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Steady State Analysis of an M/M/1 Queue with Coxian-2 Server Vacation and Breakdown

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Abstract

This paper deals with an M/M/1 queue which has a coxian-2 server vacations and random breakdowns..Atthe moment of service completion, the serverdecides to go for vacation with probability potherwise continue service in the system with probability 1-p. The vacation phases follow coxian-2 distribution, which hascompulsory first phase and optional second phase. The system suffers random breakdowns with Poisson rate. The flow balance equations corresponding to the model have been derived and closed form expression of system size probability has been generated.

Keywords; *Coxian-2 server vacation, server Breakdown, Bernoulli schedule vacation, single vacation policy, Probability generating functions*

I. INTRODUCTION

Extensive research is being carried out in vacation queueing models as it has umpteen applications in telecommunications, data networks, manufacturing sectors and supply chain systems. As an extension of classical queueing theory, vacation theory was developed in 1970's, so that the server may take vacations rather than serving customers who arrive at random. Survey papers on the works carried out in vacation queueing models, in the last decades, were reported by Doshi[8], Ke[7] and many others, in which Doshi[8] had reported the works on single server queues with vacations, extensively. Levy and Yechiali[9] has done an extensive work on multiserverMarkovian queues with vacation. The concept of breakdown during service has been investigated by umpteen researchers including K.C.Madan[1], where he studies a queue with bulk service that suffers random breakdowns. The interruptedservice,of a customer, due to breakdown will resume once the system starts functioning after under repair for exponential time.

Jararha and Madan[2] have analyzed single channel queue with deterministic service and coxian-2 server distribution under steady state where the vacations are optional. Gray et.al[6] studied a vacation queue subject to breakdown during service, where he extends the vacation process, service process and repair process follow PH distribution.

Our paper briefly discusses an M/M/1 queue in which the random variables corresponding to arrival time and service time follow Poisson and exponential distribution. The queue hasserver vacations which are optional with Bernoulli schedule and coxian-2 distribution, and the system is subject to random breakdowns. Once the customer's servicehas been completed,there are two options to the server, either, with probability p, it can go for a vacation or continue service to the next customer with probability 1-p. Our queue has two phases of vacation in which the first phase is mandatory and the second phase is voluntary. The server may decide to take another phase vacation with a probability parameterb or revert to the system with 1-b. Here it has been considered single vacation policy

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II. MODEL DESCRIPTIONS

Arrivals follow Poisson with mean arrival rateλ ($\lambda > 0$)

Exponential distribution is followed to service time with mean $1/\mu$ ($\mu > 0$)

After service is over for a customer, the server can go for vacation process with probability p or choose to be in the systemwith probability 1-p.

Server's vacation is of 2 phases- Phase 1, being a compulsory vacation, is exponential with mean 1/β_1 (β_1> 0). Optional Phase 2 is exponential with mean 1/β_2 (β_2> 0). Server may opt to go for second phase ofvacation (probability b)or revert to stay in the system (probability 1-b), after taking compulsory phase 1 vacation.

System may suffer a breakdown, when the server is functioning, with Poisson rate η ($\eta > 0$). The service channel immediately undergoes a repair process which is exponential with mean γ ($\gamma > 0$). The ongoing service of a customer is suspended until repair and it resumes once the server starts functioning.

The stochastic processes involved are independent of one another.

FCFS discipline is followed.

Definitions, Notations

- $W_n(t)$: Probability for n (n > 0) customers being present in the system both in queue and in service at time t (t > 0)
- $V_n^j(t)$: Probability for the server to be in the jth phase of vacation where j = 1,2 and there are n units in the system at time t (t > 0)
- $P_n(t)$: Probability for n (n > 0) customers in the system at time t (t > 0) without regard to the system's state.
- $R_n(t)$: Probability that the system is under repair and there are n (n > 0) units in the system at time t.

$$P_n(t) = W_n(t) + \sum_{j=1}^{2} V_n^j(t) + R_n(t)$$

Define the steady state PGF

$$W(z) = \sum_{n=0}^{\infty} W_n z^n \quad (1)$$

$$V^j(z) = \sum_{n=0}^{\infty} V_n^j z^n, \; j = 1,2 \quad (2)$$

$$R(z) = \sum_{n=0}^{\infty} R_n z^n \quad (3)$$

$$P(z) = \sum_{n=0}^{\infty} P_n z^n \quad (4)$$

Steady state equationsof the system

$$(\lambda + \mu + \eta)W_n = (1-p)\mu W_{n+1} + (1-p)\lambda W_{n-1} + (1-b)\beta_1 V_n^1 + \beta_2 V_n^2 + \gamma R_n, n \geq 1 \quad (5)$$

$$(\lambda + \eta)W_0 = (1-p)\mu W_1 + (1-b)\beta_1 V_0^1 + \beta_2 V_0^2 + \gamma R_0, n = 0 \quad (6)$$

$$(\lambda + \beta_1)V_n^1 = \lambda V_{n-1}^1 + p\mu W_{n+1} + p\lambda W_{n-1}, \; n \geq 1 \quad (7)$$

$$(\lambda + \beta_1)V_0^1 = p\mu W_1, n = 0 \quad (8)$$

$$(\lambda + \beta_2)V_n^2 = \lambda V_{n-1}^2 + b\beta_1 V_n^1, n \geq 1 \quad (9)$$

$$(\lambda + \beta_2)V_0^2 = b\beta_1 V_0^1, n = 0 \quad (10)$$

$$(\lambda + \gamma)R_n = \lambda R_{n-1} + \eta W_n, n \geq 1 \quad 11)$$

$$(\lambda + \gamma)R_0 = \eta W_0, \; n = 0 \quad (12)$$

2.2 System size probability generating functions

Multiplying (5) by z^{n+1} and summing over from 1 to ∞ together with (6) yields,

$$W(z) = \frac{z\mu W_0 + z\beta_1(1-b)V^{(1)}(z) + z\beta_2 V^{(2)}(z) + z\gamma R(z) - (1-p)\mu W_0}{(\lambda+\mu+\eta)z - \lambda(1-p)z^2 - \mu(1-p)} \quad (13)$$

Similarly multiplying (7) by z^{n+1} and summing over from 1 to ∞ together with (8) yields,

$$V^{(1)}(z) = \frac{(p\lambda z^2 + p\mu)W(z) - p\mu W_0}{(\lambda+\beta_1)z - \lambda z^2} \quad 14)$$

Multiplying (9) by z^n and summing over from 1 to ∞ together with (10) yields,

$$V^{(2)}(z) = \frac{b\beta_1 V^{(1)}(z)}{\lambda + \beta_2 - \lambda z} \quad (15)$$

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Similarly multiplying (11) by z^n and summing over from 1 to ∞ together with (12) yields,

$$R(z) = \frac{\eta W(z)}{\lambda+\gamma-\lambda z} \qquad (16)$$

Using equations (13),(14),(15) and (16) we solve for $W(z), V^{(1)}(z)$, $V^{(2)}(z)$, R(z)

$$W(z) = \frac{A}{E(z)} \qquad (17)$$

$$V^{(1)}(z) = \frac{B}{E(z)} \qquad (18)$$

$$V^{(2)}(z) = \frac{C}{E(z)} \qquad (19)$$

$$R(z) = \frac{D}{E(z)} \qquad (20)$$

Where,

$$A = \mu z w_0 g_2(z)g_3(z)g_4(z) - \mu(1-p)w_0 g_2(z)g_3(z)g_4(z) - z\beta_1(1-b)p\mu w_0 g_3(z)g_4(z) - z\beta_1\beta_2 bp\mu w_0 g_4(z)$$

$$B = p\lambda z^3\mu g_3(z)g_4(z)w_0 + p\mu^2 z g_3(z)g_4(z)w_0 - \mu(1-p)p\lambda z^2 g_3(z)g_4(z)w_0 - p\mu^2(1-p)g_3(z)g_4(z)w_0 - p\mu g_1(z)g_3(z)g_4(z)w_0 + \gamma z\eta p\mu g_3(z)w_0$$

$$C = b\beta_1[p\lambda z^3\mu g_2(z)g_4(z)w_0 + p\mu^2 z g_2(z)g_4(z)w_0 - \mu(1-p)p\lambda z^2 g_2(z)g_4(z)w_0 - p\mu^2(1-p)g_2(z)g_4(z)w_0 - p\mu g_1(z)g_2(z)g_4(z)w_0 + \gamma z\eta p\mu g_2(z)w_{0]}$$

$$D = \eta[\mu z w_0 g_2(z)g_3(z) - \mu(1-p)w_0 g_2(z)g_3(z) - z\beta_1(1-b)p\mu w_0 g_3(z) - z\beta_1\beta_2 bp\mu w_0]$$

$$g_1(z) = (\lambda+\mu+\eta)z - \lambda(1-p)z^2 - \mu(1-p)$$

$$g_2(z) = (\lambda+\beta_1)z - \lambda z^2$$

$$g_3(z) = \lambda+\beta_2-\lambda z$$

$$g_4(z) = \lambda+\gamma-\lambda z$$

$$E(z) = g_1(z)g_2(z)g_3(z)g_4(z) - z^3\beta_1(1-b)p\lambda g_3(z)g_4(z) - z\beta_1(1-b)p\mu g_3(z)g_4(z) - z^3\beta_1\beta_2 bp\lambda g_4(z) - z\beta_1\beta_2 bp\mu g_4(z) - \eta z\gamma g_2(z)g_3(z)$$

Now, adding equations (17),(18),(19) and (20) we get,

$$P(z) = W(z) + V^1(z) + V^{(2)}(z) + R(z) \qquad (21)$$

To find the only unknown w_0 which appears on the numerator on the RHS of equation (21), it has been used P(1) = 1 to check for the normality condition. But, when z=1, W(z),$V^1(z)$, $V^{(2)}(z)$, R(z) are indeterminate $(\frac{0}{0} form)$which implies P(z) is indeterminate. Hence we apply L's Hospital rule on (17), (18), (19) and (20) and simplifying,

$$W(1) = \sum_{z\to 1} W(z) = \frac{[\mu\beta_1\beta_2\gamma-\mu p\lambda\beta_2\gamma-\mu p\lambda b\beta_1\gamma]w_0}{\beta_1\beta_2[\mu\gamma-\lambda\gamma-\eta\lambda]-(\lambda+\mu)[p\gamma\beta_2\lambda+b\beta_1 p\lambda\gamma]} \qquad (22)$$

$$V^{(1)}(1) = \sum_{z\to 1} V^{(1)}(1) = \frac{p\mu\lambda\beta_2[2\gamma+\eta]w_0}{\beta_1\beta_2[\mu\gamma-\lambda\gamma-\eta\lambda]-(\lambda+\mu)[p\gamma\beta_2\lambda+b\beta_1 p\lambda\gamma]} \qquad (23)$$

$$V^{(2)}(1) = \sum_{z\to 1} V^{(2)}(1) = \frac{bp\beta_1\mu\eta\lambda w_0}{\beta_1\beta_2[\mu\gamma-\lambda\gamma-\eta\lambda]-(\lambda+\mu)[p\gamma\beta_2\lambda+b\beta_1 p\lambda\gamma]} \qquad (24)$$

$$R(1) = \sum_{z\to 1} R(z) = \frac{[\beta_1\beta_2\mu\eta-p\lambda\mu\eta(\beta_2+b\beta_1)]w_0}{\beta_1\beta_2[\mu\gamma-\lambda\gamma-\eta\lambda]-(\lambda+\mu)[p\gamma\beta_2\lambda+b\beta_1 p\lambda\gamma]} \qquad (25)$$

Now, summing (22),(23),(24) and (25) and equating it to 1, we have,

$$w_0 = \frac{\beta_1\beta_2[\mu\gamma-\lambda\gamma-\eta\lambda]-(\lambda+\mu)[p\gamma\beta_2\lambda+b\beta_1 p\lambda\gamma]}{\beta_1\beta_2\mu(\gamma+\eta)+p\lambda\mu\gamma(\beta_2-b\beta_1)} \qquad (26)$$

Equation (26) is the idle time probability of the server. The steady state stability condition is given by,

$$(\lambda+\mu)[p\gamma\beta_2\lambda+b\beta_1 p\lambda\gamma] < \beta_1\beta_2$$

Using w_0 in (23), (24), (25) and (26) we obtain

$$W(1) = \frac{\mu\beta_1\beta_2\gamma-\mu p\lambda\beta_2\gamma-\mu p\lambda b\beta_1\gamma}{\beta_1\beta_2\mu(\gamma+\eta)+p\lambda\mu\gamma(\beta_2-b\beta_1)} \qquad (27)$$

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$$V^{(1)}(1) = \frac{p\mu\lambda\beta_2[2\gamma+\eta]}{\beta_1\beta_2\mu(\gamma+\eta)+p\lambda\mu\gamma(\beta_2-b\beta_1)} \quad (28)$$

$$V^{(2)}(1) = \frac{bp\beta_1\mu\eta\lambda}{\beta_1\beta_2\mu(\gamma+\eta)+p\lambda\mu\gamma(\beta_2-b\beta_1)} \quad 29)$$

$$R(1) = \frac{\beta_1\beta_2\mu\eta-p\lambda\mu\eta(\beta_2+b\beta_1)}{\beta_1\beta_2\mu(\gamma+\eta)+p\lambda\mu\gamma(\beta_2-b\beta_1)} \quad (30)$$

Equations (27), (28), (29) and (30) give the probability for the server to be available in the system, is on phase-1 vacation, is on phase-2 vacation and the server is under repair, respectively in steady state.

To find the utilization factor ρ, that is, the busy period, we subtractw_0 from W(1), since W(1) includes idle periodw_0

$$\rho = W(1) - w_0 = \frac{\beta_1\beta_2\lambda(\gamma+\eta)+p\lambda^2\gamma[\beta_2+b\beta_1]}{\beta_1\beta_2\mu(\gamma+\eta)+p\lambda\mu\gamma(\beta_2-b\beta_1)} \quad (31)$$

By substituting for $w_0$from (26) into equations (17), (18), (19) and (20), we have determined W(z), $V^{(1)}(z)$, $V^{(2)}(z)$and R(z) completely and explicitly.

2.3 System performance measures

The PGF of the steady-state probability distribution for the system size is given by

$$P(z) = W(z) + V^1(z) + V^{(2)}(z) + R(z) \quad (32)$$

- By differentiating (32) w.r.t z, and substituting for z as 1, we get the average system size

$$L = \sum_{z\to 1} \frac{d}{dz}[P(z)] \quad (33)$$

- The average queue size is obtained as,

$$L_q = L - \rho \quad (34)$$

whereρ is given by (31)

- The mean system time W, and the mean waiting time in the queue W_q, is derived from Little's formula

$$W = \frac{L}{\lambda} \quad (35)$$

$$W_q = \frac{L_q}{\lambda} \quad (36)$$

Special case

M/M/1 queue without vacations and breakdown

For this case we let, p=0 in (31), (26), (17), (18), (19) and (20) and get,

$$V^1(z) = V^{(2)}(z) = R(z) = 0$$

$$W(z) = \frac{\mu z w_0 - \mu w_0}{[\lambda + \mu + \eta]z - \lambda z^2 - \mu}$$

Which on simplifying yields,

$$W(z) = \frac{\mu - \lambda}{\mu - \lambda z} = \frac{1-\rho}{1-\rho z}$$

Equations (26) and (31) yields,

$$w_0 = 1 - \rho$$

$$\rho = \frac{\lambda}{\mu}$$

Which corresponds to the results of the conventional M/M/1 queueing model.

III. CONCLUSION

In our paper, a single server queue with coxian-2 server vacation and server breakdown has been analyzed. We have derived PGF for the system size explicitly. We have also derived average system size and queue size. System's performance measures have been found.This work can be extended to find waiting time distribution function in steady state or the entire model can be extended for transient state solutions.

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