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Analysis of Erlangian Phased Service Queuing Model: Fuzzy Randomness Approach

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Article Info
Volume 83
Page Number: 7415 - 7419
Publication Issue:
March - April 2020

Article History
***Article Received**: 24 July 2019*
***Revised**: 12 September 2019*
***Accepted**: 15 February 2020*
***Publication**: 07 April 2020*

Abstract

Waiting line models are prominent structures to navigate the behaviour of complex systems in modernistic environs. A parametric computing in accordance with Zadeh's extension principle is consumed for enumerating the membership functions of the performance measures in Erlangian service model grounded on Randomness Fuzziness consistency principle. The inter-arrivals and exponential service rates are blended in phase service queues FM/Ek/1 and M/FEk/1. The appropriacy of the proposed model is outlined by concrete illustrations.

Keywords; *Queuing model, membership functions, Zadeh's extension principle, parametric programming.*

I. INTRODUCTION

In traditionalistic queuing models, Poisson input and exponential service times were commonly assumed and only single phase was utilized to complete all the arriving customers. But in recent years, many real life oriented circumstances are closely interlinked with phase service queuing models specifically in industrial scenarios and communication networks. In a predestinate world of challenges and congestion disputes, queuing models sprung forth with phase services to depict the urge of queuing situations.

The most widely used distributions to generate phase type services are Erlangian distribution, Hyper – exponential distribution, phase Type distribution, Two-phase essential and Optimal service, multiphase essential and optimal service for imparting solution to queuing problems in varied backgrounds incorporating different techniques.

Fuzzy randomness stimulates fuzzy outcomes in terms of probability when the stochasticity can't be sighted with precision. Many researchers have extensively studied on fuzzy queues Li and Lee [7], Negi and Lee [9], Kao et al [6], Nithyangini Jhala, Pravin Bhathawala on mathematical multiphase single server and multi phase multiserver queuing system. Dhruba Das, Hemanta K. Baruah [4] investigated on Fuzzy randomness principle. This study focuses on two Erlangian queuing models to construct their membership functions derived from the specified principle.

Definition:

A normal imprecised number F_N = [a, b, c] combined with its membership $\mu_{FN}(x)$ is defined as [3],

$$\mu_{FN}(x) = \begin{cases} \phi_1(x) & , \quad a \le x \le b \\ \phi_2(x) & , \quad b \le x \le c \\ 0 & , \quad othewise \end{cases}$$

where ϕ_1 (x) and ϕ_2 (x) are continuous increasing and decreasing functions in [a, b] and [b, c] with

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ISSN: 0193-4120 Page No. 7415 - 7419

$\phi_1(a) = \phi_2(c) = 0$; $\phi_1(b) = \phi_2(b) = 1$ and the fuzzy number is characterized as $\{x, \mu_{FN}(x), o : x \in R\}$.

In the work proposed by Dubois and Prade, the terms $\phi_1(x)$ and $\phi_2(x)$ are called leftward and rightward reference functions. A normal fuzzy number is produced based on the functions, namely, super-imposition of sets and Glivenko – Cantelli theorem (Loeve. M, 1977). Baruah (2010, 2012) established the missing link between ill-defined and possibility concept where $\phi_1(x)$ and $\phi_2(x)$ indicate the distribution and complementary function in [a, b] and [b, c] .According to the above principle, the Dubois and Prade leftward reference function is a distribution function and rightward reference function is complementary distribution function.

II. MODEL DESCRIPTION

(FM/E$_k$/1): (FCFS/∞/∞) MODEL:

Consider a single server queuing facility, in which the inter-entry rate is poisson with fuzzy parameter $\bar{\lambda}_A$ and administration rate (service) follows Erlangian conveyance which is made up of 4 exponential phases where the task takes place in succession at distinct counters.

The fuzzy entry level is $\tilde{\lambda}_A = \{(\Omega, \phi_{\tilde{\lambda}_A}(\Omega))/\Omega \in R^+\}$.

The membership function is $\chi_{f(\tilde{\lambda}_A,\mu)}(\phi) = \sup_{\Omega\in R^+} \{\phi_{\tilde{\lambda}_A}(\Omega)/\phi = f(\Omega,\mu)\}$. The alpha – cuts of $\tilde{\lambda}_A$ is

$$\tilde{\lambda}_A = \left[\Omega_\alpha^L, \Omega_\alpha^U\right] = \left[\min_{\Omega\in R^+}\left\{\Omega/\phi_{\tilde{\lambda}_A}(\Omega) \ge \alpha\right\}, \max_{\Omega E R^+}\left\{\Omega/\phi_{\tilde{\lambda}_A}(\Omega) \ge \alpha\right\}\right]$$

which lies at possibility level α.

The Fuzzy Erlangian queue can be reduced to a family of ordinary queue M/E$_k$|1. The supremum and infimum bounds are $\Omega^L_{\tilde{\lambda}_A}(\alpha) = \min.\phi^{-1}_{\tilde{\lambda}_A}(\alpha)$ and $\Omega^U_{\tilde{\lambda}_A}(\alpha) = \max.\phi^{-1}_{\tilde{\lambda}_A}(\alpha)$. To find the constraints of the interval for $f\left(\tilde{\lambda}_A, \mu\right)$ at probable level α

$$\Omega^L_{f(\alpha)} = \min.f(\Omega,\mu);\ \text{subject to: } \Omega^L_{\lambda(\alpha)} \le \Omega \le \Omega^U_{\lambda(\alpha)}$$

$$\Omega^U_{f(\alpha)} = \max.f(\Omega,\mu);\ \text{subject to: } \Omega^L_{\lambda(\alpha)} \le \Omega \le \Omega^U_{\lambda(\alpha)}$$

If $\Omega^L_{f(\alpha)}$ and $\Omega^U_{f(\alpha)}$ are subjected to inversion relating to α, then $L(\phi) = \Omega^{L^{-1}}_{f(\alpha)}$ and $R(\phi) = \Omega^{U^{-1}}_{f(\alpha)}$ are called leftward and rightward reference functions of the membership functionality $\chi_{f(\tilde{\lambda}_A,\mu)}$.

$$\chi_{f(\tilde{\lambda}_A,\mu)}(\phi) = \begin{cases} L(\phi_1) & , \quad \phi_1 \le \phi \le \phi_2 \\ R(\phi) & , \quad \phi_2 \le \phi \le \phi_3 \\ 0 & , \quad otherwise \end{cases}$$ such that

$L(\phi_1) = R(\phi_3) = 0$ and $L(\phi_2) = R(\phi_2) = 0$

The membership functionality for $\tilde{L}_q$ and $\tilde{W}_q$

(i)

$$\phi_{\tilde{L}_q}(\chi) = \sup_{\tilde{\lambda}_A,\,\mu\in R^+ and \frac{\tilde{\lambda}_A}{\mu}<1} \left\{\phi_{\tilde{\lambda}_A}(\Omega) : \chi = (k+1)\lambda\rho / 2k\mu(1-\rho)\right\}$$

(ii)

$$\phi_{\tilde{W}_q}(\chi) = \sup_{\tilde{\lambda}_A,\,\mu\in R^+ and \frac{\tilde{\lambda}_A}{\mu}<1} \left\{\phi_{\tilde{\lambda}_A}(\Omega) : \chi = (k+1)\lambda\rho / 2k\mu(1-\rho)\right\}$$

where $\phi_{\tilde{\lambda}_A}(\Omega)$ is the membership function of $\tilde{\lambda}_A$, $\tilde{W}_q$, represents the expected waiting time in the queue and $\tilde{L}_q$, is the expected system length in the queue.

Numerical Example:

Consider a queue with fuzzy arrival $\tilde{\lambda} = [22, 25, 28]$ and the service rate is $\mu = 29$ with Erlang conveyance made up of 4 phases.

The interval of confidence at α-cut level is [3α+22,28-3α].

The parametric constraints for $\tilde{L}_q$

Published by: The Mattingley Publishing Co., Inc.

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(i) $\chi^{L}_{\tilde{L}_q} = \min.\frac{5}{8}\frac{\lambda^2}{\mu(\mu-\lambda)}$; subject to : 3α + 22 ≤ λ ≤ 28 – 3α

(ii) $\chi^{U}_{\tilde{L}_q} = \min.\frac{5}{8}\frac{\lambda^2}{\mu(\mu-\lambda)}$; subject to : 3α + 22 ≤ λ ≤ 28 – 3α

when λ reaches its lower bound $\frac{5}{8}.\frac{\lambda^2}{\mu(\mu-\lambda)}$ pocures its minimum and the optimum solution is $\chi^{L}_{\tilde{L}_q(\alpha)} = \frac{5}{232}.\frac{(3\alpha+22)^2}{7-3\alpha}$. When λ acquires its upper bound $\frac{5}{8}.\frac{\lambda^2}{\mu(\mu-\lambda)}$ attains its maximum and the optimum solution is $\chi^{U}_{\tilde{L}_q(\alpha)} = \frac{5}{232}.\frac{(28-3\alpha)^2}{1+3\alpha}$. The inverse functions of $\chi^{L}_{\tilde{L}_{q(\alpha)}}$ and $\chi^{U}_{\tilde{W}_q(\alpha)}$ exists, which produces the left and right reference function of $\chi^{U}_{\tilde{L}_{q(\alpha)}}$ as

$$\mu_{\tilde{L}_q}(\chi) = \begin{cases} L(\chi) = \frac{3(232\chi+220)-4(30276\chi^2+75690\chi)^{1/2}}{90} & ,1.49 \le \chi \le 3.36 \\ R(\chi) = \frac{-2(420-348\chi)+4(30276\chi^2+75690\chi)^{1/2}}{90} & ,3.36 \le \chi \le 16.89 \\ 0 & ,\ otherwise \end{cases}$$

On the basis of Randomness – Fuzziness Consistency Principle the left and the right references are distribution and complementary distribution functions.

The parametric constraints for $\tilde{W}_q$

(i) $\chi^{L}_{\tilde{W}_q} = \min.\frac{5}{8}\frac{\lambda}{\mu(\mu-\lambda)}$; subject to: 3α + 22 ≤ λ ≤28-3α

(ii) $\chi^{U}_{\tilde{W}_q} = \min.\frac{5}{8}\frac{\lambda}{\mu(\mu-\lambda)}$; subject to: 3α + 22 ≤ λ ≤28-3α

when λ reaches its lower bound, $\frac{5}{8}.\frac{\lambda}{M(\mu-\lambda)}$ attains its minimum and the optimal solution is $\chi^{L}_{\tilde{W}_q(\alpha)} = \frac{5}{232}.\frac{3\alpha+22}{7-3\alpha}$. When λ reaches its upper bound, $\frac{5}{8}.\frac{\lambda}{M(\mu-\lambda)}$ attains its maximum and the optimal solution is $\chi^{U}_{\tilde{W}_q(\alpha)} = \frac{5}{232}.\frac{28-3\alpha}{1+3\alpha}$. The inverse function exists, which yields the left and right reference functions of $\mu_{\tilde{W}_q}(\chi)$ as:

$$\mu_{\tilde{W}_q}(\chi) = \begin{cases} L(\chi) = \frac{1624\chi-110}{696\chi+15}, & 0.068 \le \chi \le 0.135 \\ R(\chi) = \frac{140-232\chi}{696\chi+15}, & 0.135 \le \chi \le 0.603 \\ 0 & ,\ otherwise \end{cases}$$

Consequently, the left and right reference functions are called as distribution and complementary distribution functions on the basis of Fuzzy Randomness Consistency Principle.

(M/ FE$_k$ /1): (FCFS/∞/∞) MODEL:

Consider a single server queuing model, in which arrivals follow poisson process with crisp parameter λ and service rate follows Erlangian process with fuzzy parameter $\tilde{\mu}_s$ with 4 phases of service. The fuzzy service rate $\tilde{\mu}_s$ is defined as $\tilde{\mu}_s = \left\{\Omega, \phi_{\tilde{\mu}_s}(\Omega) / \Omega E R^+\right\}$. The performance measure $\chi_{f(\lambda,\tilde{\mu}_s)}(\phi) = \sup_{\Omega \in R^+} \left\{\phi_{\tilde{\mu}_s}(\Omega) / \phi = f(\lambda,\Omega)\right\}$. The alpha level set of $\tilde{\mu}_s$ is

$$\tilde{\mu}_s = \left[\Omega^L_\alpha, \Omega^U_\alpha\right] = \left[\min_{\Omega E R^+}\left\{\Omega / \phi_{\tilde{\mu}_s}(\Omega) \ge \alpha\right\}, \max_{\Omega E R^+}\left\{\Omega / \phi_{\tilde{\mu}_s}(\Omega) \ge \alpha\right\}\right]$$

at possibility level α.

The intervals are $\Omega^L_{\tilde{\mu}_s}(\alpha) = \min.\phi^{-1}_{\tilde{\mu}_s}(\alpha)$ and $\Omega^U_{\tilde{\mu}_s}(\alpha) = \max.\phi^{-1}_{\tilde{\mu}_s}(\alpha)$.

Published by: The Mattingley Publishing Co., Inc.

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The mathematical programs are:

$\Omega^{L}_{f(\alpha)}(\alpha) = \min.\ f(\lambda,\Omega)$ such that $\Omega^{L}_{\lambda(\alpha)} \le \Omega \le \Omega^{U}_{\lambda(\alpha)}$

$\Omega^{U}_{f(\alpha)}(\alpha) = \max.\ f(\lambda,\Omega)$ such that $\Omega^{L}_{\lambda(\alpha)} \le \Omega \le \Omega^{U}_{\lambda(\alpha)}$

If $\Omega^{L}_{f(\alpha)}$ and $\Omega^{U}_{f(\alpha)}$ are invertible with respect to α, then $L(\phi) = \Omega^{L\ -1}_{f(\alpha)}$ and R(ϕ) = $\Omega^{U\ -1}_{f(\alpha)}$ are called leftward and rightward reference functions of $\chi_{f(\lambda,\tilde{\mu}_s)}$. Based on Randomness fuzziness consistency principle, L(ϕ) is called the distribution function and R(ϕ) is complementary distribution function.

$$\chi_{f(\lambda,\tilde{\mu}_s)}(\phi) = \begin{cases} L(\phi), & \phi_1 \le \phi \le \phi_2 \\ R(\phi), & \phi_2 \le \phi \le \phi_3 \\ 0 & , otherwise \end{cases}$$

such that $L(\phi_1) = R(\phi_3) = 0$ and $L(\phi_2) = R(\phi_2) = 1$

The membership functions for $\tilde{L}_q$ and $\tilde{W}_q$

$$\phi_{\tilde{L}_q}(\chi) = \sup_{\lambda,\tilde{\mu}_s \in R^+, \frac{\lambda}{\tilde{\mu}_s} < 1} \left\{ \phi_{\tilde{\mu}_s}(\Omega) : \chi = (k+1)\lambda\rho / 2k\mu(1-\rho) \right\}$$

$$\phi_{\tilde{W}_q}(\chi) = \sup_{\lambda,\tilde{\mu}_s \in R^+, \frac{\lambda}{\tilde{\mu}_s} < 1} \left\{ \phi_{\tilde{\mu}_s}(\Omega) : \chi = (k+1)\lambda\rho / 2k\mu(1-\rho) \right\}$$

Numerical Example:

Consider a queue M/FE$_4$/1, in which arrivals follow poisson process with crisp parameter λ = 25 and Erlang service rate with crisp parameter λ = 25 and Erlang service rate with $\tilde{\mu}_s$ = [26, 28, 34]. The confidence level α is [2α+26, 34-6α].

The parametric constraints for $\tilde{L}_q$ are:

(i) $\chi^{L}_{\tilde{L}_q} = \min.\frac{5}{8}.\frac{\lambda^2}{\mu(\mu-\lambda)}$; Subject to: 2α + 26 ≤ λ ≤ 34 – 6α

(ii) $\chi^{U}_{\tilde{L}_q} = \min.\frac{5}{8}.\frac{\lambda^2}{\mu(\mu-\lambda)}$; Subject to: 2α + 2b ≤ λ ≤ 34 – 6α

When λ reaches its lower bound, $\frac{5}{8}.\frac{\lambda^2}{\mu(\mu-\lambda)}$ produces its minimum and the optimum solution is: $\chi^{L}_{\tilde{L}_q(\alpha)} = \frac{3125}{(16\alpha+208)\,(2\alpha+1)}$. When λ reaches its upper bound, $\frac{5}{8}.\frac{\lambda^2}{\mu(\mu-\lambda)}$ procures its maximum and the optimum solution is $\chi^{U}_{L_q(\alpha)} = \frac{3125}{(272-48\alpha)(9-6\alpha)}$. The inverse functions exists which gives rise to left and right reference functions of $\mu_{\tilde{L}_q}(\chi)$ as:

$$\mu_{\tilde{L}_q}(\chi) = \begin{cases} L(\chi) = \frac{1200\chi - 24(-2396\chi^2 + 6250\chi)^{1/2}}{576z}, & 1.28 \le \chi \le 4.65 \\ R(\chi) = \frac{-432\chi + 40(100\chi^2 + 250\chi)^{1/2}}{64z}, & 4.65 \le \chi \le 15.02 \\ 0 & , otherwise \end{cases}$$

The parametric constraints for $\tilde{W}_q$

(i) $\chi^{L}_{\tilde{w}_q} = \min.\frac{5}{8}.\frac{\lambda}{\mu(\mu-\lambda)}$; subject to: 2α + 26 ≤ λ ≤ 34-6α

(ii) $\chi^{U}_{\tilde{w}_q} = \max.\frac{5}{8}.\frac{\lambda}{\mu(\mu-\lambda)}$; subject to: 2α + 26 ≤ λ ≤ 34-6α

When λ reaches its lower bound $\frac{5}{8}.\frac{\lambda}{\mu(\mu-\lambda)}$ attains its minimum and the optimal solution is $\chi^{L}_{\tilde{w}_{q(\alpha)}} = \frac{125}{(16\alpha+208)(2\alpha+1)}$. When λ reaches its upper bound, $\frac{5}{8}.\frac{\lambda}{\mu(\mu-\lambda)}$ attains its maximum and the optimal solution is $\chi^{U}_{\tilde{w}_q(\alpha)} = \frac{125}{(272-48\alpha)(9-6\alpha)}$.

Published by: The Mattingley Publishing Co., Inc.

March - April 2020
ISSN: 0193-4120 Page No. 7415 - 7419

The inverse function exists which gives rise to left and right reference functions of

$$\mu_{\tilde{W}_q}(\chi)=\begin{cases} L(\chi)=\dfrac{1200\chi-24(-2396\chi^2+250\chi)^{1/2}}{576\chi}, & 0.05\le\chi\le 0.08 \\ R(\chi)=\dfrac{-432\chi+40(100\chi^2+10\chi)^{1/2}}{576\chi}, & 0.186\le\chi\le 0.60 \\ 0, & otherwise \end{cases}$$

Where L(χ) and R(χ) are distribution and Complementary distributions.

III. CONCLUSION

Queuing models with phase service has enormous multifaceted applications in the performance and production of manufacturing and networking sectors in all dimensions. In the last few years, focus has been made extensively by many investigators in surveying phase service queuing models for practical applicability. The consumption of the Erlang for phased services yields greater adaptability to shape the real world perplexities. This paper studies two fuzzy queues with Erlangian phase and attains the membership functionality with the help of parametric constraints based Fuzzy randomness with their respective ranges.

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