

Versatility of Analytical Hierarchy Process as a Decision Making Tool

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Abstract

Day to day life run on the basis of small or big decisions, if the decision is correct the result is fruitful that means making correct decision is very important task for the successful life. The degree of success in our life is depends on the decision power and its absoluteness. In this study we discuss a decision making support method Analytical Hierarchy Process for various problems in our day to day life. This method is applicable for such problems in which number of criteria and sub criteria are involved. The decision making is not simple task when there are many criteria are to be checked. Particularly when there are many criteria are to be checked, compared, it becomes very complex task for decision makers. It is not only complex but also time taking and sometimes costly. So we need some decision making method which will save our efforts, time and cost and will give optimal results. Analytical Hierarchical Process is one of the Multicriteria Decision Analysis method (MCDA) and was first developed by Thomas L. Saaty in 1970. Now day's machines are expertise for decision making, in such cases AHP can be used to train the machines for decision making purpose.

Keywords: Decision, Decision making, Criteria, Analytical Hierarchy Process, Comparison, priority, Rank, Complexity. Multicriteria, Decision maker, MCDA

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1. Introduction to Decision making and AHP

Correct decisions are important in completion of successful tasks. Behind any successful achievement there are number of decision makers who intellectually work to sort out the problem by analyzing the various aspects of the problem. Decision making is become essential for any organization and its management. Particularly problems are solved, situations and crises are handled and conflicts are solved using the decision making. The decision making process involve the finding and defining the problem, comparison of available data, and using exact action. Decision making is a process of choosing optimal criteria and ranking them on the basis of their importance. In the process of decision making decision makers have to judge the various

available options and minimize the options. Decision making is a human conceptual and conscious process for individual as well as for social phenomena on the basis of facts and the values with the choice of activity among one or more existing alternatives to fulfill the desired outcome [1]. Decision making is a process of judging available all alternatives and finalize the appropriate alternative by deducting the less appropriate alternatives. Also he has maintained the definition of decision making of Kreitner (1966) which says that, "Decision making is a process of identifying and choosing an alternative course of action in a manner appropriate to the demand of the situation"[2]. The process of decision making may involve many criteria and subcriteria to rank the available alternatives [3]. [4] Can be referred to know that how the decision-making process is carried out in everyday life[5]

is a review paper in which he maintained two types of decision making as Individual decision making techniques and group decision making techniques. The personal decision making techniques are cons and pros, Simple Prioritization of options and find the satisfaction level and so on. Consensus decision-making, Voting-based methods, Democratic decision-making, are the techniques under group decision making. Also he has maintained seven steps of decision making as Creation of common space, Perception, Interpretation, Judgment, Motivation, action and Reflection in action. Also he has discussed applications of decision making like Decision-making in military and martial environments, Decision-making in therapeutic environments, Islamic decision-making.

Decision making is not easy task as it involves number of alternatives to be compare, while decision making comparison of each alternatives is to be done with every alternative and assign the importance or weight to every alternative on the basis of comparison. As the problem is having number of alternatives the number of comparison increases and the process becomes more and more complex. Handling such problems becomes difficult for decision makers that's why the decision making process is a complex, critical and time taking process for human. Many times other aspects like situation, time, cost, capacity of decision makers and the analysis of vast data will also affect the decision making process, all these are to be considered while decision making. The decision making can be affected by heuristic, overconfidence, hindsight bias, illusory correlation etc. also [6]. In [7] the author listed types of multi attribute decision making methods and its sub types when alternatives and criteria are finite. 1. Cost-benefit analysis, 2. Elementary methods- Pros and cons analysis, Maximin and maximax methods, Conjunctive and disjunctive methods, Lexicographic method, 3. MAUT methods -

Simple multiattribute rating technique (SMART), Generalized means, The Analytic Hierarchy Process, 4. Outranking methods- The ELECTRE methods, The PROMETHEE methods, 5. Group decision making, 6. Sensitivity analysis

In this study Analytic hierarchy process is studied as decision making process in relation of handling multiattribute problems and its scope in various applications. One of the view behind this study is that the AHP technique can be used to train the machines for the decision making and will speed up, save time, cost and efforts.

2. AHP history, uses and application areas

Most of the problems in the world are tackled with the 'AHP' mathematical and psychological scientific technique for analyzing the complex decision making approach devised by Thomas L. Saaty (July 18, 1926 – August 14, 2017) in 1970, who was a professor at Joseph M. Katz Graduate School of Business of Pittsburgh University. Before it, he was working as professor of Operation Research at the Wharton School of the University of Pennsylvania and he was leading a project of U.S. Department of state for Arms Control and Disarmament. He was very disappointed by the team of his three people who is working for the project. Even years later when he was working as a teacher at Wharton School motivated to devise the AHP. He has invented Analytic Hierarchy Process (AHP) to help in solving the decision making problems to ordinary people when he found that it is difficult for the scientists and lawyers too to sort out the problem where decisions are to be taken on the basis of available alternatives. [8], [9], [10]. Generally AHP works in following steps.

1. Modeling of the problem in hierarchy along with goal, alternatives and evaluation criteria.
2. Prioritization of the elements of hierarchy on the basis of judgments by doing pairwise comparisons.

3. Find the overall priority on the basis of elements and judgments
4. Check the consistency for the judgments
5. Conclude the final decision [11].

AHP is used in solving variant problems and the list of applications is not limited like government sectors, business and industry, medical, construction, education, research, military and so on. If we refer the Wikipedia about the AHP, we will get broad list of AHP applied in various problems like.

1. Selection of nuclear reactors
2. Deciding how best to reduce the impact of global climate change
3. Quantifying the overall quality of software system
4. Faculty selection process in University and colleges
5. Deciding locations for various plants
6. Risk assessment for various areas
7. More Effectively Define and Evaluate SAP Implementation Approaches
8. Construction sector.

From the above list it is clear that AHP is applicable for any area and any situation where the operations like Selection of Choice, Ranking, Prioritizing, Resource allocation, Bench marking, Medical and Health care, Public policy, Strategic planning, Quality management and Conflict resolution are to be performed. Also Forecasting, total quality management, business process reengineering, quality function deployment, and the balanced scorecard are areas where AHP is applicable. So the list of applications of AHP is not small and there are such applications which are not maintained anywhere for its confidentiality [12], [13], [14].

The AHP? : The literature study gives number of definitions of AHP, some are listed below.

The AHP is a method that can be used to establish measures in both the physical and social domains. In using the AHP to model a problem one needs a hierarchic or a network structure to represent that problem and pairwise comparisons to establish relations within the structure. The Analytic Hierarchy Process (AHP) is a theory of measurement through pairwise comparisons and relies on the judgments of experts to derive priority scales [3]. The Analytic Hierarchy Process (AHP) is a general theory of measurement. It is used to derive ratio scales from both discrete and continuous paired comparisons. These comparisons may be taken from actual measurements or from a fundamental scale which reflects the relative strength of preferences and feelings [15]. AHP is multiattribute decision making method [16]. Broadly speaking, the AHP is a theory and methodology for relative measurement [17]. The Analytic Hierarchy Process (AHP) is a methodology for structuring, measurement, and synthesis [18].

In the study it is found that the AHP is also useful in intangible criteria and alternatives. This feature of AHP extends it to handle the multi-criteria decision making problems i.e. MCDA and it can be said that AHP belongs to the multi-criteria decision making methods group. [17], [19].

Analytical Hierarchy process involves three basic functions which help to understand it; Structuring Complexity, Measurement, and Synthesis, also it helps to know the ease of handling complexity of AHP problem and its vast applicability to variety of problems [13].

Structuring Complexity: Thomas L. Saaty comes to know that how people dealt with complexity after examining numerous examples and it was the Hierarchical Structuring of Complexity into homogeneous clusters of factors. That is the problem can be arranged in

hierarchical levels. Particularly at first level, goal – overall objectives, at second level criteria or constraints and at final level alternatives, if we consider the structure is of three level.

Measurement: Thomas L.Saaty's AHP method optimally measure the factor in hierarchy with the help of Ratio Scale Measurement. In AHP to derive ratio scale measurement the judgments of the ratio of each pair of criteria are used. AHP requires ratio scale priorities for elements because the priorities or weights of the elements at any

level in hierarchy can be determined by having the product of priorities of the elements in that level by the priorities of the parent element. Even AHP allows to measure the ratio scale at lowest level of the hierarchy.

The comparison in this step can be performed using the generalized scale provided in the following tabulated data. This table shows the importance of one element over another or revers versa with respect to criterion, with respect which they are compared [11].

Table 1. The basic scale for absoluteness

Serial no.	Intensity of importance on an absolute scale	Defamation	Details
1	1	Equal Importance	Two activities contribute equally to the objective
2	3	Moderate importance of one over another	Experience and judgment strongly favor one activity over another
3	5	Essential or strong importance	Experience and judgment strongly favor one activity over another
4	7	Very strong importance	An activity is strongly favored and its dominance demonstrated in practice
5	9	Extreme importance	The evidence favoring one activity over another is of the highest possible order of affirmation
6	2,4,6,8	Intermediate values between the two adjacent judgments	When compromise is needed
7	Reciprocals	If activity i has one of the above numbers assigned to it when compared with activity j, then j has the reciprocal value when compared with i	
8	Ratios arising from the scale		If consistency were to be forced by obtaining n numerical values to span the matrix

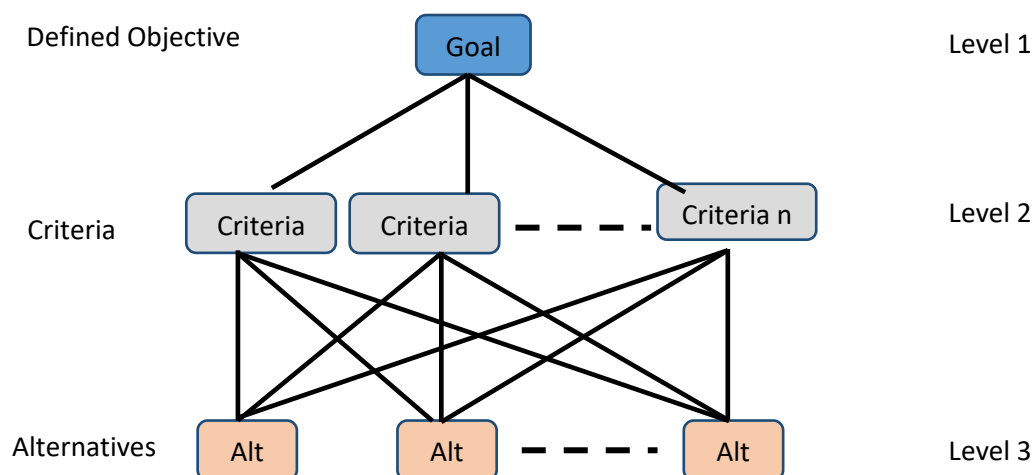
Synthesis: Synthesis function in AHP involves the process to combine into one. The synthesis

function helps to combine multi-dimensional elements comparison scale together.

AHP workflow: As per Thomas L. Saaty an Analytical Hierarchy process involve the below four steps in sort of any problem of decision making.

1. Define the problem and determine the kind of knowledge sought.
2. Structure the decision hierarchy from the top with the goal of the decision, then the objectives from a broad perspective, through the intermediate levels (criteria on which subsequent elements depend) to the lowest level (which usually is a set of the alternatives).
3. Construct a set of pairwise comparison matrices. Each element in an upper level is used to compare the elements in the level immediately below with respect to it.
4. Use the priorities obtained from the comparisons to weigh the priorities in the level immediately below. Do this for every element. Then for each element in the level below add its weighed values and obtain its overall or global priority. Continue this process of weighing and adding until the final priorities of the alternatives in the bottom most level are obtained[11].

The step 1 explains that to understand problem deeply and decide what exactly is to be find or achieve i.e. goal. After defining the goal in first step the second step built up the hierarchy structure in which the goal is placed at the top of hierarchy then decide the objectives or criteria of the problem from all aspects and put them below the goal in hierarchy and at lowest level the available alternatives will be there in hierarchy which is also known as AHP hierarchy structure. After completion of forming the hierarchy structure a matric is to be constructed which will hold the pairwise comparison of elements. Here every element in the level is compared with the element present in the level exactly below it. In the last step weights are assigned to elements as per the priorities decided on the basis of comparisons to the level exactly below. The same process is followed for every element. Then calculate the global priority by adding the weighted values with each element in the level above it. This priority calculation is to be continued i.e. weighting and adding till the final priorities of the alternatives at lowest level is obtained. The structure of decision hierarchy will look like as drawn in the below figure.



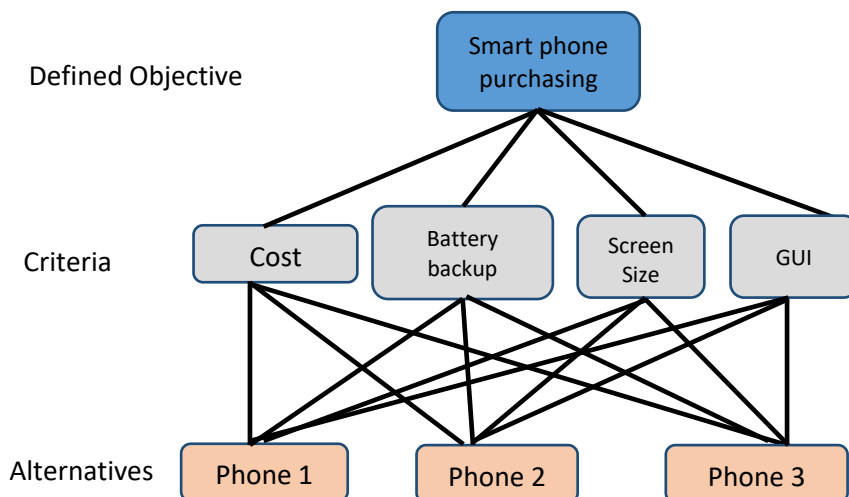
Decision Hierarchy Structure figure

The level of structure may be extended up to any number as per the requirement of problem and its size.

The in detail of AHP can be best explain with a small example in which a problem of smartphone purchasing is evaluated.

Step 1. Construction of hierarchy of problem:As per theory of AHP in handling the

problem of preaching a smart phone is that to develop the hierarchy structure which will be structured as below.



The hierarchy of the problem

In this example the main goal is that to purchase a best smart phone among the available three alternative phones with respect to the four features cost, battery backup, screen size and graphical user interface criterion.

Step 2. Construction of Pairwise comparison matrix:

Now as per steps in AHP the second phase deals with deriving weightmatrix also known as pair-wise comparison matrix of the criteria. In this example the weights of criterion cost, battery backup, screen size and GUI on the basis of preference (importance of one criteria over another) are given by the decision maker. To decide the criteria weight we reference the table 1. Here decision maker's judgment for comparison of criteria is having much importance. To maintain the pair-wise comparison matrix the below matrix is constructed.

	Cost	Battery Backup	Screen size	GUI
Cost	1	7-Jan	5-Jan	3-Jan
Battery Backup	7	1	2	3
Screen size	5	2-Jan	1	3
GUI	3	3-Jan	3-Jan	1

Pair-wise comparison matrix 1

and in matrix form it is shown as below

$$\begin{bmatrix} 1 & 1/7 & 1/5 & 1/3 \\ 7 & 1 & 2 & 3 \\ 5 & 1/2 & 1 & 3 \\ 3 & 1/3 & 1/3 & 1 \end{bmatrix}$$

A1

In pair-wise comparison matrix when the criteria is compared with itself then its importance is equal importance and is noted as 1, hence in our case the comparison of cost to cost, battery

2.1 Pairwise Comparison Matrix Construction

backup to battery back , screen size to screen size and GUI to GUI is 1 and shown in matrix diagonally. Now we move towards the comparison of battery backup to cost i.e. 2, 1 the location in matrix. The decision makers importance intensity regarding battery backup is 'Very strong' over the cost and hence it is recorded as 7 as per the scale in table1. Now when we consider the comparison of cost to battery backup i.e. at location 1, 2 it is to be recorded as reciprocal as 1/7. When a decision maker think about the comparison of screen size to cost then his judgment is that screen size is 'little more important than cost' and hence it is recorded as 5 and for cost to screen size it is 1/5. In case of GUI to cost the decision maker gives very less to cost and it is recorded as 3 and for cost to GUI it will be 1/3. If the decision maker moved for Battery backup to screen size and he judged it as very less and recorded as 2 and automatically it will be 1/2 for screen size to battery backup. Consider 3 is the comparison of battery backup to GUI by decision maker and 1/3 in case of GUI to battery backup and so on the matrix is full filled as constructed in above matrix 1.

2.2 Finding weight of Criterion: The weight of criterion is computed by having the row wise product of importance of criterion followed by its $1/n^{\text{th}}$ power and computing its total, where n is total number of criterion. The same can be shown with following equation 1.

2.2.1 Product of criterion importance and its $1/n^{\text{th}}$ power

$$C_1 = (i_1 * i_2 * \dots * i_n)^{1/n} \quad 1.$$

Where i indicates the importance for criterion assigned by the decision maker.

$$\text{Cost importance} = (1 * 1/7 * 1/5 * 1/3)^{1/4} = 0.31239$$

$$\text{Battery backup importance} = (7 * 1 * 2 * 3)^{1/4} = 2.5457$$

$$\text{Screen size importance} = (5 * 1/2 * 1 * 3)^{1/4} = 1.6549$$

$$\text{GUI importance} = (3 * 1/3 * 1/3 * 1)^{1/4} = 0.75984$$

We obtain the matrix as

$$\begin{matrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{matrix} \begin{bmatrix} 0.31239 \\ 2.5457 \\ 1.6549 \\ 0.75984 \end{bmatrix}$$

Where C_1 is the first criteria i.e. Cost and i indicates the importance of criteria with respect to other criterion.

2.2.2 Calculating sum of criterion importance

Take sum of all criteria importance as shown below.

$$TC = C_1 + C_2 + \dots + C_n \quad 2.$$

Where TC is sum of row (column sum) and is equal to 5.2728

2.2.3 Dividing row sum by sum of row

Now to obtain the weight matrix for criterion by divide the row sum by sum or row as in following expression

$$3. \quad W = \begin{bmatrix} C_1 / TC \\ C_2 / TC \\ - \\ - \\ - \\ C_n / TC \end{bmatrix}$$

Where W is the criterion weight matrix Likewise the following results will be obtained

$$W = \begin{bmatrix} 0.059246 \\ 0.4828 \\ 0.31385 \\ 0.1441 \end{bmatrix} \quad A2$$

2.2.4 Check for sum of weights is equal to 1

Here in above weight matrix it is considered that if the sum of weights is equal to 1 then our decision making problem is on right track and the result is 1.

$$0.059246 + 0.4828 + 0.31385 + 0.1441 = 1.$$

Step 3: Consistency Ratio calculation

Next in this process it is important to check that the weights are consistent or not, that means the assigned weights are correct or not. To understand the inconsistency level in the problem an example is to be discussed here.

Suppose there are 3 criteria such as A, B and C and a decision maker has given the preference 2 to A with respect to B, and 3 to B with respect to C then as per mathematics the preference of criteria A should be 6 with respect to C, but actually the decision maker may assign the importance 4, 5 or 7 and there will be certain level of inconsistency here in the comparison matrix. By considering the possibility of presence of inconsistency in the problem, what level of inconsistency is acceptable in the problem can be decided by the following random index which is introduced by Thomas L. Saaty.

Table 2. Random index

Criterion	3	4	5	6	7	8	9	10
RI	0.52	0.89	1.11	1.25	1.35	1.4	1.45	1.49

Now to find the consistency ratio CR we find the ratio of product of A1, A2 to A2 matrix like below.

3.1 Product:

$$\begin{bmatrix} 1 & 1/7 & 1/5 \\ 7 & 1 & 2 \\ 5 & 1/2 & 1 \end{bmatrix} \times \begin{bmatrix} 0.059246 \\ 0.4828 \\ 0.31385 \\ 0.1441 \end{bmatrix} = \begin{bmatrix} 0.23902 \\ 1.9575 \\ 1.2838 \\ 0.58739 \end{bmatrix}$$

The result is considered as matrix A3 Product of A1 and A3

3.2 Ratio

Now take the ratio of A3 and A2 as A4 as below

$$A4 = A3/A2$$

$$\begin{bmatrix} 0.059246 \\ 0.4828 \\ 0.31385 \\ 0.1441 \end{bmatrix} \div \begin{bmatrix} 0.23902 \\ 1.9575 \\ 1.2838 \\ 0.58739 \end{bmatrix} = \begin{bmatrix} 4.0344 \\ 4.0545 \\ 4.0905 \\ 4.0476 \end{bmatrix} \quad A4$$

If we see the A4 matrix it is find that the numberof criteria and the each element of matrix showing same value which is 4.

3.4 Average of elements of A4 ($\lambda_{\max}$)

Now have the average of elements in the A4 matrix and which is equals to

$$(4.0344 + 4.0545 + 4.0905 + 4.0476) / 4 = 4.0639$$

which is known as $\lambda_{\max}$

3.5 Calculation of CI

The Consistency Index is obtained using the following expression

$$CI = (\lambda_{\max} - n) / (n - 1)$$

Where in the above expression n is the number of criterion.

So it gives

$$CI = 4.0639 - 4 / 4 - 1 = 0.0213$$

3.6 Calculation of CR

The consistency Ratio CR is having the expression as

$$CR = CI / RI$$

Where RI is random index from table 2.

Now put the RI for four criterion from the random index table and it is 0.89 and can be taken as 0.9 in the above expression.

$$CR = 0.0213 / 0.9 = 0.023667$$

3.7 Check for CR

Now compare the CR value with 0.1. If the CR is below 0.1 then the pairwise comparison weights are best that indicates 10 percentage inconsistency is considerable.

$CR < 0.1 = \text{True}$ and hence the assigned weights best.

Step 4. Synthesizing:

In our case the matrix A2 exhibits that cost has the least priority, the highest priority is for

battery backup then the priority goes to screen size and then to GUI.

To get better understand consider the following real time example in case of purchasing mobile.

	Cost	Battery Backup	Screen size	GUI
Phone1	16000	5000	6.6	Better - 5
Phone2	12000	4000	7	Poor - 3
Phone3	17000	3500	4.2	Excellent - 8

4.1. Normalization: As per the decision maker's decisions normalize the above table. Consider the decision maker wants the phone of heavy battery backup, very least cost, small screen size and best GUI then the normalization is as below

	Cost	Battery Backup	Screen size	GUI
Phone1	16000/12000	5000/5000	6.6/4.2	8-May
Phone2	12000/12000	4000/5000	7/4.2	8-Mar
Phone3	17000/12000	3500/5000	4.2/4.2	8-Aug

The result is as below

	Cost	Battery Backup	Screen size	GUI
Phone1	1.33	1	1.57	0.63
Phone2	1	0.8	1.67	0.38
Phone3	1.42	0.7	1	1

4.2 Final Decision:

Now get the sum of product of matrix A2 and A5 to obtain the final weights of phones as shown below.

$$\text{Phone1} = (0.059246 * 1.33) + (0.4828 * 1) + (0.31385 * 1.57) + (0.1441 * 0.63) = 1.1451$$

$$\text{Phone2} = (0.059246 * 1) + (0.4828 * 0.80) + (0.31385 * 1.67) + (0.1441 * 0.38) = 1.0244$$

$$\text{Phone3} = (0.059246 * 1.42) + (0.4828 * 0.70) + (0.31385 * 1) + (0.1441 * 1) = 0.8800$$

Finally the importance preference for phone is

Phone1	1.1451
Phone2	1.0244
Phone3	0.88

So the matrix shows, phone1 is best as per the AHP calculations, phone2 is the second best and phone3 is having the lowest priority in selection phone for purchasing.

3. Conclusion

AHP is a Multicriteria Decision Analysis method in which multiple criteria are easily compared against each other and easily assign the criteria weight. The AHP process helps to sort out the problems of decision making where alternatives are present in large quantity, as the problem can be constructed in hierarchy and it will work as decision support tool. The ease ness of AHP makes it able to handle the complex and versatile problems of decision making. The AHP can also be the future to train the machines as a need of time. If the inconsistency is minimized up to some extent then the speed of problem solving can be increase as well accuracy in decision making can be enhanced. The fuzzy theory can also be used in AHP to expand the applicability of the AHP. AHP helps the decision makers to sort out the decision making problems in day to day life by saving time, efforts and cost also. It is observed that the AHP is totally depends on the weights provided by the decision makers, hence while solving the important, sensitive and big problems the criteria must be compared without partiality by the decision makers then and then the results will be real, this is the area where IT or computer science has the scope that the weights can be assigned by the machines by training them.

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