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On the Solution of a Generalized Equation in a Reaction Problem with Diffusion with Distributed Parameters

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Abstract:
In the proposed article, we analyze the effects of results to a primary issue, that are inadequately analyzed, hence stimulating to trace a development of the reaction exercise with dissemination. The main results of numerical and analytical studies of nonlinear mathematical models describing population processes in a homogeneous and inhomogeneous medium with nonlinear diffusion by quasilinear parabolic equations were presented. We studied such categories of nonlinear differential equations where an unspecified function and imitative of this function penetrate in a power-law manner, thus actual physical exercises were nonlinear, and non-linear numerical models should be used to describe them adequately.

I. Introduction

To solve the Cauchy problem of the reaction-diffusion equation, self-similar solutions satisfying the ordinary differential equation are of great interest.

Studies of the self-similar problem carried out in [1-12]. The one-dimensional case well studied. In particular, it proved that there are a finite number of solutions to the self-similar problem on the line.

Two-dimensional problem is less studied. In its study, usually using numerical methods, and there are certain difficulties. The differential problem posed on the entire plane and it is necessary to choose the domain of numerical integration so that the constructed solution is sufficiently small on its boundary. In addition, when using purely implicit difference schemes, the Newton method usually used for iterations over nonlinearity. Therefore, it is necessary to learn how to build good initial approximations, that is, it is necessary to imagine in advance the form of the desired solution.

In [1] considered, the following Cauchy problem in $Q_T = R^N \times (0,T), N \geq 1$:

$$\frac{\partial}{\partial t}\left(\rho(|x|)u\right) = \sum_{i=1}^{N}\frac{\partial}{\partial x_i}\left(u^{m-1}|Du|^{p-2}u_{x_i}\right), \quad (1)$$

$$u(x,0) = u_0(x), \quad x \in R^N, \quad u_0(x) \geq 0, \quad (2)$$

$$supp u_0 \in B_{R_0} \equiv \{|x| < R_0\}, \quad u_{0\infty R^N} < \infty, \quad (4)$$

$$\rho = (1+|x|)^{-l}, \quad l > 0. \quad (5)$$

In [2], the mentioned equation is considered:

$$u_t - div\left(u^{m-1}|Du|^{p-2}Du\right) = 0 \quad в \quad Q_T = \Omega \times (0,T). \quad (6)$$

$$u^{m-1}|Du|^{p-2}\frac{\partial u}{\partial \vec{n}} = 0 \quad на \quad \partial\Omega \times (0,T,) \quad (7)$$

$$u(x,0) = u_0(x), \quad x \in \Omega \quad (8)$$

where $\Omega \in R^N$, $N \geq 2$ - at $m + p - 3 < 0$ (6) describes a fast diffusion process.

In [3], the Cauchy problem of the following form is considered:

$$u_t = div\left(u^{\alpha}|Du|^{m-1}Du\right) + u^p,$$

thus $0 < m + \alpha \leq 1$.

In [4], the existence and non-existence in time for the result to the following Cauchy complication of an initial data slowly tending to zero are established. Considered:

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$$u_t = div\left(u^{\alpha}\left|Du\right|^{m-1}Du\right)+u^p, \quad (9)$$

$$\left(x,t\right)\in Q_T = \mathrm{R}^N\times\left(0,T\right),\ T>0,\ N\geq 1,$$

$$u\left(0,x\right)=u_0\left(x\right),\quad x\in\mathrm{R}^N, \quad (10)$$

where $m+\alpha\leq 1, m>0, m+\alpha>\max\left\{0, 1-\frac{m+1}{N}\right\}, p>1$.

At $m+\alpha-1<0$, equality (9) refers to the equations of fast diffusion, the case $m=1$ arises in plasma physics [3].

In [5] it is shown that if $p<p^*=1+2/N$, then any non-negative solution to problem (9) - (10) "explodes" at definable duration. In case of exacerbation $p>p^*$.

II. Formulation of the problem.

Under $\mathrm{Q}=\{(\mathrm{t},\mathrm{x}):\ 0<\mathrm{t}<\infty, \mathrm{x}\in\mathrm{R}^N\}$ consider the cross-diffusion system of a biological population

$$\begin{cases}\dfrac{\partial u_1}{\partial t}=\nabla\left(\left|x\right|^n\left|\nabla u_1^k\right|^{p-2}\nabla u_2^{m_1}\right)+k_1\left(u_1-u_1^{\beta_1}\right),\\ \dfrac{\partial u_2}{\partial t}=\nabla\left(\left|x\right|^n\left|\nabla u_2^k\right|^{p-2}\nabla u_1^{m_2}\right)+k_1\left(u_2-u_2^{\beta_2}\right),\end{cases} \quad (11)$$

$u_1\big|_{t=0}=u_{10}(x)$, $u_2\big|_{t=0}=u_{20}(x)$,

where $m_1, m_2, n, p>0$, $\beta_1,\beta_2\geq 0$, $u_1=u_1(t,x)\geq 0$, $u_2=u_2(t,x)\geq 0$.

In this paper, analyze the effects of results from equation (11) depend upon a self-identical way by the practice of nonlinear distributing [12].

Replace in (11) $u_1(t,x)=e^{-k_1t}v_1(\tau(t),x),\quad u_2(t,x)=e^{-k_2t}v_2(\tau(t),x)$:

$$\begin{cases}\dfrac{\partial v_1}{\partial\tau}=\nabla\left(\left|x\right|^n\left|\nabla v_1^k\right|^{p-2}\nabla v_2^{m_1}\right)-a_1\tau^{b_1}v_1^{\beta_1},\\ \dfrac{\partial v_2}{\partial\tau}=\nabla\left(\left|x\right|^n\left|\nabla v_2^k\right|^{p-2}\nabla v_1^{m_2}\right)-a_2\tau^{b_2}v_2^{\beta_2},\end{cases} \quad (12)$$

$$v_1\big|_{t=0}=v_{10}(x),\quad v_2\big|_{t=0}=v_{20}(x).$$

Here

$$\tau(t)=\frac{e^{[m_1k_2+(p-2)kk_1-k_1]t}}{m_1k_2+(p-2)kk_1-k_1}=\frac{e^{[m_2k_1+(p-2)kk_2-k_2]t}}{m_2k_1+(p-2)kk_2-k_2},$$

$$b_1=\frac{k_1\beta_1-(p-2)kk_1-m_1k_2}{m_1k_2+(p-2)kk_1-k_1},$$

$$b_2=\frac{k_2\beta_2-(p-2)kk_1-m_2k_1}{m_2k_1+(p-2)kk_1-k_2}.$$

Subsequently the result of structure (2) is pursued in the format

$$v_1(t,x)=\bar{v}_1(\tau)w_1(\tau(t),\varphi(|x|)),$$
$$v_2(t,x)=\bar{v}_2(\tau)w_2(\tau(t),\varphi(|x|)),$$
$$\bar{v}_1(\tau)=(T_0+\tau)^{-\gamma_1},\ \bar{v}_2(\tau)=(T_0+\tau)^{-\gamma_2},\ T_0>0, \quad (13)$$

Where at $b_1=0,\ b_2=0$: $\gamma_1=\dfrac{1}{\beta_1-1}$, $\gamma_2=\dfrac{1}{\beta_2-1}$, and at $b_1\neq 0,\ b_2\neq 0$: $\gamma_1=\dfrac{b_1+1}{\beta_1-1}$, $\gamma_2=\dfrac{b_2+1}{\beta_2-1}$.

After for $w_i(\tau,\varphi(|x|)),\ i=1,2$ obtain the structure of equations:

$$\begin{cases}\dfrac{\partial w_1}{\partial\tau}=\varphi^{1-s}\dfrac{\partial}{\partial\varphi}\left(\varphi^{s-1}\left|\dfrac{\partial w_1^k}{\partial\varphi}\right|^{p-2}\dfrac{\partial w_2^{m_1}}{\partial\varphi}\right)+\psi_1(w_1-w_1^{\beta_1}),\\ \dfrac{\partial w_2}{\partial\tau}=\varphi^{1-s}\dfrac{\partial}{\partial\varphi}\left(\varphi^{s-1}\left|\dfrac{\partial w_2^k}{\partial\varphi}\right|^{p-2}\dfrac{\partial w_1^{m_2}}{\partial\varphi}\right)+\psi_2(w_2-w_2^{\beta_2}),\end{cases} \quad (14)$$

where $\psi_1=\dfrac{1}{(1-\gamma_1[(p-2)k-1]-\gamma_2m_1)\tau_1}$ и

$$\psi_2=\frac{1}{(1-\gamma_2[(p-2)k-1]-\gamma_1m_2)\tau_2}.$$

Here at $p>n$: $\varphi(|x|)=|x|^{p_1}/p_1$, $p_1=(p-n)/p$, $s=pN/(p-n)$,

And at $p=n$: $\varphi(|x|)=\ln(|x|)$.

Self-identical outcome of structure (14) have a format

$$w_i(\tau(t),\varphi)=f_i(\xi),\quad \xi=\varphi(|x|)/\tau^{1/p}. \quad (15)$$

Then substituting (15) in (14) for $f_i(\xi)$ get a system of self-similar equations

$$\begin{cases}\xi^{1-s}\dfrac{d}{d\xi}(\xi^{s-1}\left|\dfrac{df_1^k}{d\xi}\right|^{p-2}\dfrac{df_2^{m_1}}{d\xi})+\dfrac{\xi}{2}\dfrac{df_1}{d\xi}+\mu_1(f_1-f_1^{\beta_1})=0,\\ \xi^{1-s}\dfrac{d}{d\xi}(\xi^{s-1}\left|\dfrac{df_2}{d\xi}\right|^{p-2}\dfrac{df_1^{m_2}}{d\xi})+\dfrac{\xi}{2}\dfrac{df_2}{d\xi}+\mu_2(f_2-f_2^{\beta_2})=0.\end{cases} \quad (16)$$

where $\mu_1=\dfrac{1}{(1-\gamma_1[(p-2)k-1]-\gamma_2m_1)}$ and

$$\mu_2=\frac{1}{(1-\gamma_2[(p-2)k-1]-\gamma_1m_2)}.$$

System (16) has an approximate solution of the form

$$\bar{f}_1=A(a-\xi)_+^{n_1},\ \bar{f}_2=B(a-\xi)_+^{n_2},$$

where

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$$n_1 = \frac{(p-1)[k(p-2)-(m_1+1)]}{[k(p-2)-1]^2 - m_1 m_2},$$
$$n_2 = \frac{(p-1)[k(p-2)-(m_2+1)]}{[k(p-2)-1]^2 - m_1 m_2}.$$

III. PARABOLIC

Structure of two quasilinear reaction-diffusion equations. Observe within an area $Q=\{(t,x):\ 0<t, x\in R\}$. parabolic structure of two quasilinear reaction-diffusion equations:

$$\begin{cases} \dfrac{\partial u_1}{\partial t} = \dfrac{\partial}{\partial x}\left(D_1 u_2^{m_1-1}\left|\dfrac{\partial u_1}{\partial x}\right|^{p-2}\dfrac{\partial u_1}{\partial x}\right) + l(t)\dfrac{\partial u_1}{\partial x} + k_1(t)u_1\left(1-u_2^{\beta_1}\right), \\ \dfrac{\partial u_2}{\partial t} = \dfrac{\partial}{\partial x}\left(D_2 u_1^{m_2-1}\left|\dfrac{\partial u_2}{\partial x}\right|^{p-2}\dfrac{\partial u_2}{\partial x}\right) + l(t)\dfrac{\partial u_2}{\partial x} + k_2(t)u_2\left(1-u_1^{\beta_2}\right), \end{cases} \tag{17}$$

$$u_1\big|_{t=0} = u_{10}(x),.$$

Diffusion coefficients are equal $D_1 u_2^{m_1-1}\left|\dfrac{\partial u_1}{\partial x}\right|^{p-2}$, $D_2 u_1^{m_2-1}\left|\dfrac{\partial u_2}{\partial x}\right|^{p-2}$ and convective transport with speed $l(t)$, where $m_1, m_2, p, \beta_1, \beta_2$ - favorable real numerals, $u_1 = u_1(t,x)\ge 0,\ u_2 = u_2(t,x)\ge 0$ - pursued solutions.

Next, evaluated the quality measurement elements of the equation under consideration by building a self-identical structure of equations of (17).

We develop the self- identical structure of equations through the technique of nonlinear distributing [1].

Substitution in (17):

$$u_1(t,x) = e^{-\int_0^t k_1(\zeta)d\zeta} v_1(\tau(t),\eta),\ \eta = x - \int_0^t l(\zeta)d\zeta,$$

$$u_2(t,x) = e^{-\int_0^t k_2(\zeta)d\zeta} v_2(\tau(t),\eta),\ \eta = x - \int_0^t l(\zeta)d\zeta$$

will lead (1) to the type

$$\begin{cases} \dfrac{\partial v_1}{\partial \tau} = \dfrac{\partial}{\partial \eta}\left(D_1 v_2^{m_1-1}\left|\dfrac{\partial v_1}{\partial \eta}\right|^{p-2}\dfrac{\partial v_1}{\partial \eta}\right) - k_1(t)e^{[(2-p)k_1+(\beta_1-m_1+1)k_2]t} v_1 v_2^{\beta_1}, \\ \dfrac{\partial v_2}{\partial \tau} = \dfrac{\partial}{\partial \eta}\left(D_2 v_1^{m_2-1}\left|\dfrac{\partial v_2}{\partial \eta}\right|^{p-2}\dfrac{\partial v_2}{\partial \eta}\right) - k_2(t)e^{[(\beta_2-m_2+1)k_1+(2-p)k_2]t} v_1^{\beta_2} v_2, \end{cases} \tag{18}$$ as

$$v_1\big|_{t=0} = v_{10}(\eta),\ v_2\big|_{t=0} = v_{20}(\eta).$$

If $(m_1-1)k_2 + (p-2)k_1 = (m_2-1)k_1 + (p-2)k_2$, then choosing

$$\tau(t) = \frac{e^{[(m_1-1)k_2+(p-2)k_1]t}}{(m_1-1)k_2+(p-2)k_1} = \frac{e^{[(m_2-1)k_1+(p-2)k_2]t}}{(m_2-1)k_1+(p-2)k_2},$$

we get the mentioned structure of calculations:

$$\begin{cases} \dfrac{\partial v_1}{\partial \tau} = \dfrac{\partial}{\partial \eta}\left(D_1 v_2^{m_1-1}\left|\dfrac{\partial v_1}{\partial \eta}\right|^{p-2}\dfrac{\partial v_1}{\partial \eta}\right) - a_1(t)\tau^{b_1} v_1 v_2^{\beta_1}, \\ \dfrac{\partial v_2}{\partial \tau} = \dfrac{\partial}{\partial \eta}\left(D_2 v_1^{m_2-1}\left|\dfrac{\partial v_2}{\partial \eta}\right|^{p-2}\dfrac{\partial v_2}{\partial \eta}\right) - a_2(t)\tau^{b_2} v_1^{\beta_2} v_2, \end{cases} \tag{19}$$

where

$$a_1 = k_1\left((p-2)k_1 + (m_1-1)k_2\right)^{b_1},$$
$$b_1 = \frac{(2-p)k_1 + (\beta_1 - m_1 + 1)k_2}{(p-2)k_1 + (m_1-1)k_2},$$
$$a_2 = k_2\left((m_2-1)k_1 + (p-2)k_2\right)^{b_2},,$$
$$b_2 = \frac{(\beta_2 - m_2 + 1)k_1 + (2-p)k_2}{(m_2-1)k_1 + (p-2)k_2}.$$

If $b_i = 0$ following $a_i(t) = const,\ i=1,2,$ the structure will use this type

$$\begin{cases} \dfrac{\partial v_1}{\partial \tau} = \dfrac{\partial}{\partial \eta}\left(D_1 v_2^{m_1-1}\left|\dfrac{\partial v_1}{\partial \eta}\right|^{p-2}\dfrac{\partial v_1}{\partial \eta}\right) - a_1 v_1 v_2^{\beta_1}, \\ \dfrac{\partial v_2}{\partial \tau} = \dfrac{\partial}{\partial \eta}\left(D_2 v_1^{m_2-1}\left|\dfrac{\partial v_2}{\partial \eta}\right|^{p-2}\dfrac{\partial v_2}{\partial \eta}\right) - a_2 v_1^{\beta_2} v_2. \end{cases}$$

The Cauchy calculation of structure (19) in this scenario while $b_1 = b_2 = 0$, analyzed in [2-6], where a presence of ripple universal results and blow-up outcomes was also determined.

In order to obtain a self-identical for the structure of calculations (19), it first located the result to the system of normal differential calculations:

$$\begin{cases} \dfrac{d\bar{v}_1}{d\tau} = -a_1 \bar{v}_1 \bar{v}_2^{\beta_1}, \\ \dfrac{d\bar{v}_2}{d\tau} = -a_2 \bar{v}_1^{\beta_2} \bar{v}_2, \end{cases}$$

In the form

$$\bar{v}_1(\tau) = c_1(\tau + T_0)^{-\gamma_1},\ \bar{v}_2(\tau) = c_2(\tau+T_0)^{-\gamma_2},,\ T_0 > 0,$$

where

$$c_1 = 1,,\ \gamma_1 = \frac{1}{\beta_2},\ c_2 = 1,\ \gamma_2 = \frac{1}{\beta_1}.$$

And further the outcome to structure (17) was pursued

$$v_1(t,\eta) = \bar{v}_1(t) w_1(\tau,\eta),$$
$$v_2(t,\eta) = \bar{v}_2(t) w_2(\tau,\eta),$$

and $\tau = \tau(t)$ is chosen like this:

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$$\tau_1(\tau)=\int_0^{\tau}\overline{v}_1^{(p-2)}(t)\overline{v}_2^{(m_1-1)}(t)dt=$$

$$=\begin{cases}\dfrac{1}{1-[\gamma_1(p-2)+\gamma_2(m_1-1)]}(T+\tau)^{1-[\gamma_1(p-2)+\gamma_2(m_1-1)]}, \\ \qquad \text{если } 1-[\gamma_1(p-2)+\gamma_2(m_1-1)]\neq 0, \\ \ln(T+\tau), \quad \text{если } 1-[\gamma_1(p-2)+\gamma_2(m_1-1)]=0, \\ (T+\tau), \quad \text{если } p=2 \text{ и } m_1=1,\end{cases}$$

if $\gamma_1(p-2)+\gamma_2(m_1-1)=\gamma_2(p-2)+\gamma_1(m_2-1)$.

After for $w_i(\tau,x),\ i=1,2$ we obtain the structure of equations:

$$\begin{cases}\dfrac{\partial w_1}{\partial \tau}=\dfrac{\partial}{\partial \eta}\left(D_1 w_2^{m_1-1}\left|\dfrac{\partial w_1}{\partial \eta}\right|^{p-2}\dfrac{\partial w_1}{\partial \eta}\right)+\psi_1(w_1 w_2^{\beta_1}-w_1) \\ \dfrac{\partial w_2}{\partial \tau}=\dfrac{\partial}{\partial \eta}\left(D_2 w_1^{m_2-1}\left|\dfrac{\partial w_2}{\partial \eta}\right|^{p-2}\dfrac{\partial w_2}{\partial \eta}\right)+\psi_2(w_2 w_1^{\beta_2}-w_2)\end{cases}, \tag{20}$$

where

$$\psi_1=\begin{cases}\dfrac{1}{(1-[\gamma_1(p-2)+\gamma_2(m_1-1)])\tau}, & \text{если } 1-[\gamma_1(p-2)+\gamma_2(m_1-1)>0, \\ \gamma_1 c_1^{-(1-[\gamma_1(p-2)+\gamma_2(m_1-1)])}, & \text{если } 1-[\gamma_1(p-2)+\gamma_2(m_1-1)=0,\end{cases}$$

$$\psi_2=\begin{cases}\dfrac{1}{(1-[\gamma_2(p-2)+\gamma_1(m_2-1)])\tau}, & \text{if } 1-[\gamma_2(p-2)+\gamma_1(m_2-1)]>0, \\ \gamma_2 c_1^{-(1-[\gamma_2(p-2)+\gamma_1(m_2-1)])}, & \text{if } 1-[\gamma_2(p-2)+\gamma_1(m_2-1)]=0.\end{cases}$$

The description of structure (18) within a kind of (20) suggests that with $\tau\to\infty$ и $\psi_i\to 0$ the solution of the last system asymptotically at the front may tend to solve the system

$$\begin{cases}\dfrac{\partial w_1}{\partial \tau}=\dfrac{\partial}{\partial \eta}\left(D_1 w_2^{m_1-1}\left|\dfrac{\partial w_1}{\partial \eta}\right|^{p-2}\dfrac{\partial w_1}{\partial \eta}\right), \\ \dfrac{\partial w_2}{\partial \tau}=\dfrac{\partial}{\partial \eta}\left(D_2 w_1^{m_2-1}\left|\dfrac{\partial w_2}{\partial \eta}\right|^{p-2}\dfrac{\partial w_2}{\partial \eta}\right).\end{cases}$$

This circumstance is used to find the initial approximation for the construction of the iterative process. If $1-[\gamma_1(p-2)+\gamma_2(m_1-1)\neq 0$, then system (20) has the wave solution:

$$w_i(\tau(t),\eta)=f_i(\xi),\ \xi=c\tau\pm\eta,\ i=1,2,$$

while c is the wave velocity along with functions $w_i(\tau(t),\eta)=f_i(\xi)$ find from the system of self-similar equations:

$$\begin{cases}\dfrac{d}{d\xi}(f_2^{m_1-1}\left|\dfrac{df_1}{d\xi}\right|^{p-2}\dfrac{df_1}{d\xi})+c\dfrac{df_1}{d\xi}+\mu_1(f_1-f_1\ f_2^{\beta_1})=0, \\ \dfrac{d}{d\xi}(f_1^{m_2-1}\left|\dfrac{df_2}{d\xi}\right|^{p-2}\dfrac{df_2}{d\xi})+c\dfrac{df_2}{d\xi}+\mu_2(f_2-f_2\ f_1^{\beta_2})=0,\end{cases} \tag{21}$$

$$\mu_i=\frac{1}{(1-[\gamma_i(p-2)+\gamma_{3-i}(m_i-1)])},\ i=1,2,$$

which has a localized solution

$$\overline{f}_1=A(a-\xi)_+^{n_1},\ \overline{f}_2=B(a-\xi)_+^{n_2},$$

$$n_1=\frac{(p-1)(p-(m_1+1))}{n},\ n_2=\frac{(p-1)(p-(m_2+1))}{n},$$

$$n=(p-2)^2-(m_1-1)(m_2-1),$$

if

$$p>2+[(m_1-1)(m_2-1)]^{1/2},\ p-(m_i+1)>0,\ i=1,2,$$

$$\beta_1=1/n_1,\ \beta_2=1/n_1.$$

As well as, coefficients A and B have been ascertain through the result of a structure of nonlinear algebraic calculations:

$$(n_1)^{p-1}A^{p-1}B^{m_1-1}=c,\ (n_2)^{p-1}A^{m_2-1}B^{p-1}=c.$$

Then taking into account the expressions

$$u_1(t,x)=e^{-\int_0^t k_1(\zeta)d\zeta}v_1(\tau(t),\eta),$$

$$u_2(t,x)=e^{-\int_0^t k_2(\zeta)d\zeta}v_2(\tau(t),\eta)$$

Include,

$$u_1(t,x)=Ae^{-\int_0^t k_1(\zeta)d\zeta}(c\tau(t)-\xi)_+^{n_1},$$

$$u_2(t,x)=Be^{-\int_0^t k_2(\zeta)d\zeta}(c\tau(t)-\xi)_+^{n_2},\ c>0.$$

Due to the fact that $[b\tau(t)-\int_0^t l(\eta)d\eta-x]=0$,

if

$$x\geq[b\tau(t)-\int_0^t l(\eta)d\eta-x]<0,\ \forall t>0,$$

then

$$u_1(t,x)\equiv 0,\ u_2(t,x)\equiv 0,\ x\geq[b\tau(t)-\int_0^t l(\eta)d\eta-x]<0,$$

$$\forall t>0.$$

Hence, the state for localizing the results of structure (17) was the conditions

$$\int_0^e l(y)dy<0,\ \tau(t)<\infty \text{ for } \forall t>0. \tag{22}$$

Condition (22) is the state of an illusion for the recent consequence — this internalization of ripple results (22). If condition (22) is not satisfied, then the circumstance of a definable velocity of propagation of a distraction will be carried out, i.e.

$$u_i(t,x)\equiv 0 \text{ at } |x|\geq b(t),\ \tau(t)=\int_0^t e^{-(m_1+p-3)\int_0^{\zeta}k_1(y)dy}d\zeta,$$

moreover, the front goes as far as the time goes by, since $\tau(t)\to\infty$ at $t\to\infty$.

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IV. SLOW DIFFUSION.

Case $n_1 > 0, n_2 > 0,\ n > 0$ (slow diffusion). Using the method [1] to solve equation (17), we obtain the following functions:

$$\bar{\theta}_1(\xi) = (a-\xi)_+^{\ n_1},\ \bar{\theta}_2(\xi) = (a-\xi\ )_+^{\ n_2},$$

where $a > 0$, $(y)_+ = \max(y, 0)$, $\xi < a$.

It is clear [1, 2] that for the universal presence of the result of calculation (17) the function $f(\xi)$ should fulfill the below mentioned inequality:

$$\begin{cases} \dfrac{d}{d\xi}\left(f_2^{m_1-1}\left|\dfrac{df_1}{d\xi}\right|^{p-2}\dfrac{df_1}{d\xi}\right) + c\dfrac{df_1}{d\xi} + \mu_1(f_1 - f_1\ f_2^{\beta_1}) \le 0, \\ \dfrac{d}{d\xi}\left(f_1^{m_2-1}\left|\dfrac{df_2}{d\xi}\right|^{p-2}\dfrac{df_2}{d\xi}\right) + c\dfrac{df_2}{d\xi} + \mu_2(f_2 - f_2\ f_1^{\beta_2}) \le 0, \end{cases}$$

and

$$\beta_1 = 1/n_2,\ \beta_2 = 1/n_1.$$

Take functions $\bar{\theta}_1(\xi),\ \bar{\theta}_2(\xi)$ and show that they will be the asymptotics of the finite solutions of system (21).

and show that they will be the asymptotics of the finite solutions of system (21). $\xi \to a_-$ has asymptotics $f_i(\xi) \sim \vartheta_i(\xi)$.

Evidence. It will seek a result to equation (21) like a subsequent type:

$$f_i = \bar{\vartheta}_i(\xi)\, y_i(\eta),\ \text{i} = 1,2, \tag{23}$$

where $\eta = -\ln(a-\xi)$, and $\eta \to +\infty$ at $\xi \to a_-$, which allows us to study the asymptotic stability $\eta \to +\infty$. Substituting (23) into (21), for $y_i(\eta)$ we get the following equation:

$$\frac{d}{d\eta}\left(y_{3-i}^{m_i-1}\left|\frac{dy_i}{d\eta} - n_i y_i\right|^{p-2}\left(\frac{dy_i}{d\eta} - n_i y_i\right)\right) + \left(\frac{e^{-\eta}}{a-e^{-\eta}} - n_i\right)\left(y_{3-i}^{m_i-1}\left|\frac{dy_i}{d\eta} - n_i y_i\right|^{p-2}\left(\frac{dy_i}{d\eta} - n_i y_i\right)\right)$$
$$+c\left(\frac{dy_i}{d\eta} - n_i y_i\right) - \mu_i\frac{e^{-\eta}}{a-e^{-\eta}}\, y_i(\eta)(1 + e^{-n_{3-i}\beta_i\eta}\, y_{3-i}^{\ \beta_i}) = 0, \tag{24}$$

where η - function defined above.

Note that studying a result of the final calculation is parallel to analyzing the results of equation (21), every of that in a particular interval $[\eta_0, +\infty)$ completes an inequality

$$y_i(\eta) > 0,\ \frac{dy_i}{d\eta} - n_i y_i \ne 0.$$

We indicate primary of all the outcomes $y_i(\eta)$ equations (21) has the definable restrain y_{0i} at $\eta \to +\infty$. We initiate the subsequent manner:

$$\omega_i(\eta) = y_{3-i}^{m_i-1}\left|\frac{dy_i}{d\eta} - n_i y_i\right|^{p-2}\left(\frac{dy_i}{d\eta} - n_i y_i\right).$$

Then equation (21) takes the form

$$\omega_i' = -\left(\frac{e^{-\eta}}{a-e^{-\eta}} - n_i\right)\omega_i - c\left(\frac{dy_i}{d\eta} - n_i y_i\right) -$$
$$-\mu_i\frac{e^{-\eta}}{a-e^{-\eta}}\, y_i(\eta)(1 + e^{-n_i\beta_i\eta}\, y_{3-i}^{\ \beta_i}).$$

To investigate the final expression, we establish a fresh auxiliary function

$$\phi(\tau,\eta) = -\left(\frac{e^{-\eta}}{a-e^{-\eta}} - n_i\right)\tau - c\left(\frac{dy_i}{d\eta} - n_i y_i\right) -$$
$$-\mu_i\frac{e^{-\eta}}{a-e^{-\eta}}\, y_i(\eta)(1 + e^{-n_i\beta_i\eta}\, y_{3-i}^{\ \beta_i}),$$

where τ - real number.

Therefore, this is simple to view for each value τ function $\phi(\tau,\eta)$ retains a mark at some interval $[\eta_1, +\infty) \subset [\eta_0, +\infty)$ and for all $\eta \in [\eta_1, +\infty)$ one of the inequalities

$$\omega_i'(\eta) > 0,\ \omega_i'(\eta) < 0.$$

In addition, of the function $\omega_i(\eta)$ there is a restrain for $\eta \in [\eta_1, +\infty)$. From the expression for $\omega_i(\eta)$ pursues as

$$\lim_{\eta\to+\infty}\omega_i'(\eta) =$$
$$= \lim_{\eta\to+\infty}\left\{-\left(\frac{e^{-\eta}}{a-e^{-\eta}} - n_i\right)\omega_i - c\left(\frac{dy_i}{d\eta} - n_i y_i\right) - \mu_i\frac{e^{-\eta}}{a-e^{-\eta}}\, y_i(\eta)(1 + e^{-n_i\beta_i\eta}\, y_{3-i}^{\ \beta_{3-i}})\right\} = 0.$$

Hence, given that

$$\xi \to (a)-h,\ \lim_{\eta\to+\infty} e^{-\eta} \to 0,\ \lim_{\eta\to+\infty} a - e^{-\eta} \to a,\ \ \omega_i' = 0,$$

we obtain the following algebraic equation:

1) $\beta_i > 1,\ i = 1, 2,$

$$(n_1)^{p-1}\, y_2^{m_1-1} y_1^{p-1} = c,$$
$$(n_2)^{p-1}\, y_1^{m_2-1} y_2^{p-1} = c.$$

The solution to the latter system gives $y_i = 1$ and by virtue (21), $f_i(\xi) \sim \bar{\vartheta}_i(\xi)$.

2) $\beta_i = 1/n_i,\ i = 1, 2,\ y_i$ should be a solution to the system

$$(n_1)^{p-1}\, y_2^{m_1-1} y_1^{p-2} + y_1^{\ n_1} y_2^{n_1(\beta_1-1)} = c,$$
$$(n_2)^{p-1}\, y_1^{m_2-1} y_2^{p-2} + y_1^{\ n_2(\beta_2-1)} y_2^{n_2} = c.$$

Thorem 1 is proved.

V. FAST DIFFUSION

Case $n_1 > 0, n_2 > 0,\ n < 0$ (fast diffusion). For (21) we have

$$\chi_1(\xi) = (a+\xi\ )^{n_1},\ \chi_2(\xi) = (a+\xi\ )^{n_2},$$

where $a > 0$.

Theorem 2. At $\xi \to +\infty$ the solution problem (21) disappearing at infinity has the asymptotics $f_i(\xi) \sim \chi_i(\xi), \quad i = 1, 2.$

Proof. To prove the theorem, we use the transformation

$$f_i = \chi_i(\xi) y(\eta), \quad i = 1, 2,$$

where $\eta = \ln(a + \xi)$, which leads (22) to the following form.

Substituting (22) into (21), for $y_i(\eta)$ we get the following equation:

$$\frac{d}{d\eta}\left(y_{3-i}^{m_i-1} \left| \frac{dy_i}{d\eta} - n_i y_i \right|^{p-2} (\frac{dy_i}{d\eta} - n_i y_i) \right) + \left(\frac{e^\eta}{a+e^\eta} + n_i \right)\left(y_{3-i}^{m_i-1} \left| \frac{dy_i}{d\eta} - n_i y_i \right|^{p-2} (\frac{dy_i}{d\eta} - n_i y_i) \right) +$$
$$+c\left(\frac{dy_i}{d\eta} + n_i y_i \right) - \mu_i \frac{e^\eta}{a+e^\eta} y_i(\eta)(1 + +e^{-n_i\beta_i\eta} y_{3-i}{}^{\beta_i}) = 0, \tag{25}$$

where η - unction defined above.

Note that studying the result of the final equation is parallel to studying those solutions of equation (21), each of which in a certain interval $[\eta_0, +\infty)$ satisfies the inequality

$$y_i(\eta) > 0, \ \frac{dy_i}{d\eta} - n_i y_i \neq 0.$$

We indicate primary of all the outcomes $y_i(\eta)$ equations (22) has the definable restrain y_{0i} at $\eta \to +\infty$. We initiate the subsequent manner:

$$\omega_i(\eta) = y_{3-i}^{m_i-1} \left| \frac{dy_i}{d\eta} - n_i y_i \right|^{p-2} (\frac{dy_i}{d\eta} - n_i y_i).$$

Then equation (22) takes the form

$$\omega_i' = -\left(\frac{e^{-\eta}}{a - e^{-\eta}} - n_i \right)\omega_i - c\left(\frac{dy_i}{d\eta} - n_i y_i \right) - \mu_i \frac{e^{-\eta}}{a - e^{-\eta}} y_i(\eta)(1 + e^{-n_i\beta}$$

To investigate the final expression, we establish a fresh auxiliary function

$$\phi(\tau, \eta) = -\left(\frac{e^{-\eta}}{a - e^{-\eta}} - n_i \right)\tau - c\left(\frac{dy_i}{d\eta} - n_i y_i \right) - \mu_i \frac{e^{-\eta}}{a - e^{-\eta}} y_i(\eta)(1 + e$$

where τ - real number.

Therefore, this is simple to view for every value τ function $\phi(\tau, \eta)$ retains a mark at some interval $[\eta_1, +\infty) \subset [\eta_0, +\infty)$ и при всех $\eta \in [\eta_1, +\infty)$ one of the inequalities

$$\omega_i'(\eta) > 0, \ \omega_i'(\eta) < 0.$$

Hence, for function $\omega_i(\eta)$ there is a restrain for $\eta \in [\eta_1, +\infty)$. From the expression for $\omega_i(\eta)$ follows that

$$\lim_{\eta \to +\infty} \omega_i'(\eta) =$$
$$= \lim_{\eta \to +\infty} \left\{ \begin{array}{l} -\left(\frac{e^\eta}{a - e^\eta} - n_i \right)\omega_i - c\left(\frac{dy_i}{d\eta} - n_i y_i \right) - \\ \mu_i \frac{e^\eta}{a - e^\eta} y_i(\eta)(1 + e^{-n_i\beta_i\eta} y_{3-i}{}^{\beta_i}) \end{array} \right\} = 0.$$

From here, with $\xi \to \infty$, $\beta_i > 1$, $i = 1, 2$, we obtain the subsequent algebraic equation:

$$(-n_1)^{p-1} y_2^{m_1-1} y_1^{p-1} = c,$$
$$(-n_2)^{p-1} y_1^{m_2-1} y_2^{p-1} = c.$$

Calculating the last equation gives $y_i = 1$ and by virtue (7), $f(\xi) \sim \chi_i(\xi)$..

Theorem 2 is proved.

VI. Computational Experiment

To numerically solve calculation (17), we develop a consistent grid and a temporary grid [13-14].

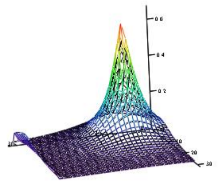
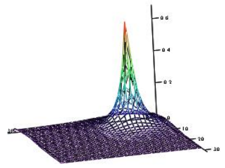
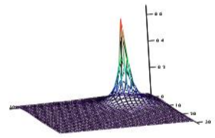
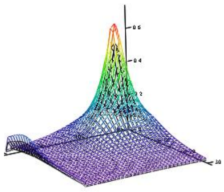
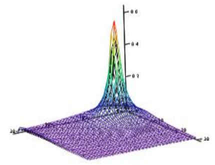
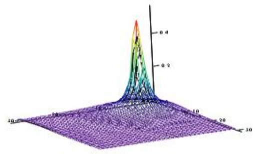
We substitute equation (17) with an complete distinction plan and get a distinction calculation with an delusion $O(h^2 + h_1)$.

As it is clear, the major obstacle for the numerical outcome in nonlinear calculations is the suitable option of the primary estimation along with the technique of linearizing the structure (17).

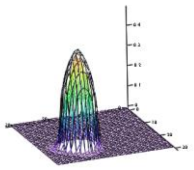
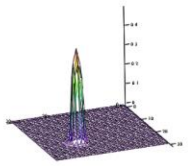
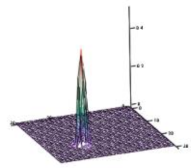
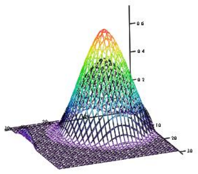
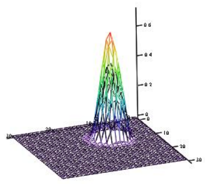
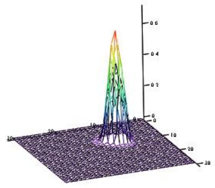
The created application authorizes you to visually monitor the development of the procedure for various values of parameters as well as data.

Numerical computations indicate that in the scenario of arbitrary values $m, p > 0, \beta > 0$ qualitative elements of outcomes will not alter. Furnished were the outcomes of numerical examinations for several parameter values (Fig. 1-2).

1 . Fast diffusion

Parameter values	$x1=1;\ x2=1;$ $\lvert x\rvert=\sqrt{2}$	$x1=30;\ x2=30;$ $\lvert x\rvert=30\sqrt{2}$	$x1=30;\ x2=40;$ 50
$m_1=0.8,\ m_2=0.7,\ p=2.1$ $eps=10^{-3}$ $\beta_1=2\ \ \beta_2=5$ $n<0$			
$m_1=0.4,\ m_2=0.5,\ p=2.2$ $eps=10^{-3}$ $\beta_1=2\ \ \beta_2=2$ $n<0$			

2 . Slow diffusion

Parameter values	$x1=1;\ x2=1;$ $\lvert x\rvert=\sqrt{2}$	$x1=30;\ x2=30;$ $\lvert x\rvert=30\sqrt{2}$	$x1=30;\ x2=40;$ 50
$m_1=1.9,\ m_2=5,\ p=2.5$ $eps=10^{-3}$ $\beta_1=1.5\ \ \beta_2=2$ $n>0$			
$m_1=1.5,\ m_2=2,\ p=2.5$ $eps=10^{-3}$ $\beta_1=1.5\ \ \beta_2=2$ $n>0$			

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VII. Conclusion

Above properties are established on the basis of the solution comparison theorem, the asymptotics of self-similar solutions are obtained, including for the case of fast diffusion. Based on the solutions found, numerical calculations are performed.

Evolution of solutions developing in an aggravated regime involves several stages. In particular, a quasistationary stage (slow solution growth) and an explosive growth stage are necessarily present. Using the example of finite solutions, it is possible to study the rarely considered stages, when the solution first decreases in amplitude, its carrier increases ("spreading of the solution"), then localization occurs (the carrier ceases to change) and only then the solution begins to grow. Our goal was to find out the relationship between the form of the initial perturbation and the duration of these stages.

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