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Exponential Weighted Moving Average (EWMA) Control Chart for Air Pollution Index at SK Batu Muda, WP Kuala Lumpur

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Abstract:

Since transboundary haze pollution often occurs and it is a major threat to Malaysia, thus very important for the government to monitor the air quality status from time to time. SK Batu Muda is one of the locations for pollution data observation by Department of Environment Malaysia (DOEM). SK Batu Muda was the most affected location with highest the Air Pollutant Index (API) reading in 2015 compared to year 2016 and 2017, this is considered as anomalies in API reading. To overcome the issue, two types of Exponential Weighted Moving Average (EWMA) control chart are proposed to accurately monitor the air pollution levels in SK Batu Muda named as classical EWMA and iterative re-estimation EWMA. This is to prevent the inflation ofthe control limits which consequently will cause the power to detect changes in the data to be reduced in Phase II. Based on average run length (ARL) results, the iterative re-estimation EWMA control chart in Phase II has tighter control limits compared to classical. As conclusion, the iterative re-estimation EWMA chart is more efficient to handle the anomalous situation for API reading.

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1 Introduction

Haze episodes are common in South East Asia since 1983 which is a big threat to Malaysia. This is due to the transboundary haze pollution that once occurred in 1997–1998 and 2006–2007 (Kusumaningtyas & Aldrian, 2016). Since then, the awareness of the environmental has gained attention to the government and established Malaysian Air Quality Guidelines to handle this issue (Afroz, Hassan, & Ibrahim, 2003). Haze is defined as the presence of dry particles and smoke in air when the humidity level is lower than 80% and strict down the visibility less than 10 km(Kusumaningtyas & Aldrian, 2016). Air pollution is caused by the release of pollutants to atmosphere which can cause harm to human health and environmental(Mackenzie, 2016). Several substances that cause air pollution are carbon monoxide (CO), nitrogen dioxide (NO_2), Sulphur dioxide (SO_2), ozone and particulate matter (PM_{10})(Zamzuri and Razali, 2017).

Haze phenomenon is likely manmade where the occasion of transboundary haze was from Sumatra Island in Indonesia to deforestation by open burning on a large-scale.The purpose to do that was to clear the space for foreign palm oil plantations. This has caused tension to Malaysia and Singapore as the victim for the reckless action. The latest occurrence of South East Asia's haze was in 2015 which was the

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worst recorded in decades.Several countries including Malaysia suffered from this incident which led to temporary shutdown of schools in areas with hazardous API reading. In Kuala Lumpur, Batu Muda recorded the worst (hazardous)air pollutant index (API)reading(Malik, 2015). The API is a standard system to determine the air pollutant level that ranged as shown in Table 1.

Table 1. Air Pollutant Index (API) along with descriptor.

API	Descriptor
0 – 50	Good
51 – 100	Moderate
101 – 200	Unhealthy
201 – 300	Very Unhealthy
> 300	Hazardous

Haze is associated with high level of air pollutants. However, a high concentration of an unusual pollutant, a phenomenon known as anomaly, does not necessarily signify a true contaminated air quality status. Possibly, it is caused by a faulty air pollution sensor (Cong, 2015; Shaadan, Jemain, Latif, and Deni, 2014). Anomaly refers to the patterns in the data that do not conform to the notion of a normal pattern (Chandola, Banerjee, and Kumar, 2009; Cong, 2015). Anomaly detection is a process of identifying unusual observations in the data set, called outliers.It is applicable in various domains such as cyber security, medicine, banking and manufacturing (Aleskerov, Freisleben, and Rao, 2002; Kumar, 2005; Spence, Parra, and Sajda, 2001). In analyzing environmental data set concerning the occurrence of haze, anomaly detection would be very useful as it allows proper identification between real event and faulty sensor or machine.

This paper presents two approaches based on Exponentially Weighted Moving Average (EWMA) control chart for identifying data anomaly, i.e. outliers, in air pollution data set from SK Batu Muda. The data is available at the website of the Department of Environment Malaysia (DOEM). In this study, the two approaches are identified as classical EWMA and iterative re-estimation EWMA which later be used to understand the outlier's pattern in the dataset (SK Batu Muda). The goal is to find which approach perform best in terms of monitoring air quality status when the environmental data may or may not be contaminated.

This paper is structured into the following sections. Section 1.1 introduces control chart in general. Section 2 focuses on the methodology studied in this paper. The results of this study are discussed in Section 3. Finally, the conclusion is given in Section 4.

1.1 Control Chart

Control chart is one of Statistical Process Control (SPC) tool that is widely used in practice for quality management. Traditionally, control chart has been used in manufacturing sector to achieve process stability. When the parameters that represent some quality characteristics of the process are unknown, control chart can be applied in two distinct stages namely Phase I and Phase II. In Phase I, control chart is used retrospectively for attaining in-control data set, for instance, the observations which are representative of the process. Using this Phase I data set, the parameters of the process are then estimated, and control limits are constructed for use in Phase II. In Phase II, control chart is used prospectively to monitor significant changes in the data.

Generally, the Shewhart control chart is widely used in manufacturing.However, the limitation of this chart is unable to identify out-of-control process when a non-random pattern exists in the process (Montgomery, 2009). The presence of non-random pattern in a sequence indicates the process is out of control, for instance of the non-random pattern is including cyclicals, trends, small shifts, stratification and mixtures (Rakitzis & Bersimis, 2006). Failing to detect the existence of any non-random pattern in the process will cause the Shewhart control chart to be less sensitive to a small process mean shift (Klein,

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2000; Koutras, Bersimis, & Maravelakis, 2007; Montgomery, 2009).As alternatives, Exponential Weighted Moving Average (EWMA)and Cumulative Sum (CUSUM) control charts were introduced by Lucas and Saccucci (1990) and Page (1954), respectively. Both EWMA and CUSUM control charts are able to detect small shifts. However, this study is focusing on the development of EWMA chart only. The main reason is that, the EWMA has a performance quite close to that of the CUSUM (Montgomery, 2009; Srivastava and Wu, 1997).

In Phase I, the presence of outliers will cause miscalculation of the control limits for the chart(Mason & Young, 2002). In other words, outliers in Phase I may seriously affect estimation of the process parameters and therefore, the estimation of the control limits for use in Phase II**.** Consequently, the use of control chart for process monitoring (Phase II) is no longer reliable. The chart may signal more often than expected when the process is in-control. It is also likely that the chart may experience detection delay when the process is out-of-control. To mitigate the problem, this studyemploys two different approaches to set up the control limits for the EWMA chart in Phase I. The first approach is to estimate the control limits using standard estimators based on the sample mean and standard deviation without checking for outliers in Phase I. The estimated control limits are then used for Phase II monitoring. This approach is known as the classical EWMA chart. The second approach is to screen for outliers using Phase I EWMA chart. Only when the Phase I data set are free from outliers (for example, all points contain within the iterative control limits), the parameters of the process are estimated using the sample mean and the standard deviation and the control limits are constructed for use in Phase II. This approach is known as iterative re-estimation EWMA. There are two different historical datasets used to construct Phase I that has anomalous feature and non-anomalous feature to compare the effectiveness of the performance in two different EWMA control charts i.e. classical and iterative re-estimation.

To evaluate the performance of control chart, average run length (ARL) was used. The ARL is referring to the expected number of plotted chart statistics before a signal is observed. In addition, ARL is another way to evaluate the decision related to sample size and sampling frequency used in designing a control chart.

Materials and Methods

Data

The air pollution data was obtained from the Department of Environment Malaysia (DOEM). The location of the study is at SekolahKebangsaan (SK) Batu Muda, Wilayah Persekutuan (WP) Kuala Lumpur, Malaysia. The recorded API data was in daily series from year 2015 to 2017as displayed in Figure 1. From the graph, it can be seen that the highest API reading is at 300 API level (very unhealthy) in 2015, meanwhile, in 2016 and 2017, the highest peak is below than 100 API level (moderate). By the end of year 2017, the graph shows a decaying pattern in API level, suggesting a good API reading. To conduct this study, there aretwo different historical datasets in Phase I thatused to setup the control limits. The first dataset has anomalous features starting from year 2015 to 2016. The second dataset has non-anomalous features in which 2016 was treated as historical dataset. As for Phase II, year 2017 dataset is used for process monitoring both cases of anomalous and non-anomalous datasets.

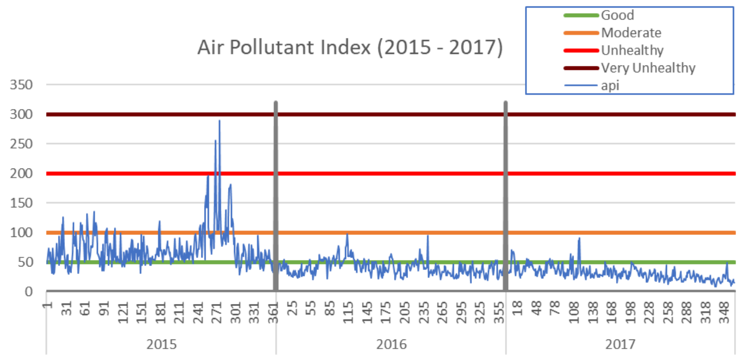

Fig. 1. The trend of API reading from 2015 to 2017.

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2.2 EWMA Control Chart

Exponential Weighted Moving Average (EWMA) control was first introduced byRoberts (1959). EWMA is statistics specialised in monitoring process where it averages the data by giving less weight as the data is older in time (Ahmad, 2015). EWMA control chart is defined as follows:

$$z_i = \lambda x_i + (1-\lambda) z_{i-1} \tag{1}$$

where λ is a suitable constant that ranged from $0 < \boldsymbol{\lambda} \leq 1$, $\boldsymbol{z_i}$ is statistic of EWMA at time $\boldsymbol{i}$, $\boldsymbol{x_i}$ is the observation of air pollution index at time *i*. The target value of the process is$\boldsymbol{\mu_0}$. The starting value is defined as $\boldsymbol{z_0} = \boldsymbol{\mu_0}$. The EWMA control chart signals out-of-control when $\boldsymbol{z_i}$ value is over the specified control limits. The control limitsare defined as follows:

$$UCL = \mu_0 + L\sigma\sqrt{\frac{\lambda}{(2-\lambda)}\left[1-(1-\lambda)^{2i}\right]} \tag{2}$$

$$Centerline = \mu_0 \tag{3}$$

$$LCL = \mu_0 - L\sigma\sqrt{\frac{\lambda}{(2-\lambda)}\left[1-(1-\lambda)^{2i}\right]} \tag{4}$$

where $\boldsymbol{L}$ is a positive coefficient that determines the width of the EWMA limits and $\boldsymbol{\sigma}$ is the process standard deviation. Based on Montgomery (2009), the $\boldsymbol{\lambda}$ values ranged from $0.05 \leq \boldsymbol{\lambda} \leq 0.25$ work well in practice. This study used $\boldsymbol{\lambda} = 0.05$, $\boldsymbol{\lambda} = 0.10$, and $\boldsymbol{\lambda} = 0.20$ since these values are frequently employed in control charting literature and have been showed to perform well in various conditions. The appropriate values of *L* for the chosen $\boldsymbol{\lambda}$follow Lucas and Saccucci (1990).

2.3 Development of Control Chart

The time frame specified for Phase I and Phase II data were clarified in the previous section. The procedure to perform control chart for Phase I is using two different approaches named as classical and iterative re-estimation to produce the reliable control limits for Phase II. The classical approach is using the preliminary observations to construct the trial control limits without take any action on the out of control points. Then, use the control limits from Phase I to construct Phase II chart.

The iterative re-estimation technique is using the preliminary observations as well to establish a reliable control limits for Phase I by excluding the out-of-controlpoints from the calculation of control limits and repeat the same step for the next iteration till there is no points are out-of-control (stable). After Phase I data is in-control, the established control limits from Phase I are in Phase II for process monitoring. Like any other control chart, it is important to ensure that the Phase I is in-control before proceeding to the Phase II.

In the initial stage of developing the control chart, the data is often assumed to be independent. However, in any environmental data, the tendency of independent observation tends to be violated to construct statistical process control(Ahmad, 2015; Pan & Chen, 2008). Even with the small level of autocorrelation could bring a bad influence on the statistical properties in the control chart. Thus, there were two approaches to solve the autocorrelation as mentioned by Ahmad (2015). First, is by adjusting the control limits where it also known as modified control chart,and second is the series of fitted Auto-Regressive Integrated Moving Average (ARIMA) by using the residuals to construct the control chart.

The following are the steps to apply in constructing the control chart.

Classical approach:

1. Use historical data to construct control limits in Phase I.
2. If the points are out of control, do not take action.
3. Continue with the Phase I control limits to construct EWMA statistics for Phase II data.

Iterative re-estimation approach:

1. Use historical data to construct control limits in Phase I.

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2. If there are points fallen over control limits, exclude the points and re-establish the control limits again.
3. Repeat the process till all of the points in Phase I is in-control (stable).
4. Continue with the control limits from Phase I to construct EWMA statistics for Phase II data.

3 Statistical Analyses

3.1 ARIMA Model

In this study, the second approach, which is using residuals to construct the control chart, was employed to handle autocorrelated data. Since the data is not independent, the data has downward pattern where the first differencing was initiated to perform ARIMA model. There are several numbers of lag were identified by using autocorrelation function (ACF) and partial autocorrelation function (PACF) and it showed the most fitted ARIMA model is

ARIMA (1,1,2) fitted for 2015–2016:

$$(1-B)api_i = -0.0205 + \frac{1-0.37004B}{1-0.75197B-0.13811B^2}\varepsilon_i$$

ARIMA (1,1,1) fitted for 2016:

$$(1-B)api_i = 0.01665 + \frac{1-0.5006B}{1-0.91147B}\varepsilon_i$$

The residuals from the fitted ARIMA models are normally distributed. To create the Phase II data, substitute the values of API data from 2017 into these equations to proceed with process monitoring.

3.2 Phase I

The notations used in Phase I is defined as estimator or smoothing value (**λ**), upper control limit (**UCL**), lower control limit (**LCL**), Mean (**μ**), Standard deviation (**σ**), and iteration numbers (**n**). The control limits of the residuals used from year 2015 to 2016 (Table 2) showed that the number of iterations required are between three to five. Meanwhile, the results in Table 3 indicate that for larger value of **λ**, more iterations are needed before the final control limits for use in Phase II can be attained. For example, for**λ**= 0.20, the control limits have to be re-established four times in total, but, for **λ**= 0.10, only single iteration is needed.

(5)

Table 2. Constructing control limits for anomalous dataset.

Phase I: 2015–2016						
Approach	λ	UCL	LCL	μ	σ	n
Classical	0.05	7.53	-7.43	0.05	17.86	0
	0.10	11.58	-11.48	0.05	17.86	0
	0.20	17.68	-17.59	0.05	17.86	0
Iterative	0.05	6.13	-6.55	-0.14	14.81	3
Re-estimation	0.10	9.25	-9.58	-0.17	14.58	5
	0.20	17.68	-17.59	-0.16	15.18	3

Table 3. Constructing control limits for non-anomalous dataset.

Phase I: 2016						
Approach	λ	UCL	LCL	μ	σ	n
Classical	0.05	3.93	-3.70	0.12	9.11	0

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	0.10	6.00	-5.76	0.12	9.11	0
	0.20	9.11	-8.88	0.12	9.11	0
Iterative Re-estimation	0.05	3.93	-3.70	0.12	9.11	0
	0.10	5.85	-5.77	0.04	9.00	1
	0.20	8.20	-8.09	0.05	8.25	4

3.3 Phase II

The notations used in the control chart for **c** is defined as classical and **i** is defined as iterative re-estimation. The control chart visualisation for classical EWMA and iterative re-estimation EWMA are being merged into one control chart for each **λ** values to distinguish the difference from both approaches to give better insights. Using the anomalous control limits in Figure 2, the process monitoring in 2017 showed that the iterative re-estimation EWMA have several points that exceed the control limits when compared to the classical. As the **λ** value is increasing from 0.05 to 0.20, the control limits are getting tighter for the iterative EWMA. On the other hand, the classical EWMA showed in-control process for each **λ** values.

In Figure 3, non-anomalous control limits were used to monitor process for year 2017. When **λ** = 0.10, the classical EWMA indicates the out-of-control processfor more than 10 periods, meanwhile, when the first iteration executed, the out-of-control points is reduced to four periods. As **λ** value increases to 0.20, the iterative EWMA control limits arenoticeably tighter than that with **λ** = 0.10.

Based on different historical control limits, on each different **λ** values, the out-of-control points are mostly ranged from 10th to 12th periods for iterated process and stay in-control for the rest of the year except when **λ** = 0.20, one to two points are indicate the process is out-of-control at 115th and 117th for the iterated process.

As shown in both figures, when there is iteration procedure in process monitoring, it made the control limits become significantly tighter. The tighter the control limits, the more restrict the process monitoring in control chart. As a result, the process monitoring has become more sensitive and detected more out-of-control points than the classical approach.

3.4 Average Run Length

Table 4 lists the ARL values obtained from the classical and iterative re-estimation approaches. These ARL values, when compared to the ARL values suggested in the work of Lucas and Saccucci (1990), are quite close. Focusing on the anomalous historical data, noticeably when the shift in mean is less than 1σ, the iterative re-estimation approach gives smaller ARL than the classical approach, suggesting faster detection. On the other hand, for non-anomalous historical data, the classical approach always gives smaller ARL than the iterative re-estimation approach except when the shift in mean is at 0.5σ. By comparing the performance of the two approaches, clearly, the iterative re-estimation approach is more suitable when Phase I datasetconsisted of anomalies feature. This is because of the procedure involves the screening of outliers and thus, giving more robust control limits for use in Phase II. Therefore, more effective monitoring can be achieved.

4 Conclusion

In conclusion, anomalous and non-anomalous historical dataset give different insights how the outliers affect the width of the control limits in process monitoring (Phase II) based on the classical EWMA and the iterative re-estimation EWMA. The iterative re-estimation EWMA for non-anomalous control

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limits performed a little bit slow in detection compared to anomalous control limits. Meanwhile, the classical EWMA does not give good detection in Phase II for anomalous control limits compared to non-anomalous control limits. This has caused high false alarm where the given points were claimed to be in-control but in real case, the chart showed several points were plotted outside the control limits. As supported by the ARL results, the iterative re-estimation approach is more sensitive to the presence of the anomalous patterns in Phase I dataset when compared to the classical approach. Therefore, the iterative re-estimation approach can be used reliably to monitor the air quality status. In the occurrence of haze, the proposed approach can accurately differentiate the anomalous and non-anomalous pattern in the dataset.

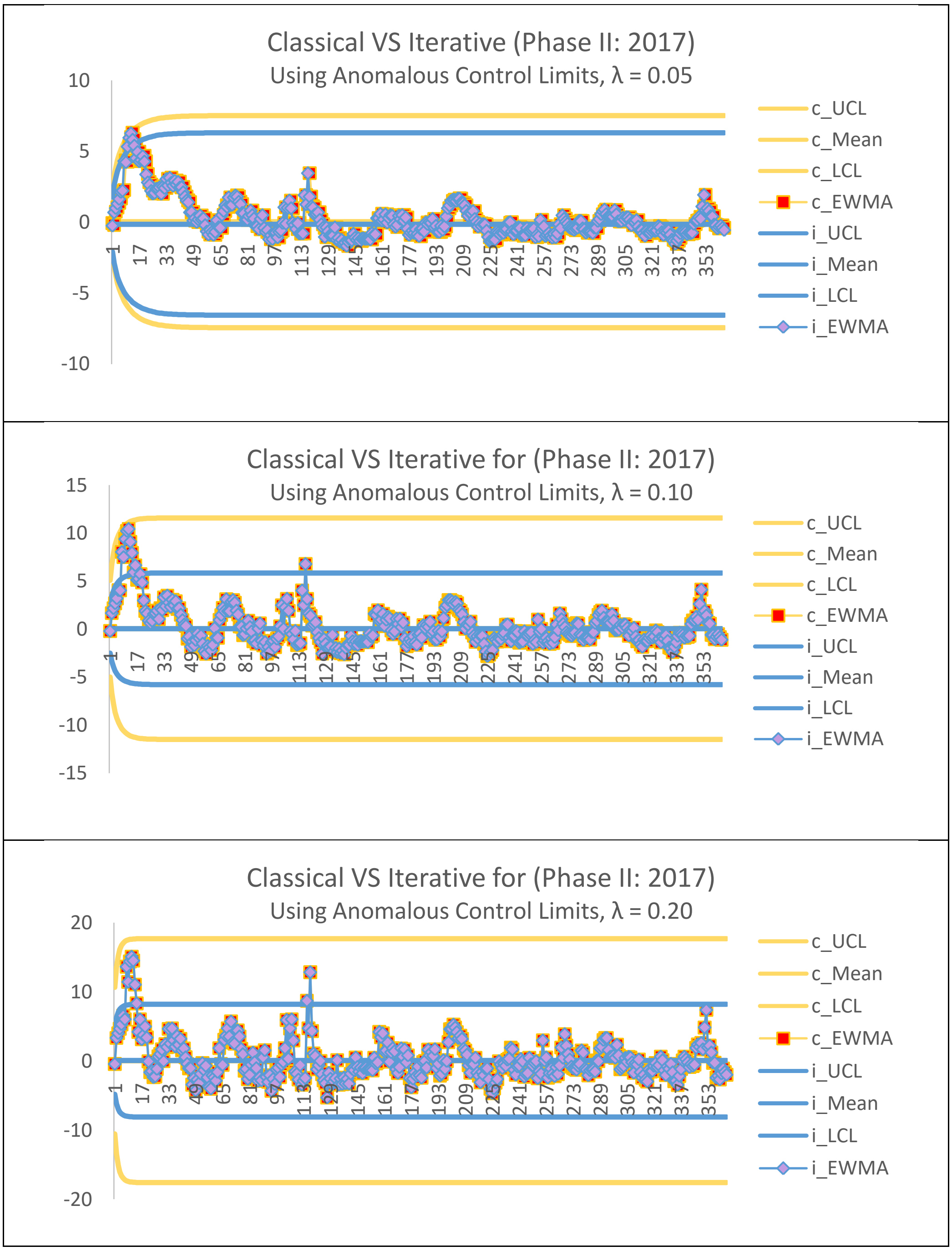

Fig. 2.EWMA control charts based on anomalous control limits.

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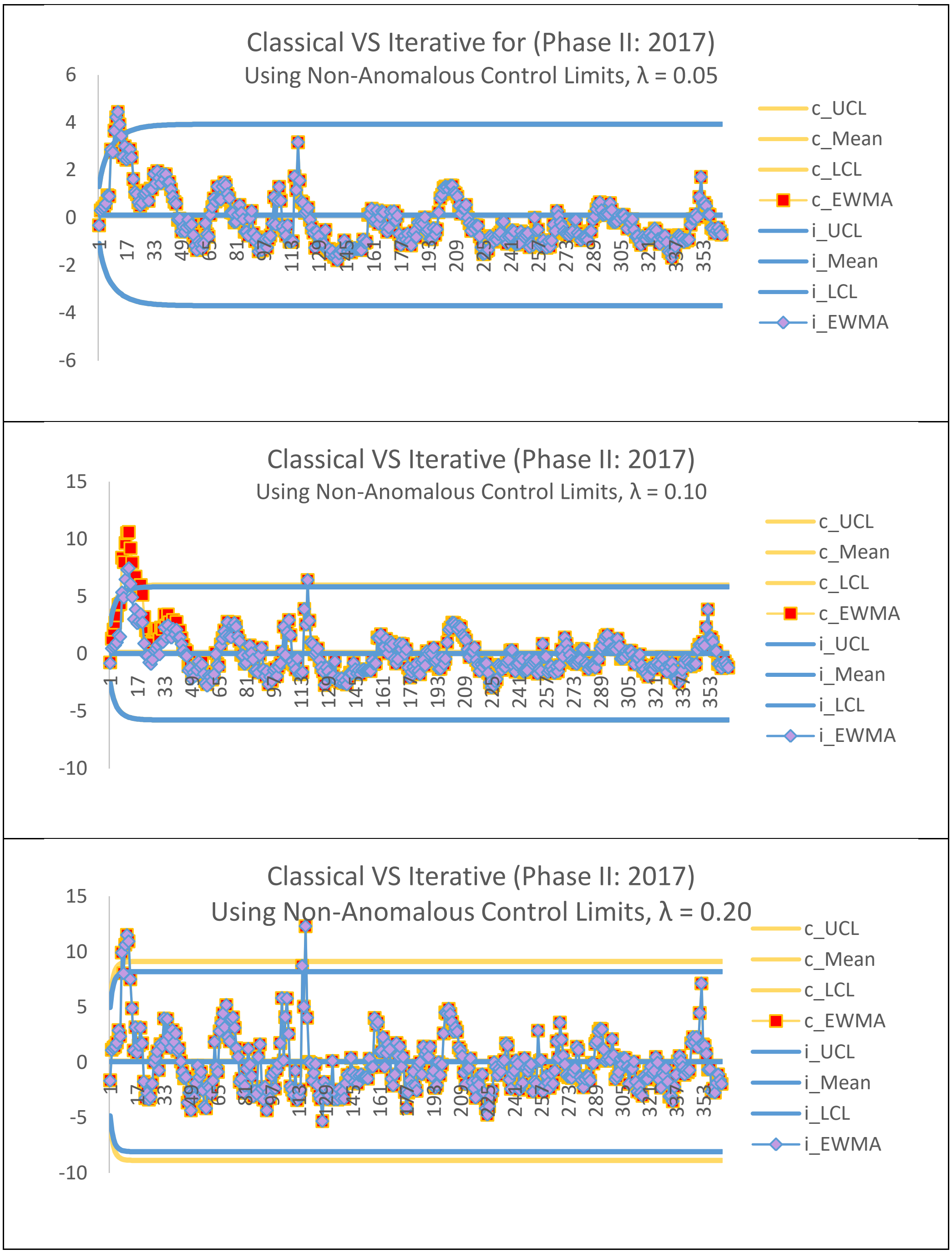

Fig. 3.EWMA control charts based on non-anomalous control limits.

Table 4. Average Run Length (ARL).

Phase I: 2015, 2016 | Phase II: 2017			Phase I: 2016 | Phase II: 2017		
Shift in Mean	Classical	Iterative Re-estimation	Shift in Mean	Classical	Iterative Re-estimation
	λ = 0.05			λ = 0.05	
0	500	500	0	500	500
0.10	273.7585	265.6791	0.10	272.4264	272.4264
0.25	84.2154	83.6858	0.25	85.0563	85.0563
0.50	28.7502	28.7763	0.50	29.1031	29.1031
0.75	16.4086	16.3608	0.75	16.3760	16.3760
1.00	11.3850	11.3869	1.00	11.3297	11.3297
1.50	7.1125	7.1148	1.50	7.1132	7.1132
2.00	5.2244	5.2265	2.00	5.2361	5.2361
3.00	3.4962	3.4956	3.00	3.4920	3.4920
4.00	2.6947	2.6944	4.00	2.6960	2.6960
	λ = 0.10			λ = 0.10	
0	500	500	0	500	500
0.10	320.3598	315.0062	0.10	319.0877	320.2242
0.25	106.6007	106.3608	0.25	106.0634	106.3608
0.50	31.2797	31.3098	0.50	31.3459	31.3098
0.75	15.8875	15.8531	0.75	15.8106	15.8531
1.00	10.3330	10.3281	1.00	10.3262	10.3281
1.50	6.0842	6.0839	1.50	6.0848	6.0839
2.00	4.3619	4.3624	2.00	4.3636	4.3624
3.00	2.8680	2.8679	3.00	2.8583	2.8679
4.00	2.1932	2.1931	4.00	2.1931	2.1931
	λ = 0.20			λ = 0.20	
0	500	500	0	500	500
0.10	372.711	371.9106	0.10	371.6573	371.9106
0.25	150.5824	150.1287	0.25	149.8770	150.1287
0.50	41.7368	41.7313	0.50	41.8399	41.7313
0.75	18.2075	18.1617	0.75	18.0960	18.1617
1.00	10.5447	10.5446	1.00	10.5359	10.5446
1.50	5.5006	5.5005	1.50	5.5013	5.5005
2.00	3.7431	3.7429	2.00	3.7448	3.7429
3.00	2.3809	2.3808	3.00	2.3729	2.3808
4.00	1.8644	1.8644	4.00	1.8643	1.8644

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