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Centroid Based Ranking of Intutionistic Fuzzy Geometric Programming Approach for An Economic Production Quantity Model with the Integration of Environmental Costs

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Abstract

In this paper, we discuss about an economic production quantity inventory model is formulated with the inclusion of environmental costs in treatment and disposal of the waste generated from various stages in the production process using centroid based ranking of intutionistic fuzzy geometric programming approach

Keywords; *Economic Production Quantity, Environmental costs, Intutionistic fuzzy numbers, Ranking of Intutionistic fuzzy number and Fuzzy Geometric programming.*

I. INTRODUCTION

We keep in mind that one of the most well-known and productive method to tackle Non-Linear Programming (NLP) difficulty is Geometric Programming (GP). Truly, via the examination of rate minimization techniques for building and making plans difficulty [34, 35] Zenner supplied the concept of GP. In fluffy set hypothesis, we don't forget the extent of enrollment artwork for purpose capacities and necessities. As of late, fluffy set hypothesis has been usually advanced and a few alterations have confirmed up. Atanassov [2] created Intuitionistic Fuzzy (IF) set hypothesis, in which we endure in thoughts non-participation paintings alongside enrollment paintings for unfastened facts. Angelov [1] created enhancement technique in IF situation. Pramanik and Roy [26] dissected vector operational trouble the usage of IF geometric programming.

Positioning of TIFN primarily based mostly on esteem report to equivocalness report is proposed with the aid of Li [9] and tackled a multiattribute dynamic trouble. Dubey et al. [6] broadened the definitions given by means of Li [9] to the recently characterised TIFNs. Nehi [15] proposed some other positioning approach, wherein the enrollment paintings and non-participation ability of IFNs are treated as fluffy amounts.

Right now, acquaint any other method with positioning of intutionistic fluffy numbers with geometric programming method for finding the monetary introduction quantity with the mixture of ecological costs .

II. FUNDAMENTALS

This phase present a few definitions and essential thoughts recognized with Geometric Programming strategy, Intuitionistic fluffy numbers , Ranking of Intutionisitc fluffy variety and centroid based totally positioning of Intutionistic fluffy wide variety.

2.1Geometric programming problem:

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Primal problem: Primal Geometric Programming(PGP) problem is

$$\text{Minimize } g_0(t) = \sum_{k=1}^{T_0} C_{0k} \prod_{j=1}^{m} t_j^{\alpha_{0kj}}$$

$$\text{Subject to } \sum_{k=1}^{T_0} C_{rk} \prod_{j=1}^{m} t_j^{\alpha_{rkj}} \leq 1 \text{ , (r= 1,2,\ldots..l) , j=}$$

(1,2,3…..m) $t_j>0$

Where $C_{0k} > 0$ (k=1,2,…T_0) C_{rk} and α_{rk} are real numbers. It is constrained polynomial geometric problem. The number of term each polynomial constrained functions varies and it is denoted by T_r for each r= 0,1,2….. Let T= $T_0+T_1+T_2+\ldots\ldots+T_l$ be the total number of terms in the primal program. The Degree of difficulty is (DD) = T – (m+1)

Dual Problem:

$$\text{Maximize } = \prod_{r=0}^{l} \prod_{k=1}^{T^r} \left(\frac{C_{rk}}{\delta_{rk}}\right)^{\delta rk} \left(\sum_{s=1+T_{r+1}}^{T} (\delta_{rs})^{\delta rk} \right)$$

$$\text{Subject to } \sum_{k=1}^{T_0} \delta_{0k} = 1 \text{ (Normality condition)}$$

$$\sum_{r=0}^{l} \sum_{k=1}^{T_r} \alpha_{rkj} \delta_{rk} = 0 \text{ (Orthogonality conditions)}$$

$\delta_{rk} > 0$, (Positive constant)

2.2 Intutionistic Fuzzy set:

Let X is a non-empty set, An Intutionisitic fuzzy set $\bar{A}$ = { < x,$\mu_A(x)$, $v_A(x)$ > / x ϵX } where

$\mu_A(x)$ and $v_A(x)$ are membership and non-membership function such that $\mu_A(x)$, $v_A(x)$: X →[0,1] and

0≤,$\mu_A(x) + v_A(x) \leq 1$ for all xϵX.

2.3 Intutionisitic Fuzzy Number:

An Inutionistic Fuzzy subset $\bar{A}$ = { < x,$\mu_A(x)$, $v_A(x)$ > / x ϵX } of the real line R is called an intutionistic fuzzy number if the following conditions hold.

* There exists mϵR such that $\mu_A(m) = 1$ and $V_A(m) = 0$

* $\mu_A(x)$ is continuous function from R →[0,1] such that 0≤,$\mu_A(x) + v_A(x) \leq 1$ for all xϵX.

2.4 Trapezoidal Intutionistic Fuzzy number:

A trapezoidal intutionistic fuzzy number is denoted by $\bar{A}$ = (a_1,a_2,a_3,a_4) , (a_1', a_2, a_3,a_4') where

$a_1' \leq a_2 \leq a_3 \leq a_4'$ where the membership and non-membership function of $\bar{A}$ are in the following form

$$\mu_{\bar{A}}(x) = \begin{cases} 0 : x < a_1 or x > a_4 \\ \dfrac{(x-a_1)}{(a_2-a_1)} : a_1 \leq x \leq a_2 \\ 1 : a_2 \leq x \leq a_3 \\ \dfrac{(x-a_4)}{(a_3-a_4)} : a_3 \leq x \leq a_4 \end{cases}$$

$$V_{\bar{A}}(x) = \begin{cases} 0 : x < a_1 or x > a_4 \\ \dfrac{(x-a_1')}{(a_2-a_1')} : a_1' \leq x \leq a_2 \\ 1 : a_2 \leq x \leq a_3 \\ \dfrac{(x-a_4')}{(a_3-a_4')} : a_3 \leq x \leq a_4' \end{cases}$$

2.5Accuracy function for Defuzzification:

The accuracy function of Intutionistic fuzzy number is

AF(A) = [(a+ 2(b+c) +d) + (a' +2(b'+c')+d')] / 12

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March - April 2020
ISSN: 0193-4120 Page No. 7434 - 7441

2.6. Generalized Trapezoidal Intutionistic Fuzzy Number:

A Fuzzy number A = ((a_1,a_2,a_3,a_4),(b_1,b_2,b_3,b_4) ,w_A, u_A) is said to be Trapezoidal Intutionistic fuzzy number , If

$$\mu_{\bar{A}}(x)=\begin{cases} 0 : x<a_1 or x>a_4 \\ w_A\dfrac{(x-a_1)}{(a_2-a_1)} : a_1\le x\le a_2 \\ w_A : a_2\le x\le a_3 \\ w_Q\dfrac{(x-a_4)}{(a_3-a_4)} : a_3\le x\le a_4 \end{cases}$$

$$v_{\bar{A}}(x)=\begin{cases} 1 : x<b_1 \\ \dfrac{(b_2-x)+u_A(x-b_1)}{(b_2-b_1)} : b_1\le x\le b_2 \\ u_A : b_2\le x\le b_3 \\ \dfrac{(x-b_3)+u_A(b_4-x)}{(a_3-a_4)} : b_3\le x\le b_4 \\ 1 : x>b_4 \end{cases}$$

Where $0<w_A<1$, $0<u_A<1$ and $0\le w_A+u_A\le 1$

III. THE CENTROID BASED RANKING METHOD FOR TRAPEZOIDAL INTUTIONISTIC FUZZY NUMBER

The membership part of the generalized trapezoidal intutionistic fuzzy number

A = ((a_1,a_2,a_3,a_4),(b_1,b_2,b_3,b_4) ,w_A, u_A) is S (μ_A) = $(\frac{2(a_1+a_4)+7(a_2+a_3)}{18})(\frac{7w_A}{18})$ and the nonmembership part of the generalized trapezoidal intutionisitc fuzzy number is

S (v_A) = $(\frac{2(b_1+b_4)+7(b_2+b_3)}{18})(\frac{11+7u_A}{18})$

Now we define the rank of A is as follows:

$$R(A) = \frac{w_A S(\mu_A)+u_A S(v_A)}{w_A+u_A}$$

IV. MATHEMATICAL MODEL

4.1 Assumptions:

1. Demand for matters from inventory is ceaseless and at a steady fee.

2. Production charge is nonstop; more prominent than request fee and it races to resume

inventory at popular interims.

3. Production set-up/soliciting for price is constant (free of quantity delivered)

4. The lead time is constant.

5. The all out allocator fees for the gushing treatment, robust waste treatment and gasoline discharges

remedy includes the allocator prices of the treatment delivered about via the losses of diverse

type in all of the duties finished in wonderful tiers associated with the creation method.

6. The coping with price includes all of the associated charges of executing all of the sports activities

engaged with the introduction procedure.

4.2. Documentations:

D call for according to unit of time

P production consistent with unit of time

x D/P

1-x the part of time the advent manner spends definitely lingering

A steady soliciting for fee/installation charge in line with introduction run

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March - April 2020
ISSN: 0193-4120 Page No. 7434 - 7441

h keeping charge in line with unit consistent with unit of time.

V processing costs for all of the activities completed in distinct levels engaged with the

introduction way.

L total fee of fluid profluent remedy

S widespread charge of strong waste treatment

G overall rate of sturdy waste treatment

X fashionable allocator fees of the effluents treatment delivered about through the loss in all the duties

Y standard allocator prices of the strong remedy and waste removal introduced about by using the loss taking all things together

the tasks.

Z preferred allocator prices of fuel emanation treatment added approximately by means of the loss in all of the sports activities.

V. CRISP MODEL

Nowadays the manufacturing firms are facing severe pressure from government in terms of legislations and regulations which drives them to carry out environmental activities. The main motive of these activities is to prevent the environment from the effects of the disposal of waste of different matters(solid, liquid and gas). Here, the environmental costs (costs of waste treatment) are taken into account by the firm. The problem of managing both the economic and environmental costs which paves way for the extension of the economic centered inventory model to environmentally sustainable model is formulated with the inclusion of the treatment costs of the waste generated at various stages to the classical EPQ model.

The equation TD(j) = $\sum_{i=1}^{n} X_{ij} * LEC + \sum_{i=1}^{n} Y_{ij} * SWC + \sum_{i=1}^{n} Z_{ij} * GEC$ is formulated by Paulo et al.,(2009) is modified and then incorporated to the classical EPQ model.

The EPQ cost per unit of time C(Q) = $\frac{AD}{Q} + \frac{hQ(1-x)}{2}$

The processing cost per cycle $C_v(Q)$ = V

The processing costs cover input costs, labor costs and energy consumption costs that are incurred at various stages of the production process.

Waste treatment and disposal cost per cycle C_{TD} (Q) = XL + YS + ZG = F (Say)

Total cost per unit of time $TC(Q) = C(Q) + \frac{C_P(Q) + C_{TD}(Q)}{T}$ where T = Q/D

TC(Q) = $\frac{AD}{Q} + \frac{hQ(1-x)}{2} + \frac{D(V+F)}{Q}$

The objective is to determine the optimal quantity .

The necessary condition is $\frac{\partial TC}{\partial Q} = 0$

The optimal solution is Q = $\sqrt{\frac{2D(A+V+F)}{h(1-x)}}$

VI. SOLUTION OF THE INVENTORY MODEL BY CRISP GEOMETRIC PROGRAMMING

We solve the proposed model by applying geometric programming and the degree of difficulty is 0.

Max G(w) = $\prod_{i=1}^{n} \left(\frac{[A+V+F]D}{Qw_{1r}}\right)^{w1r} \left(\frac{h(1-x)Q}{2w_{2r}}\right)^{w2r}$

Subject to the conditions

$w_{1r}+w_{2r}$ = 1

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$- w_{1r} + w_{2r} = 0$

Solving these conditions , we get the values of $w_{1r} = w_{2r} = 1/2$

By applying Duffin's and Peterson's theorem ,

$$(\frac{(A+V+F)D}{Q} = w_{1r}\, g(w_{1r}, w_{2r})$$

$$hQ(1-x)/2 = w_{2r}\, g(w_{1r}, w_{2r})$$

$$\frac{2(A+V+F)D}{h(1-x)} = Q^2$$

$$Q^* = \sqrt{\frac{2D[(A+V+F]}{h(1-x)}}$$

VII. SOLUTION OF THE INVENTORY MODEL BY INTUTIONISTIC FUZZY GEOMETRIC PROGRAMMING

Let $\bar{A}$ =(a,b,c,d) be the trapezoidal fuzzy number and IF(A)= (a,b,c,d), (a',b', c',d')be the trapezoidal intutionistic fuzzy number and the objective function is

$$\overline{TC(Q)} = \frac{\bar{A}\bar{D}}{Q} + \frac{\bar{h}Q(1-x)}{2} + \frac{(\bar{V}+\bar{F})\bar{D}}{Q}$$

$$= \frac{(A+\bar{V}+\bar{F})\bar{D}}{Q} + \frac{\bar{h}Q(1-x)}{2}$$

We use the accuracy function for Intutionistic fuzzy number,

$$IF(A) = \frac{a+2(b+c)+d+a'+2(b'+c')+d'}{12}$$

Here, $IF(TC) = \frac{A+IF(V)+IF(F)}{Q} IF(D) + \frac{IF(h)Q(1-x)}{2}$ ---------------(1)

Applying Geometric Programming technique for (1)

Max $G^*(w) = \prod_{r=1}^{n}(\frac{[A+IF(V)+IF(F)]IF(D)}{Qw_{1r}})^{w1r}(\frac{IF(h)(1-x)Q}{2w_{2r}})^{w2r}$

Subject to the conditions

$w_{1r}+w_{2r} = 1$

$- w_{1r} + w_{2r} = 0$

Solving these conditions , we get the values of $w_{1r} = w_{2r} = 1/2$

By applying Duffin's and Peterson's theorem ,

$$(\frac{(A+IF(V)+IF(F))IF(D)}{Q} = w_{1r}\, g(w_{1r}, w_{2r})$$

$$IF(h)\, Q(1-x)/2 = w_{2r}\, g(w_{1r}, w_{2r})$$

$$\frac{2(A+IF(V)+IF(F))IF(D)}{IF(h)(1-x)} = Q^2$$

$$Q^{**} = \sqrt{\frac{2IF(D)[(A+IF(V)+IF(F)]}{IF(h)(1-x)}} \quad \text{---------(2)}$$

VIII. CENTROID BASED RANKING METHOD OF GENERALIZED INTUTIONISTIC FUZZY GEOMETRIC PROGRAMMING

Let GIF(A) = ((a_1,a_2,a_3,a_4),(b_1,b_2,b_3,b_4) ,w_A, u_A) be the generalized trapezoidal intutionistic fuzzy number and the corresponding centroid based ranking of generalized intutionistic fuzzy total cost function is

$$GRIF(TC) = \frac{A+GRIF(V)+GRIF(F)}{Q} GRIF(D) + \frac{GRIF(h)Q(1-x)}{2}$$ ---------------(3)

Applying Geometric Programming technique for (3)

Max $G^*(w) = \prod_{r=1}^{n}(\frac{[A+GRIF(V)+GRIF(F)]GRIF(D)}{Qw_{1r}})^{w1r}(\frac{GRIF(h)(1-x)Q}{2w_{2r}})^{w2r}$

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Subject to the conditions

$w_{1r}+w_{2r} = 1$

$- w_{1r} + w_{2r} = 0$

Solving these conditions , we get the values of $w_{1r} = w_{2r} = 1/2$

By applying Duffin's and Peterson's theorem ,

$$\left(\frac{(A+GRIF(V)+GRIF(F))GRIF(D)}{Q} = w_{1r}\, g(w_{1r}, w_{2r})\right.$$

$$GRIF(h)\, Q(1-x)/2 = w_{2r}\, g(w_{1r}, w_{2r})$$

$$\frac{2(A+GRIF(V)+GRIF(F))GRIF(D)}{GRIF(h)(1-x)} = Q^2$$

$$Q^{***} = \sqrt{\frac{2GRIF(D)[(A+GRIF(V)+GRIF(F)]}{GRIF(h)(1-x)}}$$

-------------------(4)

IX. NUMERICAL EXAMPLE

Crisp Model:

Consider an inventory system with following data.

A =\$100/cycle , h = \$5 units/cycle , D = 50000 units/year , P= 75000 units/year, x=0.67, V=\$873 , F = \$852

The Optimal Production Quantity is **Q = 10517 units**.

Crisp Geometric Programming Method:

The Optimal Production Quantity is **Q^* = 10516.9 units**

Intutionistic Fuzzy Geometric Programming Method:

D= 50000 $\bar{D}$= (49600 , 49800 50200 , 50400)

IF(D) = (49600 , 49800 50200 , 50400)((49400 , 49800 50200 , 50600)

By using Accuracy function of fuzzy number $\bar{D}$,

IF(D) = 50000

h = 5 , $\bar{h}$ = (4.8, 4.9, 5.1, 5.2) IF(h) = (4.8, 4.9, 5.1, 5.2) (4.5, 4.9, 5.1, 5.5)

By using Accuracy function of fuzzy number $\bar{h}$, IF(h) = 5

V = 873 , $\bar{V}$ =(870 , 872 , 874, 876) IF(V) = (870 , 872 , 874, 876) (868 , 872 , 874, 878)

By using Accuracy function of fuzzy number $\bar{V}$, IF(V) =873

F=852 , $\bar{F}$ = (850 ,851, 853, 854) IF(F) = (850 ,851, 853, 854) (849 ,851, 853, 855)

By using Accuracy function of fuzzy number $\bar{F}$, IF(F) = 852

From (2) **,** Q^{**} = 10517.942

The Optimal Production Quantity is **Q^* = 10517.942 units**

Centroid based Ranking of Generalized Intutionistic Fuzzy Geometric Programming Method:

GIF (D) = ((49600 , 49800 50200 , 50400)((49400 , 49800 50200 , 50600) , 0.01,0.994)

S_μ (D) = 1.944 , S_V(D) = 49805.8556

By using centroid based ranking formula in section 3.

GRIF(D) = 49835.1535

GIF(h) = ((4.8, 4.9, 5.1, 5.2) (4.5, 4.9, 5.1, 5.5) ,0.01,0.98)

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S_μ (h) =0.01944 , S_V(h) =4.9611 , GRIF (h) =4.9111

GIF(V) = ((870 , 872 , 874, 876) (868 , 872 , 874, 878) , 0.01,0.98)

S_μ(V) = 3.395 , S_V(V) = 866.21 , GRIF (V) = 857.49

GIF(F) = ((850 ,851, 853, 854) (849 ,851, 853, 855) , 0.001,0.994)

S_μ (F) =0.331333 , S_V(F) = 850.012 , GRIF(F) = 849.158

From (4) , Q^{***} = 10580.12 units

The Optimal Production Quantity = 10580.12 units

X. SENSITIVITY ANALYSIS

The affectability examination is completed for checking the viability of the financial advent quantity version with scenario within your finances, adaptable and strong introduction framework the usage of Geometric programming approach. This circumstance mirrors the age of waste at one-of-a-type stages of introduction technique and waste remedy value.

XI. CONCLUSION

Right now, fluffy economic introduction quantity version with earth viable stock version which mirror the age of waste at different phases of introduction technique and waste treatment fee using centroid based positioning of intutionisitic fluffy geometric programming method is received. This proposed approach because it ought to be mirrors the uncertainity and wavering of human reasoning and it is regularly adaptable. Thus we infer that the becoming a member of of ecological costs to the inventory model has gotten required to consolidate the lawful dangers.

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