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Calculation of Natural Frequencies of Motor-Fan System Depending on Mathematical Modeling Technique

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Abstract:
Eigenvalues and Eigenvectors for rotating machines equivalent its natural frequencies and mode shape. Several techniques can be implemented to calculate Eigenvalues and Eigenvectors. In this paper a theoretical technique depending on mathematical modeling for motor-fan system used for natural frequencies and mode shape determination of that system. A motor-fan system that contains motor, coupling, gearbox and fan is required to find its mode shapes and natural frequencies. A comparison between theoretical (calculated) and measuring values of natural frequencies has done. A mathematical model for that system is implemented with its differential equations. A code created by MATLAB is used for solving these equations to get the Eigenvalues and Eigenvectors. The results of the technique were very accurate and nearest measuring natural frequencies and mode shapes values.

Keywords:*Eigenvalue, Eigenvector, Mathematical modeling, Motor-Fan system*

I. INTRODUCTION

For determination of natural frequencies and mode shapes of the structure with damping neglected, the dynamic analysis applied. Results of the work proof that, the behavior of the structure indicated its respond related to the dynamic load [1].

Natural frequencies can define as the eigenfrequency or eigenvalues of the structure, it appears when the structure naturally vibrate subjected to a disturbance. Natural frequency and mode shape changing as a result of changes happen in design, measuring device location, result of correlate test and analysis [14].

Eigenfrequencyin the context of using the mathematical technique of Eigenvalues and Eigenvectors to calculating it from the matrix of mass and matrix of stiffness describing the object. Eigenvalue is the squared value of natural frequencies in rad/s unit but eigenvectors describe the mode shapes [2].

Eigenequation define as a group of homogeneous equations containing the components of eigenvector can be used to solve eigenvalue problem. Stiffness and mass used in eigenequation to get values of natural frequencies and mode shape [3].

Determination critical speed, and natural frequency based on reliable method done through connectingbetween roll-tensioning process which applied to the structure during production process for quantification it in industrial situation [4].

Numerical analysis implemented to explain the tendencies of the natural frequency for every variable [5].Data imported in MATLAB for their numerical analysis. Split modes exist for natural frequencies.

Rotation speed where the backward frequency tends to zero can calculate theoretically. It used in determination of the critical speed and natural frequency, which depend on rotation speed [6].

Mathematical equations depending on coupled equations of motion without the forcing terms due to imbalance and gravity by calculating free coupled vibrations, natural frequencies of the coupled bending and torsional motion [7].

Natural frequency calculated by mathematical equation depending on mass and spring values of the foundation

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[8].

Deflected in shaft shape of rotating machine causes unbalance in the shaft which appeared in theoretical equations [9].

In this paper, a mathematical model motor-fan system described in detail through section 2. Equations for calculations natural frequencies, mode shape for motor-fan system in vertical and horizontal directions explained in sections 3 & 4 respectively. MATLAB code that used in natural frequencies and mode shape calculations shown in section 5. Finally, the results of the code appeared in section 6 and section 7 for conclusion.

II. MOTOR-FAN SYSTEM

Mathematical modeling technique used to calculate natural frequencies and mode shape for rotating machine. Motor-fan system studied as an example of rotating machine.It consists of motor, two flexible couplings, shaft, gearbox and fan as appeared in figure 1

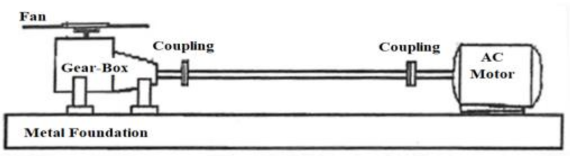

Figure 1: Motor – Fan System

Specifications of motor-fan system components explained as following;

Mass of motor ≈ 35 Kg
Mass of coupling disk ≈ 2 Kg
Mass of crown gear ≈ 15 Kg
Mass of bevel pinion gear ≈ 20 Kg
Mass of fan ≈ 150 Kg
Damping of motor foundation ≈ 1.2 N. s/m
Damping of coupling rubber ≈ 1.252 N. s/m
Damping of air ≈ 1237 N.s/m
Damping of gear box ≈ 4826.73 N. s/m
Stiffness of the motor foundation ≈ 30*106 N/m
Stiffness of the motor shaft ≈ 18.3*107 N/m
Stiffness of the coupling rubber ≈ 41*105 N/m
Stiffness of the transmission shaft ≈ 11.21*105 N/m
Stiffness of the gearbox shaft ≈ 8.67*107 N/m
Stiffness of the fan shaft ≈ 7.49*104 N/m
Stiffness of the gearbox ≈ 825*105 N/m

Natural Frequency in Vertical Direction

As in figure 2 the system' mathematical model in vertical direction with multi degree of freedom, masses, springs and dampers.

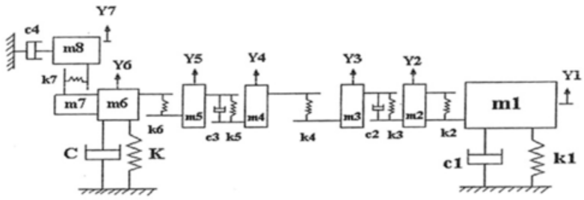

Figure 2: Mathematical Modeling of Motor-Fan System in Vertical Direction

The equivalent differential equations of the mathematical model in vertical direction (Y- axis) can be writeas following;

$$m_1 \ddot{Y}_1 = -C_1 \dot{Y}_1 - k_1 Y_1 - k_2(Y_1 - Y_2) \tag{1}$$
$$m_2 \ddot{Y}_2 = -C_2(\dot{Y}_2 - \dot{Y}_3) - k_2(Y_2 - Y_1) - k_3(Y_2 - Y_3) \tag{2}$$
$$m_3 \ddot{Y}_3 = -C_2(\dot{Y}_3 - \dot{Y}_2) - k_3(Y_3 - Y_2) - k_4(Y_3 - Y_4) \tag{3}$$
$$m_4 \ddot{Y}_4 = -C_3(\dot{Y}_4 - \dot{Y}_5) - k_4(Y_4 - Y_3) - k_5(Y_4 - Y_5) \tag{4}$$
$$m_5 \ddot{Y}_5 = -C_3(\dot{Y}_5 - \dot{Y}_4) - k_5(Y_5 - Y_4) - k_6(Y_5 - Y_6) \tag{5}$$
$$(m_6 + m_7)\ddot{Y}_6 = -C\dot{Y}_6 - KY_6 - k_7(Y_6 - Y_7) \tag{6}$$
$$m_8 \ddot{Y}_7 = -C_4 \dot{Y}_7 - k_7(Y_7 - Y_6) \tag{7}$$

Where;
m1 = Mass of motor
m2 = m3 = m4 = m5 = Mass of coupling disk
m6 = Mass of crown gear
m7 = Mass of bevel pinion gear
m8 = Mass of fan
C1 = Damping of motor foundation
C2 = C3 = Damping of coupling rubber
C4 = Damping of air
C = Damping of gear box
K1 = Stiffness of the motor foundation
K2 = Stiffness of the motor shaft
K3 = K5 = Stiffness of the coupling rubber
K4 = Stiffness of the transmission shaft
K6 = Stiffness of the gear box shaft
K7 = Stiffness of the fan shaft
K = Stiffness of the gear box

From equations (1-7) the M-Mass, C-Damping and K-Stiffness matrices can be obtained as following;

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$$M=\begin{bmatrix} m_1 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & m_2 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & m_3 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & m_4 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & m_5 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & m_6+m_7 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & m_8 \end{bmatrix}$$

$$K=\begin{bmatrix} k_1+k_2 & -k_2 & 0 & 0 & 0 & 0 & 0\\ -k_2 & k_2+k_3 & -k_3 & 0 & 0 & 0 & 0\\ 0 & -k_3 & k_3+k_4 & -k_4 & 0 & 0 & 0\\ 0 & 0 & -k_4 & k_4+k_5 & -k_5 & 0 & 0\\ 0 & 0 & 0 & -k_5 & k_5+k_6 & -k_6 & 0\\ 0 & 0 & 0 & 0 & 0 & k+k_6+k_7 & -k_7\\ 0 & 0 & 0 & 0 & 0 & -k_7 & k_7 \end{bmatrix}$$

$$C=\begin{bmatrix} c_1 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & c_2 & -c_2 & 0 & 0 & 0 & 0\\ 0 & -c_2 & c_2 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & c_3 & -c_3 & 0 & 0\\ 0 & 0 & 0 & -c_3 & c_3 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & C & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & c_4 \end{bmatrix}$$

$$\therefore M^*=\begin{bmatrix} M & 0\\ 0 & I \end{bmatrix}=\begin{bmatrix} m_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & m_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & m_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & m_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & m_5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & m_6+m_7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & m_8 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\therefore K^*=\begin{bmatrix} c_1 & 0 & 0 & 0 & 0 & 0 & 0 & k_1+k_2 & -k_2 & 0 & 0 & 0 & 0 & 0\\ 0 & c_2 & -c_2 & 0 & 0 & 0 & 0 & -k_2 & k_2+k_3 & -k_3 & 0 & 0 & 0 & 0\\ 0 & c_2 & -c_2 & 0 & 0 & 0 & 0 & 0 & -k_3 & k_3+k_4 & -k_4 & 0 & 0 & 0\\ 0 & 0 & 0 & c_3 & -c_3 & 0 & 0 & 0 & 0 & -k_4 & k_4+k_5 & -k_5 & 0 & 0\\ 0 & 0 & 0 & c_3 & -c_3 & 0 & 0 & 0 & 0 & 0 & -k_5 & k_5+k_6 & -k_6 & 0\\ 0 & 0 & 0 & 0 & 0 & C & 0 & 0 & 0 & 0 & 0 & 0 & K+k_6+k_7 & -k_7\\ 0 & 0 & 0 & 0 & 0 & 0 & c_4 & 0 & 0 & 0 & 0 & 0 & -k_7 & k_7\\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} 0$$

Equation (8) used to calculate the natural frequencies of motor-fan system in the vertical direction, which named the Eigenvalues equation [10].

$$|A-\lambda I|_Y=0 \tag{8}$$

Where;

A is system matrix

I is the unite matrix

λ is the Eigenvalues (natural frequency)

Also the system matrix (A), [12] obtained from equation (9), (10) and (11)as following;

$$A=(M^*)^{-1}K^*=\begin{bmatrix} -M^{-1}C & -M^{-1}K\\ I & 0 \end{bmatrix} \tag{9}$$

$$M^*=\begin{bmatrix} M & 0\\ 0 & I \end{bmatrix} \tag{10}$$

$$K^*=\begin{bmatrix} C & K\\ -I & 0 \end{bmatrix} \tag{11}$$

$$A=\begin{bmatrix} -\frac{c_1}{m_1} & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{k_1+k_2}{m_1} & \frac{k_2}{m_1} & 0 & 0 & 0 & 0 & 0\\ 0 & -\frac{c_2}{m_2} & \frac{c_2}{m_2} & 0 & 0 & 0 & 0 & \frac{k_2}{m_2} & -\frac{k_2+k_3}{m_2} & \frac{k_3}{m_2} & 0 & 0 & 0 & 0\\ 0 & \frac{c_2}{m_3} & -\frac{c_2}{m_3} & 0 & 0 & 0 & 0 & 0 & \frac{k_3}{m_3} & -\frac{k3+k4}{m_3} & \frac{k_4}{m_3} & 0 & 0 & 0\\ 0 & 0 & 0 & \frac{-c_3}{m_4} & \frac{c_3}{m_4} & 0 & 0 & 0 & 0 & \frac{k_4}{m_4} & -\frac{k_4+k_5}{m_4} & \frac{k_5}{m_4} & 0 & 0\\ 0 & 0 & 0 & \frac{c_3}{m_5} & \frac{-c_3}{m_5} & 0 & 0 & 0 & 0 & 0 & \frac{k_5}{m_5} & -\frac{k_5+k_6}{m_5} & \frac{k_6}{m_5} & 0\\ 0 & 0 & 0 & 0 & 0 & \frac{c}{m_6+m_7} & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{k+k_6}{m_6+m_7} & \frac{k}{m_6+m_7}\\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{-c_4}{m_8} & 0 & 0 & 0 & 0 & 0 & \frac{k_7}{m_8} & -\frac{k_7}{m_8}\\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

From equations (8), (9), (10), and (11) the Eigenvalues equation (8) can write as in equation (12)

$$0=\left[\begin{vmatrix} -M^{-1}C & -M^{-1}K\\ I & 0 \end{vmatrix}-\lambda I\right]_Y \tag{12}$$

MATLAB is used to solve Eigenvalues equation with all previous data to get motor-fan system' natural frequencies in vertical direction.

For calculation mode shapes of the system [11], the Eigenvector equation (13) used for this

$$|A-\lambda_i I||P_i|_Y=0 \tag{13}$$

Where;

λ_i is the Eigenvalues (Natural Frequencies)

P_i is the Eigenvector's (Mode Shape) for each natural frequency

```
A =1.0e+07 *

 -0.0000        0        0        0        0        0        0  -0.6086   0.5229        0        0        0        0        0
       0  -0.0000   0.0000        0        0        0        0   9.1500  -9.3550   0.2050        0        0        0        0
       0   0.0000  -0.0000        0        0        0        0        0   0.2050  -0.2611   0.0561        0        0        0
       0        0        0  -0.0000   0.0000        0        0        0        0   0.0561  -0.2611   0.2050        0        0
       0        0        0   0.0000  -0.0000        0        0        0        0        0   0.2050  -4.5400   4.3350        0
       0        0        0        0        0  -0.0000        0        0        0        0        0        0  -0.4836   0.0002
       0        0        0        0        0        0  -0.0000        0        0        0        0        0   0.0000  -0.0000
  0.0000        0        0        0        0        0        0        0        0        0        0        0        0        0
       0   0.0000        0        0        0        0        0        0        0        0        0        0        0        0
       0        0   0.0000        0        0        0        0        0        0        0        0        0        0        0
       0        0        0   0.0000        0        0        0        0        0        0        0        0        0        0
       0        0        0        0   0.0000        0        0        0        0        0        0        0        0        0
       0        0        0        0        0   0.0000        0        0        0        0        0        0        0        0
       0        0        0        0        0        0   0.0000        0        0        0        0        0        0        0
```

Natural Frequency in Horizontal Direction

As in vertical direction a mathematical model draw as in figure 3 with multi masses, stiffness, dampers and multi degree of freedom to get natural frequency and mode shape of the system in horizontal direction.

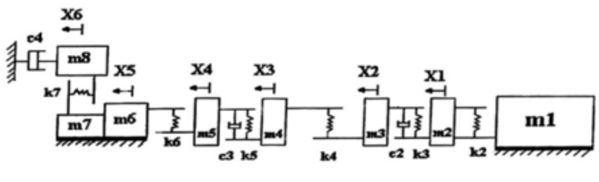

Figure 3: Mathematical Modeling of Motor-Fan System in Horizontal Direction

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According to Figure 3, the mathematical modeling of the motor-fan system in the horizontal direction has six degree of freedom. The equivalent differential equations of the mathematical model in horizontal (X- axis) direction can built as following;

$$m_2\ddot{X}_1 = -C_2(\dot{X}_1 - \dot{X}_2) - k_2(X_1) - k_3(X_1 - X_2) \tag{14}$$

$$m_3\ddot{X}_2 = -C_2(\dot{X}_2 - \dot{X}_1) - k_3(X_2 - X_1) - k_4(X_2 - X_3) \tag{15}$$

$$m_4\ddot{X}_3 = -C_3(\dot{X}_3 - \dot{X}_4) - k_4(X_3 - X_2) - k_5(X_3 - X_4) \tag{16}$$

$$m_5\ddot{X}_4 = -C_3(\dot{X}_5 - \dot{X}_4) - k_5(X_5 - X_4) - k_6(X_5 - X_6) \tag{17}$$

$$(m_6 + m_7)\ddot{X}_5 = -k_6(X_5 - X_4) - k_7(X_5 - X_6) \tag{18}$$

$$m_8\ddot{X}_6 = -C_4\dot{X}_6 - k_7(X_6 - X_5) \tag{19}$$

From equations (14-19) the M (Mass) matrix, C (Damping) matrix and K (Stiffness) matrix can obtained as following;

$$M = \begin{bmatrix} m_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & m_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & m_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & m_5 & 0 & 0 \\ 0 & 0 & 0 & 0 & m_6 + m_7 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$K = \begin{bmatrix} k_2+k_3 & -k_3 & 0 & 0 & 0 & 0 \\ -k_3 & k_3+k_4 & -k_4 & 0 & 0 & 0 \\ 0 & -k_4 & k_4+k_5 & -k_5 & 0 & 0 \\ 0 & 0 & -k_5 & k_5+k_6 & -k_6 & 0 \\ 0 & 0 & 0 & k_6 & k_6+k_7 & -k_7 \\ 0 & 0 & 0 & 0 & -k_7 & k_7 \end{bmatrix}$$

$$C = \begin{bmatrix} c_2 & -c_2 & 0 & 0 & 0 & 0 \\ -c_2 & c_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & c_3 & -c_3 & 0 & 0 \\ 0 & 0 & -c_3 & c_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & c_4 \end{bmatrix}$$

$$\therefore M^* = \begin{bmatrix} M & 0 \\ 0 & I \end{bmatrix} = \begin{bmatrix} m_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & m_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & m_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & m_5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & m_6+m_7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & m_8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\therefore K^* = \begin{bmatrix} c_2 & -c_2 & 0 & 0 & 0 & 0 & k_2+k_3 & -k_3 & 0 & 0 & 0 & 0 \\ c_2 & -c_2 & 0 & 0 & 0 & 0 & -k_3 & k_3+k_4 & -k_4 & 0 & 0 & 0 \\ 0 & 0 & c_3 & -c_3 & 0 & 0 & 0 & -k_4 & k_4+k_5 & -k_5 & 0 & 0 \\ 0 & 0 & c_3 & -c_3 & 0 & 0 & 0 & 0 & -k_5 & k_5+k_6 & -k_6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & K+k_6+k_7 & -k_7 \\ 0 & 0 & 0 & 0 & 0 & c_4 & 0 & 0 & 0 & 0 & -k_7 & k_7 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

To calculate the natural frequencies of motor-fan system in horizontal direction we will use the Eigenvalues equation (2) as shown

$$|A - \lambda I|_x = 0 \tag{20}$$

Where;
A is system matrix
I is the unite matrix
λ is the Eigenvalues (natural frequency)

$$A = (M^*)^{-1}K^* = \begin{bmatrix} -M^{-1}C & -M^{-1}K \\ I & 0 \end{bmatrix} \tag{21}$$

$$M^* = \begin{bmatrix} M & 0 \\ 0 & I \end{bmatrix} \tag{22}$$

$$K^* = \begin{bmatrix} C & K \\ -I & 0 \end{bmatrix} \tag{23}$$

From equations (21), (22) and (23) system matrix (A) obtained.

$$A = \begin{vmatrix} -\frac{c_2}{m_2} & \frac{c_2}{m_2} & 0 & 0 & 0 & 0 & -\frac{k_2+k_3}{m_2} & \frac{k_3}{m_2} & 0 & 0 & 0 & 0 \\ \frac{c_2}{m_3} & -\frac{c_2}{m_3} & 0 & 0 & 0 & 0 & \frac{k_3}{m_3} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{-c_3}{m_4} & \frac{c_3}{m_4} & 0 & 0 & 0 & \frac{k_4}{m_4} & -\frac{k_4+k_5}{m_4} & \frac{k_5}{m_4} & 0 & 0 \\ 0 & 0 & \frac{c_3}{m_5} & \frac{-c_3}{m_5} & 0 & 0 & 0 & 0 & \frac{k_5}{m_5} & -\frac{k_5+k_6}{m_5} & \frac{k_6}{m_5} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{k_6}{m_6+m_7} & -\frac{k_6+k_7}{m_6+m_7} & \frac{k_7}{m_6+m_7} \\ 0 & 0 & 0 & 0 & 0 & \frac{-c_4}{m_8} & 0 & 0 & 0 & 0 & \frac{k_7}{m_8} & -\frac{k_7}{m_8} \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{vmatrix}$$

Eigenvalues (natural frequencies of motor-fan system in horizontal direction) obtained from equation (24)

$$0 = \left[\left|\begin{matrix} -M^{-1}C & -M^{-1K} \\ I & 0 \end{matrix}\right| - \lambda I\right]_x \tag{24}$$

MATLAB software used to solve Eigenvalue equation with all previous data to get natural frequencies of motor-fan system in horizontal direction.

Calculation of mode shape (eigenvector) of the system

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done through equation (25).

$$|A - \lambda_i I||P_i|_x = 0 \quad (25$$

Where;

λ_i is the Eigenvalues (Natural Frequencies)

P_i is the Eigenvector's (Mode Shape) for each natural frequency.

```
A =

   1.0e+08 *

    0.0000   -0.0000         0         0         0         0    1.8710   -0.0410         0         0         0         0
   -0.0000    0.0000         0         0         0         0   -0.0205    0.0261   -0.0056         0         0         0
         0         0    0.0000   -0.0000         0         0         0   -0.0056    0.0261   -0.0205         0         0
         0         0   -0.0000    0.0000         0         0         0         0   -0.0205    0.4540   -0.4335         0
         0         0         0         0         0         0         0         0         0    0.4335    0.4337   -0.0002
         0         0         0         0         0    0.0000         0         0         0         0   -0.0000    0.0000
   -0.0000         0         0         0         0         0         0         0         0         0         0         0
         0   -0.0000         0         0         0         0         0         0         0         0         0         0
         0         0   -0.0000         0         0         0         0         0         0         0         0         0
         0         0         0   -0.0000         0         0         0         0         0         0         0         0
         0         0         0         0   -0.0000         0         0         0         0         0         0         0
         0         0         0         0         0   -0.0000         0         0         0         0         0         0
```

MATLAB Code

Calculation of motor-fan system natural frequency and also its mode shape a MATLAB code designed depending on mass, stiffness and damping values as inputs and eigenvalues and eigenvectorwere outputs.

Code for Vertical Direction

Values of mode shapes did not give any values because the internal forces were not included; this comes from mechanical defects in motor-fan system in the above equations and in the code.

Code for Horizontal Direction

Values of mode shapes like in the horizontal direction did not give any values because the internal forces were not included in both MATLAB cod and mathematical equations.

III. Results and Discussion

The natural frequencies of motor-fan system presented in Table 1. Both the measured and calculated frequencies are included. The considered model was only a 7-degree of freedom system. On the other hand, the actual motor-fan system is a distributed parameter system with infinite number of natural frequencies.Limited number of measured natural frequencies considered and compared with the calculated frequencies. The calculated lowest natural frequency, in both vertical and horizontal directions, are in good agreement with the measured frequencies. The lowest natural is the dominant and most important frequency. The estimates of higher frequencies are also accepted. The difference between the measured and calculated frequencies attributed to the modeling of distributed parameter system with a lumped parameter system. The MATLAB code is also included (Appendix A).

Table 1: Natural Frequencies and Mode Shapes in Vertical and Horizontal Directions

Measured Frequencies in Vertical Direction	**Natural Frequencies in Vertical Direction**	**Measured Frequencies in Horizontal Direction**	**Natural Frequencies in Horizontal Direction**
2.45	2.70	2.50	3.63
17.37	20.7	18	
25	28.54	24.53	29.72
34.44	29.33	35	
37.60	30.97	38.33	
54	57.23	55.32	60.16
79.1	70.21	80	69.26
180.53		182	78.03

( ) no calculated values of natural frequency

IV. Conclusion

The most efficient numerically accurate method usedfor computing frequencies and mode shape of any rotating machine depends on its M, C and K values. It has been pointed out that the mathematical technique that depending on Eigenvalues and Eigenvectors which used to calculate natural frequencies and mode shapes for a damped system gave a very accurate values approximately identical with the measuring values in case of taking the internal forces that comes from internal mechanical defects of motor-fan system in the mathematical model equations.

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