

Computational Analysis of Magnetic Field Effect on Flow of Blood in Stenosed Artery Having Porous Substances

Surendra Kumar, Department of Mathematics, UIET. M.D.University, Rohtak, Haryana, India
Rajkumar Yadav*, Department of Computer Sc. & Engg, I.G University, Meerpur, Rewari, Haryana, India
Gaurav Singh Sisodia, Ch. Devi Lal State Institute of Engineering & Technology, Panniwala Mota (Sirsa)

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Abstract:

In the present problem magnetic field effect on flow of blood in stenosed artery having porous substances is discussed. Model geometry, a long circular vessel with axisymmetric stenosis for blood flow is considered. The modelled problem is solved by Adomian's Decomposition method to obtain velocity field. Hartmann number effect, porosity, stenosis thickness of the stenosis on flow, shear stress, skin friction are discussed. In the last of this study generated data were analyzed by graphical representations.

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1. INTRODUCTION

The stenosis is occurred by the irregular growth in the arterial wall and is one of the frequently occurring diseases in mammalian arteries. It is well established that once an obstruction has developed, the flow of blood will be disturbed which may result into circulatory disorders. The effect of stenosis in the lumen of an artery over the flow characteristics has been studied by many mathematicians. The magnetic field influence is endorsed on the basis of the composition of blood. Since the constituents of blood are plasma and cells (RBC, WBC and platelets other particles). The red blood cells which contain, iron, that is magnetic conducting in nature, the core region, the region of RBC may be treated as magnetic fluid. Flow rate and viscous drag in arteries with pressure gradient was studied by Womersley in 1955 [21]. Two fluid blood flow model through

small diameter tubes was discussed by Chaturani and Upadhyaya in 1979 [2] and found that with the increase in tube radius effective viscosity increases. Resistive impedance, wall shear stress and immediate flow rate of oscillatory blood flow through stenosed artery was obtained numerically by Halder in 1987 [4]. Halder and Ghosh in 1994 [3] found as magnetic field increases the pressure gradient increased significantly. Effects of magnetic field and hematocrit control the velocity and point out the flattening of velocity profile at the control region of the tube. As the wall shear stress increases strength of magnetic field also increases. At higher magnetic field it increases so significantly that it may cause the stenosis to break away and there is a chance of paralysis or sudden death. Unsteady oscillatory flow of blood through indented tube with single constriction was discussed by Sanyal and Maji in 1999 [13] and found that as hematocrit value increased the pressure gradient also increases. Also in high

systolic and low diastolic pressure, peripheral blood flow will increase but coronary arterial blood flows will decrease. Srivastava in 2002 [17] studied effects of stenosis shape and hematocrit on flow of blood as particulate suspension with stenotic arteries. In this study it was noticed that as shape parameter increases the flow resistance decreases but increases with hematocrit. In 2005 Tzirtzilakis [19] studied a model for flow of blood in the presence of magnetic field and found that flow is influenced by magnetic field. In 2006 Celik, et al [1] discussed the solution of differential algebraic Eqs. by Adomian decomposition method. They obtained approximate solutions using Adomian's method. Newtonian blood flow in constricted blood vessel with uniformly applied transverse magnetic field with the help of Adomian's decomposition was discussed by Haldar in 2009 [5]. Sankar & Lee in 2010 [12] analyzed the pulsatile blood flow in catheterized blood vessel. They observed that in comparison to Casson model for blood flow the velocity of flow, flow rate and velocity distribution are higher for Herschel-Bulkley model. Kumar et al 2011 [9] described a model for stenosed artery having variable shapes and observed different effects of magnetic field, stenosis shape on the flow resistance. Unsteady pulsatile flow of blood through porous substance was studied by Sharma et al 2012 [16] and found that magnetic field and porous medium both affect transportation of blood to the organs and magnetic field is influenced by Hematocrit concentration. Sharma et al in 2013, 2015 [14] [15] formulate bio-fluid mathematical model for non-Darcy porous medium. They transformed nonlinear PDE into linear PDE and obtained numerical solutions by using finite difference method further they extended their study with heat source. The power-law & Herschel-Bulkley blood flow model are used by Kumar in 2015 [7] to study the influence of non-Newtonian characteristics of blood through an artery. They observed that fluid velocity decreases with the downstream for each value of elasticity of the vessel but in case of small value of elasticity the downstream velocity from the transition point decreases enormously compared to higher elastic value of the vessel. Kumar in 2016

[8] studied two phase blood flow in stenosed artery and analyzed magnetic field, stenosis thickness, width of plasma layer, effect on shear stress, plug flow for core and plasma. The wall shear stress and flow resistance with hematocrit level by using power-law fluid model was studied by Malek and Hoque in 2017 [10] and observed that resistance of flow is proportional to stenosis height. Jamil et al 2018 [6] presented the flow of blood with periodic acceleration through arterial stenosis and for solution of their problem they used Mathcad software. These different studies motivated for the proposed study to analyze the problem related to flow of blood through a stenosed artery having porous medium.

2. PROBLEM FORMULATION

In this paper the laminar, steady axially symmetric blood flow in a stenosed artery with magnetic field and porous medium is considered. Blood, flowing through an artery is considered as conducting Newtonian. The stenosis in the artery is mild and axially symmetric. To remove entrance end effects flow is considered in long cylindrical tube. A transverse static magnetic field is applied on the flow of blood.

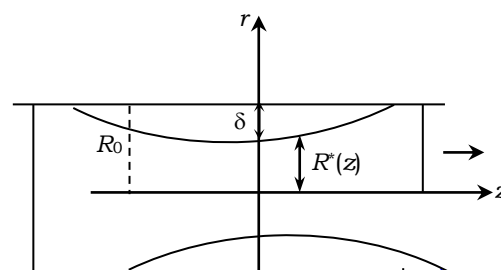


Figure 1 Flow model of the problem

Axis of artery considered as z -axis and the shape of stenosis as follows:

$$h^*(z^*) = \begin{cases} R_0 - \frac{\delta^*}{2} \left(1 + \cos \frac{\pi z^*}{L_0} \right) \\ R_0 \end{cases} \quad (1)$$

Where, R_0 : Tube radius, $R^*(z)$: Tube radius in the stenotic region and δ : Maximum height of the stenosis.

Different assumptions:

- (i) Blood is taken as electrically conducting incompressible fluid

- (ii) The arterial wall is porous
- (iii) Induced electric and magnetic field is negligible
- (iv) Viscosity of blood and density are considered constant

Considering the above assumptions the governing Eqs of motion in (r^*, θ^*, z^*) are given by

$$w^* \frac{\partial w^*}{\partial z^*} + u^* \frac{\partial w^*}{\partial r^*} = -\frac{1}{\rho} \frac{\partial p^*}{\partial z^*} + \nu \left(\frac{\partial^2 w^*}{\partial z^{*2}} + \frac{\partial^2 w^*}{\partial r^{*2}} + \frac{1}{r^*} \frac{\partial w^*}{\partial r^*} \right) - \nu \frac{w^*}{K^*} - \frac{\sigma B_0^2}{\rho} w^* \quad (2)$$

$$w^* \frac{\partial u^*}{\partial z^*} + u^* \frac{\partial u^*}{\partial r^*} = -\frac{1}{\rho} \frac{\partial p^*}{\partial r^*} + \nu \left(\frac{\partial^2 u^*}{\partial z^{*2}} + \frac{\partial^2 u^*}{\partial r^{*2}} + \frac{1}{r^*} \frac{\partial u^*}{\partial r^*} - \frac{u^*}{r^{*2}} \right) - \nu \frac{u^*}{K^*} \quad (3)$$

and the continuity Eq. is

$$\frac{\partial w^*}{\partial z^*} + \frac{\partial u^*}{\partial r^*} + \frac{u^*}{r^*} = 0 \quad (4)$$

Corresponding conditions are

$$w^* = u^* = 0 \text{ at } r^* = R^*(z^*) \quad (5)$$

$$\frac{\partial w^*}{\partial r^*} = 0, \frac{\partial u^*}{\partial r^*} = 0 \text{ at } r^* = 0 \quad (6)$$

The volumetric flux Q^* across any cross section of the tube remains constant, therefore

$$\int_0^{R^*(z^*)} 2\pi r^* w^* dr^* = Q^* \quad (7)$$

where (w^*, u^*) : Velocity components

$B_0 (= \mu_e H_0)$: Electromagnetic induction

H_0 : Transverse component of the magnetic field

K^* : Permeability of the porous medium.

3. MATHEMATICAL SOLUTION

Introducing non-dimensional parameters

$$z = \frac{z^*}{R_0}, \quad r = \frac{r^*}{R_0}, \quad w = \frac{w^*}{w_0}, \quad u = \frac{u^*}{w_0}, \quad p = \frac{p^*}{\rho w_0^2}, \quad K = \frac{K^*}{R_0^2},$$

$$R_e = \frac{w_0 R_0}{\nu}, \quad M^2 = \frac{B_0^2 \sigma R_0^2}{\mu}$$

$$\delta = \frac{\delta^*}{R_0} \quad \text{and} \quad h(z) = \frac{h^*(z^*)}{R_0} \quad (8)$$

where (w, u) : Dimensionless velocity components

w_0 : Characteristic velocity

p : Non-dimensional fluid pressure

K : Permeability factor

M : Hartmann number and

R_e : Reynolds number

The Eqs. (1) to (7), reduces into

$$h(z) = 1 - \frac{\delta}{2} \left(1 + \cos \frac{\pi z}{L_0} \right), \quad |z| \leq L_0 \quad (9)$$

$$w \frac{\partial w}{\partial z} + u \frac{\partial w}{\partial r} = -\frac{\partial p}{\partial z} + \frac{1}{R_e} \left[\frac{\partial^2 w}{\partial z^2} + \frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} \right] - \frac{1}{R_e} \frac{w}{K} - M^2 w \quad (10)$$

$$w \frac{\partial u}{\partial z} + u \frac{\partial u}{\partial r} = -\frac{\partial p}{\partial r} + \frac{1}{R_e} \left[\frac{\partial^2 u}{\partial z^2} + \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} - \frac{u}{r^2} \right] - \frac{1}{R_e} \frac{u}{K} \quad (11)$$

$$\frac{\partial w}{\partial z} + \frac{\partial u}{\partial r} + \frac{u}{r} = 0 \quad (12)$$

$$w = u = 0 \text{ at } r = h(z) \quad (13)$$

$$\frac{\partial w}{\partial r} = 0, \frac{\partial u}{\partial r} = 0 \text{ at } r = 0 \quad (14)$$

$$\frac{1}{2} = \int_0^{h(z)} r w dr \quad (15)$$

Introducing the stream function ψ given by

$$w = \frac{1}{r} \frac{\partial \psi}{\partial r} \quad \text{and} \quad u = -\frac{1}{r} \frac{\partial \psi}{\partial z} \quad (16)$$

The Eqs. (10) and (11), can be combined to yield the following Eq. in the stream function ψ Mazumdar in 1992 [11]

$$R_e \left[\frac{1}{r} \frac{\partial (\nabla_1^2 \psi, \psi)}{\partial (r, z)} - \frac{2}{r^2} \nabla_1^2 \psi \frac{\partial \psi}{\partial z} \right] = \nabla_1^4 \psi - R_e M^2 r \frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial \psi}{\partial r} \right)$$

$$- \left(\frac{1}{K} \right) r \frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial \psi}{\partial r} \right) + \left(\frac{1}{K} \right) r \frac{\partial}{\partial z} \left(\frac{1}{r} \frac{\partial \psi}{\partial z} \right) \quad (17)$$

The corresponding boundary conditions are

$$\frac{1}{r} \frac{\partial \psi}{\partial r} = 0, \quad -\frac{1}{r} \frac{\partial \psi}{\partial z} = 0 \quad \text{and} \quad \psi = \frac{1}{2} \text{ at } r = h \quad (18)$$

$$\frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial \psi}{\partial r} \right) = 0, \quad \psi = 0 \text{ at } r = 0 \quad (19)$$

$$\text{where} \quad \nabla_1^2 = \frac{\partial^2}{\partial r^2} - \frac{1}{r} \frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2} \quad (20)$$

$$\text{and} \quad \frac{\partial (\nabla_1^2 \psi, \psi)}{\partial (r, z)} \quad (21)$$

the Jacobian.

The Eq. (17), is a non-linear partial differential Eq. To obtain its solution with Adomian decomposition method, rewriting it in the following form

$$R_e N_\psi = \nabla_1^2 (\nabla_1^2 \psi) - R_e M^2 \left[\frac{\partial^2 \psi}{\partial r^2} - \frac{1}{r} \frac{\partial \psi}{\partial r} \right]$$

$$-\frac{1}{K}\left[\frac{\partial^2\psi}{\partial r^2}-\frac{1}{r}\frac{\partial\psi}{\partial r}\right] + \frac{1}{K}\left(\frac{\partial^2\psi}{\partial z^2}\right) \quad (22)$$

where

$$N_\psi = \frac{1}{r} \frac{\partial(\nabla_1^2\psi, \psi)}{\partial(r, z)} - \frac{2}{r^2} \nabla_1^2\psi \frac{\partial\psi}{\partial z} \quad (23)$$

Taking the linear operator

$$L = \frac{\partial^2}{\partial r^2} - \frac{1}{r} \frac{\partial}{\partial r} \quad (24)$$

The Eq. (22), reduces into

$$L^2\psi = R_e N_\psi - 2 \frac{\partial^2}{\partial z^2} L\psi - \frac{\partial^4\psi}{\partial z^4} + M^2 R_e L\psi + \frac{1}{K} L\psi - \frac{1}{K} \frac{\partial^2\psi}{\partial z^2} \quad (25)$$

If ψ_0 is the solution of the homogenous Eq. $L^2\psi = 0$ then the general solution of the Eq.(25) be

$$\psi = \psi_0 + L^{-2} \left[R_e N_\psi - \frac{\partial^4\psi}{\partial z^4} - 2 \frac{\partial^2}{\partial z^2} (L\psi) + M^2 R_e L\psi + \frac{1}{K} L\psi - \frac{1}{K} \frac{\partial^2\psi}{\partial z^2} \right] \quad (26)$$

The solution of the homogenous Eq. $L^2\psi_0 = 0$ is given by

$$\psi_0 = \frac{1}{16} A(z) r^4 + B(z) L_1^{-1} r \log r + \frac{1}{2} C(z) r^2 + D(z) \quad (27)$$

Considering regular decomposition of ψ and N_ψ in the following forms

$$\psi = \sum_{n=0}^{\infty} \lambda^n \psi_n, \quad N_\psi = \sum_{n=0}^{\infty} \lambda^n P_n \quad (28)$$

where P_n are Adomian's special polynomials, given by

$$P_0 = \frac{1}{r} \frac{\partial(\nabla_1^2\psi_0, \psi_0)}{\partial(r, z)} - \frac{2}{r^2} \frac{\partial\psi_0}{\partial z} \nabla_1^2\psi_0 \quad (29)$$

$$P_1 = \frac{1}{r} \left[\frac{\partial(\nabla_1^2\psi_1, \psi_0)}{\partial(r, z)} + \frac{\partial(\nabla_1^2\psi_0, \psi_1)}{\partial(r, z)} \right] - \frac{2}{r^2} \left[\frac{\partial\psi_0}{\partial z} \nabla_1^2\psi_1 + \frac{\partial\psi_1}{\partial z} \nabla_1^2\psi_0 \right] \quad (30)$$

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The corresponding conditions on boundary conditions:

$$r=0: \frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial\psi_n}{\partial r} \right) = 0, \quad \psi_n = 0 \quad (31)$$

$$r=h: \frac{1}{r} \frac{\partial\psi_0}{\partial r} = 0, \quad \psi_0 = \frac{1}{2},$$

$$\frac{1}{r} \frac{\partial\psi_n}{\partial r} = 0, \quad \psi_n = 0; n \geq 1 \quad (32)$$

Using Eqs. (28), (29) in (26), we get

$$\psi_{n+1} = \psi_{0,n+1} + L^{-2} \left[R_e P_n - \frac{\partial^4\psi_n}{\partial z^4} - 2 \frac{\partial^2}{\partial z^2} (L\psi_n) + M^2 R_e L\psi_n + \frac{1}{K} L\psi_n - \frac{1}{K} \frac{\partial^2\psi_n}{\partial z^2} \right] \quad (33)$$

Further taking parameterized decomposition of ψ_0 , $A(z)$, $B(z)$, $C(z)$ and $D(z)$ as defined by

$$\psi_0 = \sum_{n=0}^{\infty} \lambda^n \psi_{0,n}, \quad A(z) = \sum_{n=0}^{\infty} \lambda^n A_n(z),$$

$$B(z) = \sum_{n=0}^{\infty} \lambda^n B_n(z), \quad C(z) = \sum_{n=0}^{\infty} \lambda^n C_n(z),$$

$$D(z) = \sum_{n=0}^{\infty} \lambda^n D_n(z) \quad (34)$$

Using above double decomposition, we get

$$\psi_{0,n+1} = \frac{1}{16} A_{n+1}(z) r^4 + B_{n+1}(z) L_1^{-1} r \log r + \frac{1}{2} C_{n+1}(z) r^2 + D_{n+1}(z) \quad (35)$$

Under the prescribed boundary conditions $A(z)$, $B(z)$, $C(z)$, $D(z)$ and $A_1(z)$, $B_1(z)$, $C_1(z)$, $D_1(z)$ are obtained, where

$$A(z) = -\frac{8}{h^4}, \quad B(z) = 0, \quad C(z) = \frac{2}{h^2}, \quad D(z) = 0$$

$$A_1(z) = -16h^2(4F_1h^4 + 3F_2h^2 + 2F_3), \quad B_1(z) = 0,$$

$$C_1(z) = 2h^4(3F_1h^4 + 2F_2h^2 + F_3), \quad D_1(z) = 0$$

Invoking above values in the eq. (27) and (35), ψ_0 and $\psi_{0,1}$, are given by

$$\psi_0 = \frac{1}{2h^4} (-r^4 + 2h^2r^2)$$

$$\psi_{0,1} = \frac{1}{16} r^4 [-16h^2(4F_1h^4 + 3F_2h^2 + 2F_3)] + \frac{1}{2} r^2 [2h^4(3F_1h^4 + 2F_2h^2 + F_3)]$$

Therefore, the solution for ψ_1 from the Eq. (33), is given by

$$\psi_1 = (-4h^6r^4 + 3h^8r^2 + r^{10}) F_1 + (-3h^4r^4 + 2h^6r^2 + r^8) F_2 + (-2h^2r^4 + h^4r^2 + r^6) F_3$$

Taking first order approximation of ψ , the velocity w is given by

$$w = \frac{1}{r} \frac{\partial\psi_0}{\partial r} + \frac{1}{r} \frac{\partial\psi_1}{\partial r} = \frac{2}{h^4} (-r^2 + h^2) + 10F_1(z)r^8 + 8F_2(z)r^6 + 6F_3(z)r^4 - (16h^6F_1(z) + 12F_2(z)h^4 + 8F_3(z)h^2r^2 + 6h^8F_1(z) + 4h^6F_2(z) + 2h^4F_3(z))$$

$$+ 8F_3(z)h^2r^2 + 6h^8F_1(z) + 4h^6F_2(z) + 2h^4F_3(z)$$

and velocity component u is given by

$$u = -\frac{1}{r} \frac{\partial\psi_0}{\partial z} - \frac{1}{r} \frac{\partial\psi_1}{\partial z} = -2r^3h^{-5}h_1 + 2h^{-3}h_1 + 24h_1(h^5r^3 - h^7r)F_1$$

$$+ (4h^4r^3 - 3h^8r - r^9)F_1' + (12h^3r^3h_1 - 12h^5r)F_2 \\ + (3h^4r^3 - 2h^6r - r^7)F_2' + (4hh_1r^3 - 4h^3r)F_3 \\ + (2h^2r^3 - h^4r - r^5)F_3'$$

The expressions of $F_1, F_2, F_3, F_1', F_2'$ and F_3' are lengthy, so for the sake of brevity there expressions are given in Annexure.

3.1 Skin friction

Coefficient of skin friction:

$$C_f = \left(\frac{\partial w}{\partial r} \right)_{r=h} = \left[\frac{2}{h^4} (-2r) + 80F_1(z)r^7 \right. \\ \left. + 48F_2(z)r^5 + 24r^3F_3(z) \right. \\ \left. - \{16h^6F_1(z) + 12F_2(z)h^4 + 8F_3(z)\}2r \right]_{r=h}$$

4. RESULTS AND DISCUSSION

The model geometry of the problem is axisymmetric and therefore for the sake of brevity the graphs are plotted for the upper portion of the tube. In figure 2 the velocity profiles are plotted against radial distance at two locations in the tube for different values of stenosis considering other parameters as constant. We observed that at the initial position in stenosed region the up stream flow is less affected with the increase of stenosis length while at centre of the stenosis for 60% reduction of arterial cross section the change in velocity is very high and it increases proportional with the stenosis width. While at the entrance in the stenosis, the velocity profile pattern at centre line is same as that of at the middle of stenosis along the length, but near the wall of the stenosis this pattern turned reverse. From figure 3 it is observed that the axial velocity in the stenosed region increases with the increase of length of stenosis while in the proximity of the wall the velocity of the fluid decreases. Figure 4 shows that in the stenosed region the velocity profiles are affected with the Hartmann number. At the centre line the velocity decreases with the increase of Hartmann number irrespective of the location in the stenosed region, where as on moving towards tube wall a critical point is arises away from that the effect of Hartmann number turned reverse. Figure 5 demonstrate that the velocity of fluid near to the tube wall decreases with the increase of permeability parameter while on moving near the centre line the effect of stenosis is

experienced and the flow behaviour become reversed with the increase of permeability parameter. The effect of Reynolds number on the flow field is demonstrated in figure 6. The shear stress in the flow through stenosis with porous medium increases in magnitude with the increase of length of stenosis, shown in figure 7. At the surface of the stenosis there is 166% increase in shear stress with the increase of 50% in the length of stenosis, while when there is an increase of 15% in the length of the stenosis from 25% R_0 to 40% of R_0 the shear stress at the surface of the stenosis increases by 50%. The figure 8 shows the variation in skin friction along the stenosis and depict that with the increase of Hartmann number the magnitude of the skin-friction increases. There is maximum skin-friction in the proximity of the peak of the stenosis at upstream and downstream flow from the peak of the stenosis. The effects of the Reynold's number and permeability on the shear stress are shown in the figure 9 and 10 respectively. The magnitude of shear stress increases with the increase of Reynolds number. Also with the increase of permeability, the magnitude of skin friction decreases.

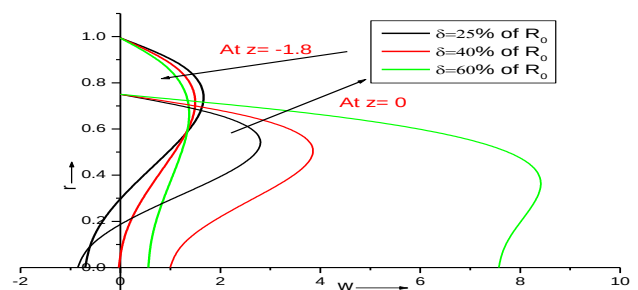


Fig. 2 Graph between flow velocity and radial distance (at $R_e = 100, M = 1, K = 0.5$)

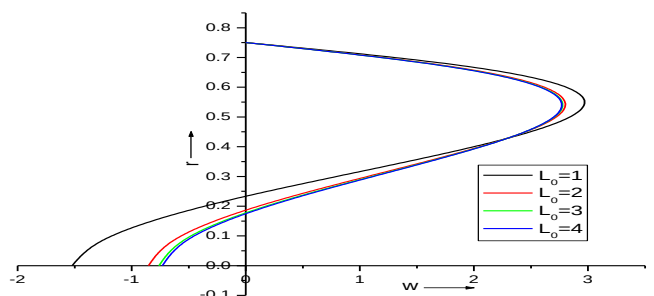


Fig. 3 Graph between flow velocity and radial distance (at $K = 0.5, M = 1, R_e = 100, \delta = 25\%$ of R_0)

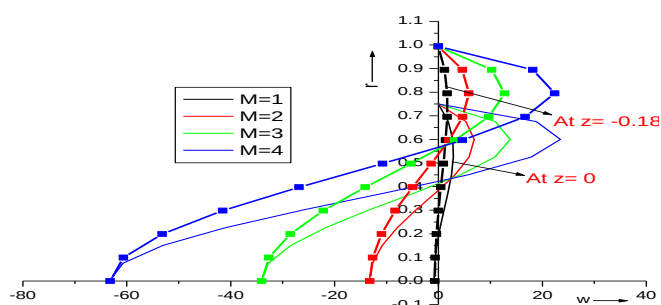


Fig. 4 Graph between flow velocity and radial distance
(at $K = 0.5$, $R_e = 100$, $\delta = 25\%$ of R_0 , $L_0 = 2$)

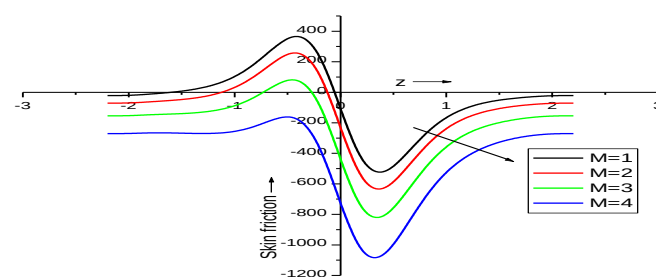


Fig. 8 Variation in skin friction on the stenosis versus axial distance
(at $K = 0.5$, $R_e = 100$, $\delta = 25\%$ of R_0)

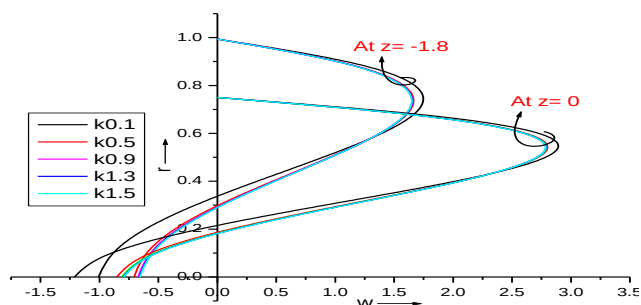


Fig. 5 Graph between flow velocity and radial distance
(at $\delta = 25\%$ of R_0 , $M = 1$, $R_e = 100$)

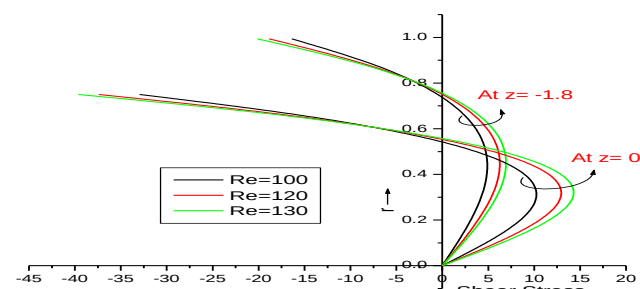


Fig. 9 Graph between shear stress and radial distance
(at $K = 0.5$, $M = 2$, $\delta = 25\%$ of R_0)

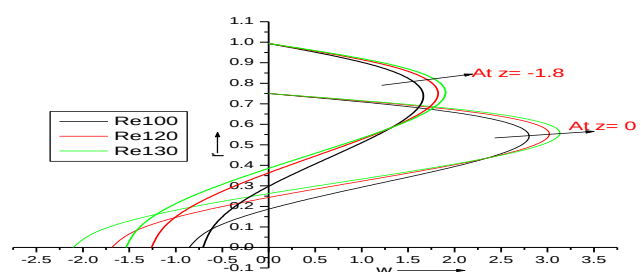


Fig. 6 Graph between flow velocity and radial distance (at $\delta = 25\%$ of R_0 , $M = 1$, $K = 0.5$)

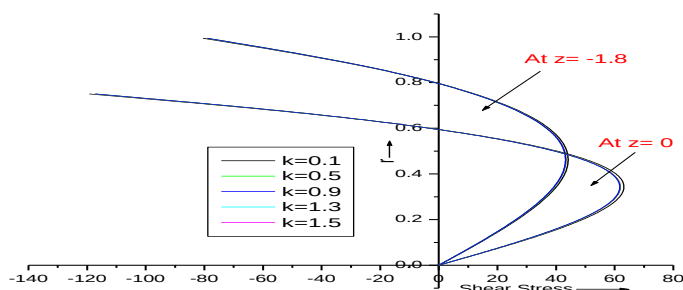


Fig. 10 Graph between stress and radial distance
(at $\delta = 25\%$ of R_0 , $M = 2$, $R_e = 100$)

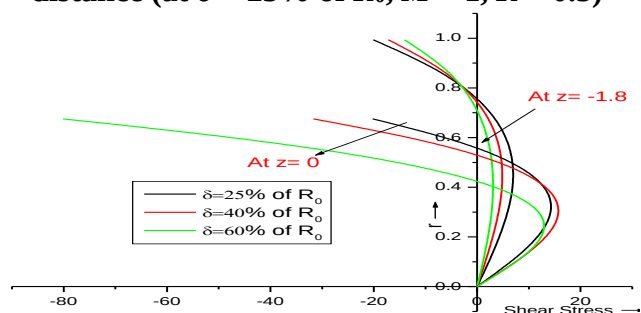


Fig. 7 Graph between shear stress and radial distance
(at $K = 0.5$, $M = 2$, $R_e = 100$)

List of Symbols

R	Radius of the normal tube
δ	Maximum height of the stenosis
w	Velocity component in the axial direction
B	Electromagnetic induction
K	Permeability of the porous medium
R_e	Reynolds number
Q	Volumetric flux
$R^*(z)$	Radius of the normal tube

(r^*, θ^*, z^*) Cylindrical polar co-ordinates

u^* Velocity component in the radial direction

M Hartmann number

L_0 Length of stenosis

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ANNEXURE

$$F_1(z) = \frac{R_e}{960} \left(\frac{1}{h^{11}} \right) (20h_1^3 - 13h_1 h_2 h + h^2 h_3)$$

$$F_2(z) = \frac{1}{48 \times 24} \left[32R_e h^{-9} h_1 - 128R_e h^{-9} h_1^3 + 88R_e h^{-8} h_1 h_2 - 8R_e h^{-7} h_3 + 420h^{-8} h_1^4 - 360h^{-7} h_1^2 h_2 + 20h^{-6} h_2 h_2 + 40h^{-6} h_1 h_3 + 10h^{-6} h_2^2 - 2h^{-5} h_4 - \frac{10}{k} h^{-6} h_1^2 + \frac{2}{k} h^{-5} h_2 \right]$$

$$F_3(z) = \frac{1}{32 \times 6} \left[-32R_e h^{-7} h_1 + 48R_e h^{-7} h_1^3 - 36h^{-6} R_e h_1 h_2 + 4R_e h^{-5} h_3 - 120h^{-6} h_1^4 + 144h^{-5} h_1^2 h_2 - 18h^{-4} h_2^2 - 24h^{-4} h_1 h_3 + 2h^{-3} h_4 + 160h^{-6} h_1^2 - 32h^{-5} h_2 - 4M^2 R_e h^{-4} - \frac{4}{k} h^{-4} + \frac{6}{k} h^{-4} h_1^2 \right]$$

$$\left[-\frac{2}{k} h^{-3} h_2 \right]$$

$$F_1' = \frac{R_e}{960} \left[-\frac{11}{h^{12}} (20h_1^3 - 13h_1 h_2 h + h^2 h_3) + \frac{1}{h^{11}} (60h_1^2 h_2 - 13h_2^2 h - 13h_1 h_3 h - 13h_1^2 h_2 + 2hh_1 h_3 + h^2 h_4) \right]$$

$$F_2' = \frac{1}{48 \times 24} \left[32R_e (-9) h^{-10} h_1^2 + 32R_e h^{-9} h_2 - 128(-9) R_e h^{-10} h_1^4 - 128R_e h^{-9} 3h_1^2 h_2 + 88(-8) R_e^{-9} h_1^2 h_2 + 88R_e h^{-8} h_2^2 + 88R_e h^{-8} h_1 h_3 - 8(-7) R_e h^{-8} h_1 h_3 - 8R_e h^{-7} h_4 + 420(-8) h^{-9} h_1^5 + 420h^{-8} 4h_1^3 - 360(-7) h^{-8} h_1^3 h_2 - 360h^{-7} 2h_1 h_2^2 - 360h^{-7} h_1^2 h_3 + 20(-6) h^{-7} h_1 h_2^2 + 20h^{-6} 2h_2 h_3 + 40(-6) h^{-7} h_1^2 h_3 + 40h^{-6} h_2 h_3 + 40h^{-6} h_1 h_4 + 10(-6) h^{-7} h_1 h_2^2 + 10h^{-6} 2h_2 h_3 - 2h^{-5} h_5 - 2(-5) h^{-6} h_1 h_{41} - \frac{10}{k} h^{-6} 2h_1 h_2 - \frac{10}{k} (-6) h^{-7} h_1^2 + \frac{2}{k} (-5) h^{-6} h_1 h_2 + \frac{2}{k} (h^{-5}) h_3 \right]$$

$$F_3' = \frac{1}{32 \times 6} \left[-32R_e h^{-7} h_2 - 32(-7) R_e h^{-8} h_1^2 + 48(-7) R_e h^{-8} h_1^4 + 48(3) R_e h^{-7} h_1^2 h_2 - 36(-6) h^{-7} R_e h_1^2 h_2 - 36h^{-6} R_e h_2^2 - 36h^{-6} R_e h_1 h_3 + 4(-5) R_e h^{-6} h_1 h_3 + 4R_e h^{-5} h_4 - 120(-6) h^{-7} h_1^5 - 120h^{-6} 4h_1^3 h_2 + 144(-5) h^{-6} h_1^3 h_2 + 144h^{-5} (2) h_1 h_2^2 \right]$$