.3) on N

Since

$$\|\psi(m)x(m)\| \leq \|\psi(m)x(m)\| +$$

$$\sum_{r=0}^{m} \left|\psi(m)Y(m)Q_1 Z^{-1}(r)\psi^{-1}(r)\right|$$

$$\sum_{v=0}^{r} \|\psi(r) f(r,v,x(v))\| +$$

$$\sum_{r=m}^{\infty} \left|\psi(m)Z(m)Q_2\, Z^{-1}(r)\psi^{-1}(r)\right|$$

$$\sum_{v=0}^{r} \|\psi(r) f(r,v,x(v))\|$$

$$\|\psi(m)x(m)\| \leq \in +$$

$$\sum_{s=0}^{t} \left|\psi(m)Z(m)Q_1 Z^{-1}(r)\psi^{-1}(r)\right|$$

$$\sum_{v=0}^{r} f(r,v)\|\psi(v)x(v)\| +$$ the x (m) is

$$\sum_{r=m}^{\infty} \left|\psi(m)Z(m)Q_2\, Z^{-1}(r)\psi^{-1}(r)\right|$$

$$\sum_{v=0}^{r} f(r,v)\|\psi(v)x(v)\|$$

Ψ-bounded on N

Further,

$$\begin{aligned} w(m) &= Y(m)Y^{-1}(0)w(0) \\ &= Y(m)[Q_1 w(0) + Q_2 w(0)] \\ &= Y(m)Q_2 w(0) \qquad (\because Q_1 = 0) \end{aligned}$$

If $Q_2 w(0) \neq 0$ from previous lemma, we conclude that $\limsup_{m\to\infty} \|\psi(m)w(m)\| = \infty$ This is contradictory Thus $Q_2 w(0) = 0$ and then $w(m) = 0$ on N.

Hence

March - April 2020
ISSN: 0193-4120 Page No. 10917 - 10925

$$x(m)=\sum_{r=0}^{m} Z(m)Q_1Z^{-1}(r)\sum_{u=0}^{s} f(r,v,x(v))-$$

$$\sum_{s=t}^{\infty} Z(m)Q_2Z^{-1}(r)\sum_{v=0}^{r} f(r,v,x(v)) \quad for\ m\geq 0$$

Similar as above, it got that

$$\|\psi(m)x(m)\|\leq LJ\,\|\psi(m)x(m)\| \quad for\ m\geq 0$$

This contradicts our assumption. So, the zero solution of (1.1) is ψ -unstable on N.

Hence proved.

References

1. Agarwal R. P. 2000, Difference Equations and Inequalities Second edition, Marcel Dekker, New York
2. Hara, T., Yoneyama, T. and Ytoh, T. (1990) Funkcialaj Ecvacioj Asymptotic Stability Criteria for Nonlinear Volterra Integro - Differential Equations, 33, 39 - 57.
3. Avramescu, C. (1968) Seria Stiinte Matematice-Fizice, Asupra comport˜arii asimptotice a solutiilor unor ecuatii functionally; Analele Universit˜atii din Timisoara vol. VI, , 41 - 55.
4. Lakshmikantham, V. and Rama Mohana Rao, M. (1987) Appl. Anal. 24, Stability in variation for nonlinear integro differential equations 165 - 173.
5. Mahfoud, W.E Proc. Amer. Math. Soc., 100(1987) Bounded ness properties in Volterra integro-differential systems 37 - 45.
6. Coppel, W.A J. London Math. Soc.38 (1963) ,On the stability of ordinary differential equations, , 255 - 260.
7. Diamandescu, A (2005), On the Ψ - stability of a nonlinear Volterra integro-differential system , Electronic .J. Diff. Eqns., Vol. 2005, No. 56, 1 - 14;
8. Diamandescu, A. (2003), On the Ψ - instability of a non-linear voltera integro differential system, Bull. Math. Soc, Sc, Math. Roumanie Tome 46(94), No: 3-4, 103-119.
9. Diamandescu, A On the Strong Stability of a nonlinear Volterra Integra-Differential System . Acta Math. Univ. Comenianae Vol. 75(2) 153-162.
10. Rao T S,. Suresh Kumar G and Murty M.S.N (2018), Ψ- Stability for nonlinear difference equations: Thai Journal of Mathematics, , Vol: 16(3). 801 – 815
11. Naga Jyothi G, Rao T S, Suresh Kumar G and Nageswara Rao T , (2019) , Strong stability of a nonlinear difference system, , International Journal of Innovative Technology and Exploring Engineering (IJITEE), Volume; 8, Issue: 8: 20-25.
12. Suresh kumar G , Srinivasa Rao T. and Murty M.S.N (2013), A note on Ψ-Bounded Solutions For Non-Homogeneous Matrix Difference Equations, Bullitian of Mathematical Analysis and Applications, , Vol 5(1), 1-9
13. Rao T.S ,.Suresh Kumar .G, and Murty M.S.N, (2013), Ψ –Asymptotic stability of non-linear difference equations, Far East journal of mathematical sciences, Vol 83 (1)-21-39
14. Srinivasa Rao T. Suresh kumar G. , and Murty M.S.N. , 2014, On Ψ –Asymptotic stability for the I[st] order nonlinear matrix difference equations, Global journal of pure and Applied Mathematics, Vol 10(3), 417-429
15. Rao T.S, G. Suresh Kumar, and Murty M.S.N (2015), Existence and uniqueness of Ψ – bounded solutions for nonlinear matrix difference equations Italian journal of pure and applied mathematics, Vol 35-453-464.