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# Predicting the Default Risk of Firm by Using the Maple Programming of the Iterated Merton Model

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***Abstract:***

This paper referred to a study that predict the default risk of firms using the iterated Merton model. Merton model is a widely used model in providing the assessment of firm probability to default. The ability of the model in predicting default risk is proven by many empirical studies. However, the iterated Merton model contains complicated mathematical structure to understand and thus, the paper enhanced the model into a maple program as a shortcut tool to predict default risk. The implementation of the iterated Merton model into a Maple programming produce several Maple coding run by the Maple 18 software. This program generates the iterated market value of asset, asset volatility and probability of default of firms as its main outputs. In this paper, samples of data of two companies are utilized as inputs to the Maple program. The result shows that the iterated market value of asset, asset volatility and probability of default converges at the second iteration. The program is tested by making sure that the predicted probability of default is consistent with the ratings of the selected companies. We found that the program able to predict the probability of default of the good rated performance company well rather than the poor rated performance company. However, the inconsistency exists is due to the failure of getting all the relevant information to predict the probability of default. Further research is recommended to improve the program into a sophisticated application.

*Keywords: Default, iteration, Maple, Merton.*

## I. INTRODUCTION

Huge changes in the development of technologies provide opportunity to firm managers to expand their business, but still we are keep getting shocked by the news of firm bankruptcy. In fact, the biggest shock that the world has come across is the bankruptcy of Toys R' Us that has filed for bankruptcy in 9 September 2017 due to default on its debt [1]. Default is occurred when a firm failed to commit their financial commitments [2]. Firms may experience default if less precautions made in predicting the default risk.

Recent studies used equity-based structural model such as Merton model to predict default risk of non-financial firm [3]. Predicting default risk using Merton model gives more accurate value [4]. The implementation of Merton Distance to Default model applied by [5] and [6] is superior and statistically significance at 1% level. Merton model also can give a continuous evaluation of the probability of default [7]. [8] stated that there is much useful information in the Merton style estimates because it can provide the firm default one year in advanced. Overall, Merton Model is considered to be the widely used model and one of the best tool in predicting probability of default.

[9] found that market value of firm assets and its volatility are the main determinants in predicting default risk. Failure in calculating the market value of assets and its volatility accurately will cause the tendency of default risk to be overestimated or underestimated. Hence, this paper used the iteration process to implement the Merton model into a Maple programming to generate the iterated market value of asset and asset volatility. The ability of the program is tested by predicting the default risk of good and poor performance companies. In this way the level of default risk predicted through the program can be validated. In the meantime, the Maple programming provides a shortcut tool for the model to predict default risk.

## II. THE ITERATED MERTON DEFAULT RISK MODEL

According to [10], iterative procedure is used to calculate the market value of firm asset and volatility. The main reason that iteration method is used is to obtain precise values of market value of asset and volatility. The iterative procedure is done until

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the volatility meets its convergence at $10^{-7}$. The paper used the iteration approach done by [11] and [12] through the following mathematical structure.

The iteration process begins by denoting $k = 0,1,2,..K$ as the number of iterations and $i = 1,2,...,n$ as number of days. Therefore, the book value of liabilities of a firm at $k+1$ iteration for day-*i*, $B_{k+1,i}$ is

$$B_{k+1,i} = A_{k,i}[1 - N(d_{k,i}^1)] + B_{0,i}e^{-r_i(Ti)}[N(d_{k,i}^2)] \quad (1)$$

where $A_{k,i}$ is the daily value of firm asset at *k* iteration, $B_{0,i}$ is the initial daily book value of liability, $r_i$ is the daily risk free rate and *N* is the standard normal cumulative density function of $d_{k,i}^1$ and $d_{k,i}^2$ distributions that are expressed as below:

$$d_{k,i}^1 = \frac{\ln\left(\frac{A_{k,i}}{B_{k,i}}\right) + \left(r_i + \frac{1}{2}\sigma_{k,i}^{\ 2}\right)T_i}{\sigma_{k,i}\sqrt{T_i}} \quad (2)$$
$$d_{k,i}^2 = d_{k,i}^1 - \sigma_{k,i}\sqrt{T_i}$$

where $\sigma_{k,i}$ is the daily asset volatility at *k* iteration. The variable $T_i$ is the time for the firm debt to mature in one-year period for each day that defined as follows:

$$T_i = T + (n - i)\Delta t, \Delta t = \frac{1}{n} \quad (3)$$

where $T$ is equal to one year, $\Delta t$ is the length of time interval.

Next is the market value of asset at $k+1$ iteration for day-*i*, $A_{k+1,i}$ that is expressed as below:

$$A_{k+1,i} = E_i + B_{k+1,i} \quad (4)$$

where *E* is the daily market value of equity. Hence, the distance to default at *k* iteration, $D_k$ can be obtained as below [9]:

$$D_k = \frac{\ln\left(\frac{A_{K,n}}{B_{K,n}}\right) + \left(\mu_K - \frac{\sigma_K^{\ 2}}{2}\right)T}{\sigma_K\sqrt{T}} \quad (5)$$

where, $\mu$ is the final iterated value of drift rate. Since $D_k$ is assumed to follow a standard normal distribution function property [11], thus the probability of default at *k* iteration, $P_k$ is expressed in the form of

$$P_k = N(-D_k) \quad (6)$$

## III. IMPLEMENTING THE MODEL INTO A MAPLE PROGRAM

The implementation of the iterated Merton default risk model into a Maple programming is executed through the "for loop" procedure. The loop stops after the asset volatility meets its convergence at $10^{-7}$. The "for loop" repeatedly executes a program statement(s) if the expression remains true. Fig. 1 shows the flowchart of the Maple programming.

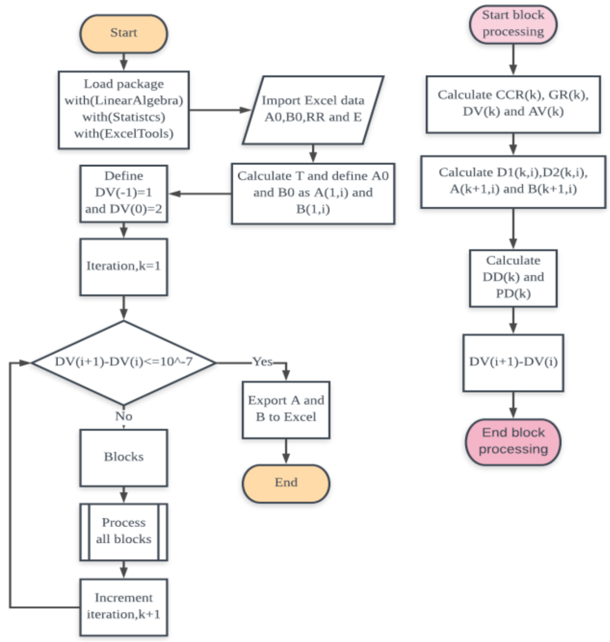


Fig. 1: The Flowchart of Maple programming

The Maple programming starts by importing the data of initial market value of asset (A0), initial book value of liability (B0), risk free rate (RR) and equity (E). Then, calculate the time maturity (T) using (3), and define A0 and B0 as $A(1,i)$ and $B(1,i)$, respectively. $A(1,i)$ and $B(1,i)$ are defined as initial market value of asset and book value of liability because programming does not recognize 0 as the initial value before entering the iterations. Hence 1 is used as the indications as initial value. The iterations process will stop if the difference of the daily volatility at day $i+1$ minus daily volatility at day-*i* equals to the tolerance value of $10^{-7}$, $DV(i+1) - DV(i) = 10^{-7}$. Thus, the daily volatility for $DV(-1) = 1$ and $DV(0) = 2$ are defined before entering the iterations process to avoid error. $DV(-1) = 1$ and $DV(0) = 2$ are defined so that the tolerance can be calculated first upon entering the iterations. If the tolerance value was not achieved, the iterations will go through a block. In the block, continuously compounded asset return (CCR), drift rate (GR), DV and annual volatility (AV) at *k*-iterations are calculated first to find the distribution of D1 and D2 as defined in (2). In addition, the new market value of asset, A(k+1), and new book value of liability, B(k+1) are calculated based on (4) and (1), respectively. Lastly the value of distance to default (DD) and probability of default (PD) at *k*-iterations are calculated based on (5) and (6), respectively. Before exiting the block, the difference between

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$DV(i+1)$ and $DV(i)$ is calculated. After exiting the block and go through an iteration increment, the process goes into the diamond shaped flowcharts again to calculate the tolerance value. If the set up tolerance is achieved, the iterations will stop and the program will export the output to Microsoft Excel.

## IV. DATA DESCRIPTION

Samples of data of Petronas Dagangan Berhad and Alam Maritim Resources Berhad in 2016 to 2017 are used in this study. The companies are categorized based on the ratings given by the Malaysian Ratings Corporation Berhad (MARC). Petronas Dagangan Berhad was rated as AAA ratings consistently on January 2016, January 2017 and January 2018 while Alam Maritim Resources Berhad was rated as BBB+ ratings on August 2016 and rated as default on July 2017. The data involves number of outstanding shares, current liability and non-current liability, collected from the companies quarterly report. The Malaysia Treasury Bill is assumed to be the risk free rate obtained from the website of Bank Negara Malaysia and daily share prices were obtained from the Yahoo Finance website.

## V. DATA IMPLEMENTATIONS

### A. Iterating the market value of asset and its volatility

The market value of asset and its volatility are iterated by finding the values of the following variables:

a. The initial daily book value of liability, $B_{0,i}$ . The initial daily book value of liabilities is calculated by adding the daily value of current liability with the daily value of non-current liability.
b. The initial daily market value of asset, $A_{0,i}$ . The initial daily market value of asset, $A_{0,i}$ is estimated by adding the daily market value of equity with the initial daily book value of liability, where the daily market value of equity is calculated by multiplying the number of outstanding share with its current daily share price.
c. The initial continuously compounding daily return, $\mu_{0,i}$
d. The initial daily value of asset volatility, $\sigma_{0,i}$ . The initial daily value of asset volatility is estimated by finding the standard deviation of the initial continuously compounding daily return.
e. The daily book value of liability at $k+1$ iteration, $B_{k+1,i}$ . The daily book value of liability at $k+1$ iteration is calculated based on the formula given in (1).
f. The daily market value of asset at $k+1$ iteration. $A_{k+1,i}$ . The daily market value of asset at $k+1$ iteration is calculated based on the equation given in (5). The market value of equity is assumed to be fixed in each iteration.

Based on the values of $A_{k+1,i}$ , the continuously compounding daily asset return is calculated by repeating the steps mentioned in c. to f. until the volatility converge at $10^{-7}$ .

### B. Predicting the probability of default

In order to evaluate the probability of default, daily volatility must be changed into yearly basis by multiplying daily volatility with square root of total trading day of that particular year. Distance to default is calculated using the equation given in (5). Since the probability of default is calculated in yearly basis, thus $T$ is defined as 1 year. After obtaining the distance to default, the probability of default can be obtained through equation (6).

## VI. RESULT AND DISCUSSION

According to the mathematical structure of the iterated Merton model and Fig. 1, several parts of Maple Commands were produced. Fig. 2 shows the part of Maple command in defining variables and calculating time maturity. Fig. 3 shows the main Maple command of the iterations process of calculating probability of default.

The Maple command in Fig. 2 and Fig. 3 were run by using the Maple18 Software to obtain the outputs that were exported into an excel spreadsheet and represented in Table 1 and Table 2.

```
> n := 248 : # define total number of trading days
> N := 248 :
> t := 1 : # number of year wants to predict
>
for i from 1 to 248 do
    # calculate time maturity and define the initial market value of asset and initial book value
    of liability
 A[1, i] := A0[i];
 B[1, i] := B0[i];
 T[i] := 1 + (n - i)/n od:
> X := RandomVariable(Normal(0, 1)) :
> DV[-1] := 1 : DV[0] := 2 : # define daily volatility (-1)=1 and daily volatility(0)=2
```

Fig. 2: Maple Command of Defining variables and Calculating Time Maturity

```
>
for k from 1 to 100 while (|DV[k - 1] - DV[k - 2]|) > 10^-7 do
    # iterations starts, k=1 and define daily volatility(k-1) and daily volatility(k-2)
 for i from 1 to 247 do;
    # calculate continously compounded asset return, drift rate, daily volatilty and annual
    volatility
 CCR[k, i] := ln(1 + A[k, i + 1]/A[k, i]);

 od;
 GR[k] := (sum(CCR[k, p], p = 1 .. n - 1))/(n - 1) :

 DV[k] := StandardDeviation(<seq(CCR[k, j], j = 1 ..247)>);
 AV[k] := DV[k]·sqrt(n);
 for i from 1 to 248 do; #calculate D1, D2, distance to default and probability of default
 D1[k, i] := (ln(A[k, i]/B[k, i]) + (RR[i] + DV[k]^2/2)·T[i])/(DV[k]·sqrt(T[i]));
 D2[k, i] := D1[k, i] - DV[k]·sqrt(T[i]);
 B[k + 1, i] := A[k, i]·(1 - CDF(X, D1[k, i])) + B[1, i]·e^(-RR[i]·T[i])·CDF(X, D2[k, i]);
 A[k + 1, i] := E[i] + B[k + 1, i];
 DD[k] := (ln(A[k, N]/B[k, N]) + (GR[k] - AV[k]^2/2)·t)/(AV[k]·sqrt(t));
 PD[k] := CDF(X,-DD[k]);

 od od;
```

Fig. 3: Maple Command of The Iterations Process of Calculating Probability of Default

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Table 1 and Table 2 shows the iterated market value of asset of Petronas Dagangan Berhad and Alam Maritim Resources Berhad for the years of 2017 and 2018. Both Tables 1 and 2 shows the market value of assets converge at the second iteration.

TABLE 1: THE ITERATED MARKET VALUE OF ASSET OF PETRONAS DAGANGAN BERHAD FOR THE YEAR 2017 AND 2018

| Year | $A_0$ (RM) | $A_1$ (RM) | $A_2$ (RM) |
|---|---|---|---|
| 2017 | 23786775206 | 23672993015 | 23672993015 |
| 2018 | 24168469040 | 24104786326 | 24104786326 |

TABLE 2: THE ITERATED MARKET VALUE OF ASSET OF ALAM MARITIM RESOURCES BERHAD FOR THE YEAR 2017 AND 2018

| Year | $A_0$ (RM) | $A_1$ (RM) | $A_2$ (RM) |
|---|---|---|---|
| 2017 | 399580165 | 251475837.2 | 251475837.2 |
| 2018 | 314912980 | 174332974.1 | 174332974.1 |

Fig. 4 shows the line graph of the iterated annual volatility of Petronas Dagangan Berhad and Alam Maritim Resources Berhad for the years of 2017 and 2018. It can be seen that the annual volatilities of both Petronas Dagangan Berhad and Alam Maritim Resources Berhad for the years of 2017 and 2018 converge at the second iterations. The annual volatility of Alam Maritim Resources Berhad is higher than the annual volatility of Petronas Dagangan Berhad. The higher the volatility, the higher the probability to default.

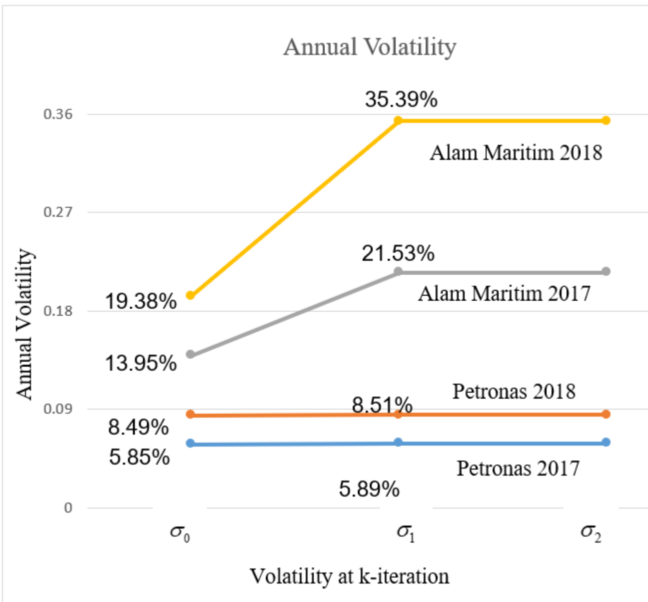


Fig. 4: The Line Graph of the Iterated Annual Volatility of Petronas Dagangan Berhad and Alam Maritim Resources Berhad for the Year 2017 and 2018

Table 3 and Table 4 represent the variables required to calculate the iterated distance to default of Petronas Dagangan Berhad and Alam Maritim Resources Berhad for the years of 2017 and 2018. Since the market value of asset and its volatility were converged at second iteration and thus the distance to default for the second and third iterations are the same. Higher distance to default gives lower probability to default

TABLE 3: THE VARIABLES REQUIRED TO CALCULATE THE ITERATED DISTANCE OF DEFAULT OF PETRONAS DAGANGAN BERHAD FOR THE YEAR 2017 AND 2018

| k-iteration | Item | Petronas Dagangan Berhad | |
|---|---|---|---|
| | | 2017 | 2018 |
| 0 | $B_0$ (RM) | 118771000 | 67275000 |
| | $A_0$ (RM) | 23786775206 | 24168469040 |
| | $\mu_0$ | 69.316% | 69.3197% |
| | $\sigma_0$ | 5.85% | 8.49% |
| | $DD_0$ | 102.4951513 | 77.46659857 |
| 1 | $B_1$ (RM) | 4988809.326 | 3592286 |
| | $A_1$ (RM) | 23672993015 | 24104786326 |
| | $\mu_1$ | 69.317% | 69.32% |
| | $\sigma_1$ | 5.89% | 8.51% |
| | $DD_1$ | 155.5829368 | 111.6177055 |
| 2 | $B_2$ (RM) | 4988809.326 | 3592286 |
| | $A_2$ (RM) | 23672993015 | 24104786326 |
| | $\mu_2$ | 69.317% | 69.32% |
| | $\sigma_2$ | 5.89% | 8.51% |
| | $DD_2$ | 155.5829368 | 111.6177055 |

TABLE 4: THE VARIABLES REQUIRED TO CALCULATE THE ITERATED DISTANCE OF DEFAULT OF ALAM MARITIM RESOURCES BERHAD FOR THE YEAR 2017 AND 2018

| k-iteration | Item | Alam Maritim Resources Berhad | |
|---|---|---|---|
| | | 2017 | 2018 |
| 0 | $B_0$ | 154598000 | 148510000 |
| | $A_0$ | 399580165 | 314912980 |
| | $\mu_0$ | 69.24% | 69.28% |
| | $\sigma_0$ | 13.95% | 71.93% |
| | $DD_0$ | 11.69986115 | 7.356412243 |
| 1 | $B_1$ | 6493672 | 7929994 |
| | $A_1$ | 251475837.2 | 174332974.1 |
| | $\mu_1$ | 69.23% | 69.27% |
| | $\sigma_1$ | 21.53% | 35.39% |
| | $DD_1$ | 20.09283505 | 10.51253445 |
| 2 | $B_2$ | 6493672 | 7929994 |
| | $A_2$ | 251475837.2 | 174332974.1 |
| | $\mu_2$ | 69.23% | 69.27% |
| | $\sigma_2$ | 21.53% | 35.39% |
| | $DD_2$ | 20.09283505 | 10.51253445 |

Table 5 and Table 6 show the yearly iterated probability of default of the Petronas Dagangan Berhad and Alam Maritim Resources Berhad for the years of 2017 and 2018.

TABLE 5: THE ITERATED PROBABILITY OF DEFAULT OF PETRONAS DAGANGAN BERHAD FOR THE YEAR 2017 AND 2018

| Year | $PD_0$ | $PD_1$ | $PD_2$ |
|---|---|---|---|
| 2017 | 0 | 0 | 0 |
| 2018 | 0 | 0 | 0 |

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TABLE 6: THE ITERATED PROBABILITY OF DEFAULT OF ALAM MARITIM RESOURCES BERHAD FOR THE YEAR 2017 AND 2018

| Year | $PD_0$ | $PD_1$ | $PD_2$ |
|---|---|---|---|
| 2017 | 6.38311E-32 | 4.26262E-90 | 4.26262E-90 |
| 2018 | 9.44595E-14 | 3.78166E-26 | 3.78166E-26 |

The probability of default for Petronas Dagangan Berhad obtained is zero from the very beginning. This can be concluded that less chances for Petronas Dagangan Berhad to bankrupt and there is no changes of probability of default as iteration procedure is applied. The probability of default of Petronas Dagangan Berhad are clearly low, it is proven that Petronas Dagangan Berhad are truly a stable company.

The probability of default of Alam Maritim Resources Berhad starts to converge at the second iteration for both years of 2017 and 2018. In the year 2017, the initial probability of default is quite low at 6.38311E-32 and becomes even lower when it converges at the second iterations at 4.26262E-90. The same goes for the year 2018, the initial probability of default also is quite low at 9.44595E-14 and becomes even lower when it converges at the second iterations at 3.78166E-26.

The ratings of Alam Maritim Resources Berhad were declining for the past 5 years. The MARC rated Alam Maritim Resources Berhad from A+ in 2013 to BBB+ in 2016 and eventually went to D in July 2017. Alam Maritim Resources Berhad were declared as default in July 2017. The Alam Maritim Resources Berhad had been inconsistent for the last 5 years. However, the probability of default of Alam Maritim in 2017 is predicted quite low as shown in Table 6. The model was unable to predict the inconsistent performance of Alam Maritim Resources Berhad due to the insufficient data. Further research needed to be done in order to predict the probability of default of Alam Maritim Resources Berhad accurately.

## VII. CONCLUSION AND RECOMMENDATION

This paper focuses on the prediction of default risk using the Maple programming of the iterated Merton model. By running the program, the iterated market values of asset are obtained. From the iterated asset, the iterated annual volatilities of Petronas Dagangan Berhad and Alam Maritim Resources Berhad were calculated. It is found that the volatilities of Petronas Dagangan Berhad for the years 2017 and 2018 are converged at the second iteration with 5.89% and 8.51%, respectively. The annual volatility of Alam Maritim Resources Berhad for the years 2017 and 2018 are also found converged at the second iteration with 21.53% and 35.39%, respectively. High volatility predicts high default risk. Hence, the probability of default of Petronas Dagangan Berhad for the years 2017 and 2018 are found extremely low that is zero, even before iteration is applied. The probability of default of Alam Maritim Resources Berhad for the years 2017 and 2018 are found converged at the second iteration with values 4.26262E-90 and 3.78166E-26, respectively.

Overall, the Maple programming of the iterated Merton model is found able to predict the default risk of Petronas Dagangan Berhad consistent as a AAA-rated company, unlike the Alam Maritim Resources Berhad case where more data needed to cope with the inconsistence performance of the company. Further research needed to be done in order to predict the probability of default of Alam Maritim Resources Berhad accurately. Future research to enhance the Maple programming of the iterated Merton model into an advanced and practical application is recommended.

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