

On Test Functions for Divergence-based Grey Wolf Optimizer

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Abstract

Nature-inspired algorithms have captured the attention of the research community in recent times. Due to the ease of implementation and the advancement in technology, these algorithms have found their niche in the field of optimization. Their applications span from the designing beam in civil engineering to the prediction of diseases in medical sciences. One such widely researched algorithm, published recently, is grey wolf optimizer (GWO); based on the behavior of the grey wolves. This grey wolf optimizer has gone through hybridization and modifications as is natural in this domain. One such recently developed variant is divergence-based grey wolf optimizer (DGWO). This paper details the working mechanism of DGWO and presents the performance based on benchmark functions. For this, 23 well-known benchmark functions implemented in python are used. Seven of the functions are unimodal and six are multimodal and ten are fixed-dimensional multimodal functions. The results for the test functions are presented by using 2D graphs. The results show that the newly developed DGWO works comparably well and is suitable for solving optimization problems.

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I. INTRODUCTION

Human beings, and for that matter all living beings, are always dependent on nature for their survival. For this survival living beings constantly interact with the nature. It can be easily observed that the living beings keep on doing activities which makes changes in the environment. And equally oppositely nature goes through its various changes, beyond the control of living beings, affecting various living beings. So, there is constant struggle wherein the living beings try to adapt to the changes in the nature so that they can survive. As per the Darwin's theory of evolution, those who can acclimatize themselves faster with the nature have a better chance of survival and those who cannot adapt to the changes in the nature are ultimately eliminated from

the population. This applies to all the living beings, from plants to small insects and organisms to animals to human beings. However, the difference is, human beings have used nature for their betterment.

History shows that human beings first used nature as a primeval source for living. They used nature as resource provider for food and shelter. As time passed, humans observed various phenomenon of the nature and started replicating those phenomenon for improving their living. For example, human observed that the friction between two hard objects, say stones or branches of tree, caused a spark and if it fell on dry grass it started fire. They replicated it for cooking and warming purposes.

As technology progressed human beings started observing nature for solving problems. Nature is understood to be a quite complex system with innumerable subsystems within it. Earlier various plants and its subsystems were studied and new species of plants were developed, some seeds were modified, colors and fragrance of flowers was modified. The experiment on seed- garden peas- was carefully studied by [1] way back in 1865 by Mendel. In last century Holland and his students extensively worked on genetic algorithms [2]-[5].

With advancement in computer technology, humans started solving many complex problems. However, it was observed that to solve some problems, for example optimization problems, traditional methods were either not giving good solutions or taking long time to reach good solutions. Again, scientist looked at nature for the possible solutions. And yet again nature came for the rescue. Researchers rigorously studied the behavior of fishes, birds and animal groups leading to a very good optimization algorithm viz. particle swarm optimization (PSO) [6]. Scientist studied ant behavior for searching food and developed the algorithm called ant colony optimization (ACO) [7], which was used for optimizing various complex problems like traveling salesman problem. Then, study of echolocation behavior of bats lead to the bat algorithm (BA) [8] and observation of intensity of light and flashing behavior of fireflies provided firefly algorithm (FF) [9]. Echolocation is used by bats to find food, avoid obstacles and to reach their homes. BA has found its applications in various domains such as solving scheduling problems, for continuous optimization and image processing. Firefly algorithm was basically developed to solve multimodal optimization problems. Artificial bee colony (ABC) algorithm based on the behavior of bees was found to be useful in solving numerical optimization problems [10]. Such nature-inspired algorithms have their special place in computational engineering to solve many real world complex problems.

This work focuses on recently invented optimization algorithm based on grey wolves and its specific variant.

The remaining paper is organized as follows: next section introduces basic GWO followed by divergence-based variant in section III, section IV introduces the renowned test functions and the experimental setup; section V presents the results for these benchmark functions followed by conclusion as section VI.

II. GWO

Seminal work by observing the behaviors of grey wolf and converting it to an optimization algorithm was presented by [11] in 2014. It has caught the attention of many researchers interested in solving optimization problems in different areas and the algorithm- called as (GWO) - has become widely famous. It has been used in large number of applications in various fields. Also, many variants of GWO have been proposed. The reason for such popularity is because of its robustness, easy implementation, less parameters and faster convergence.

Primarily the social behavior of individual grey wolves and their group hunting behavior were used for development of the algorithm.

A. Social behavior

Grey wolf leave in a pack of 5-10 individuals. The group stays together, hunts together but eats differently. The group shows a very strong command line hierarchy. There are four categories in the group i.e. alpha (α) wolf, beta (β) wolf, delta (δ) wolf and omega (ω) wolf. The leaders of the group called as alpha wolf (α) – a pair of male and female wolf- are the decision making wolves. They take decision about the selection of prey for hunting, location for rest etc. It is not necessary to have alphas to be physically strongest in the group. Beta (β) wolf are second in command and enforce the decisions of the alpha on the remaining pack. Beta are generally strong and may be the candidate for

becoming alpha in future. They respect alpha wolf but command other wolves at lower levels. Lower in the hierarchy are delta (δ) wolves, who follow all the orders of higher two, that is, alpha and beta. Their job is to protect the pack, watch boundaries, taking care of ill and wounded, and assist in hunting to

alpha and beta wolves. The lowest in the pyramid are omega wolves submitting to other three levels. Omegas are the last to be allowed to eat. They are mostly babysitters and important in terms of venting anger of the other three higher levels. This hierarchy is represented in fig. 1.

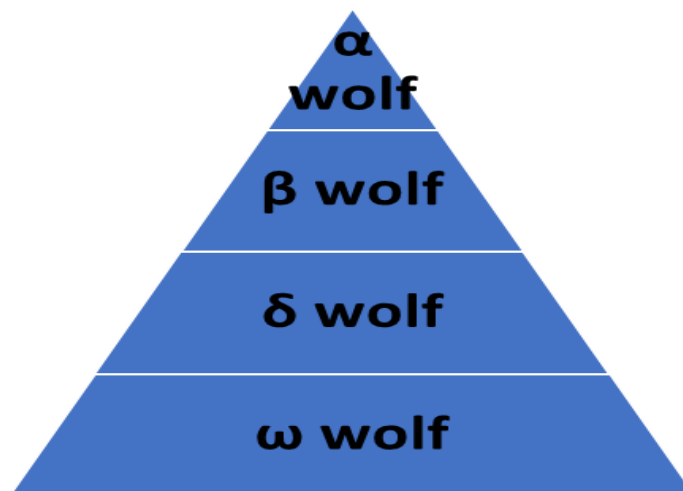


Fig. 1. Social Pyramid of Grey wolf. From top to bottom dominance decreases.

B. Hunting behavior

Grey wolves hunt in a group. In the hunting of an animal top three wolves, that is, alpha, beta and delta participate. All the commands regarding hunting are given by the alpha wolf and implemented by the beta. Grey wolf have a special

hunting behavior in which they first track the prey, chase it and approach near prey. In the second phase, they encircle the prey and harass it to the extent that it stops moving. And at last the pack of grey wolf then attacks the pray. This is well illustrated by Fig. 2 borrowed from [11].

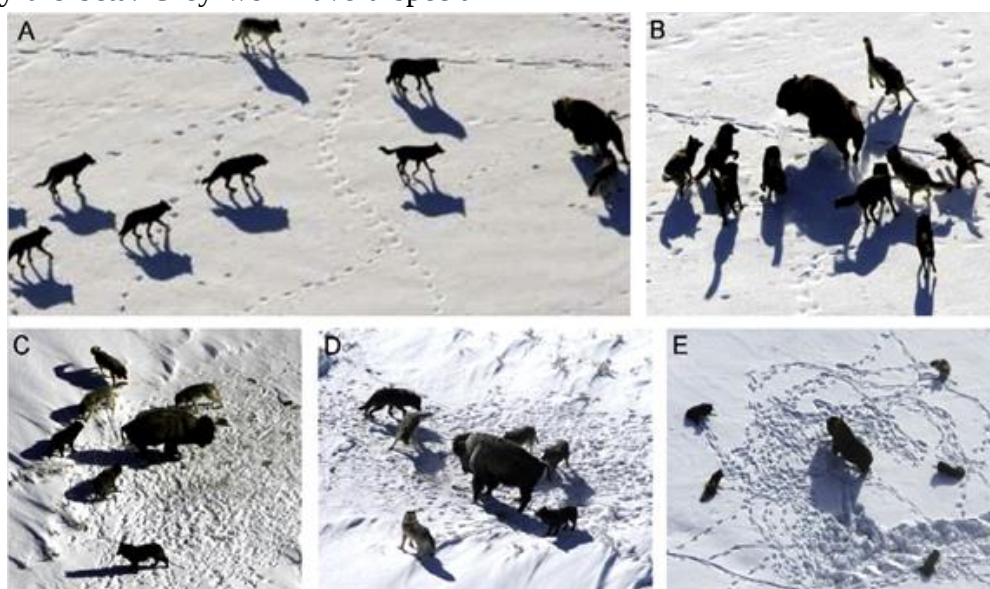


Fig. 2. Hunting behavior of grey wolves A) Tracking, chasing B) Pursuing, C) harassing and D) encircling E) stopping and attacking

C. Mathematical foundation of GWO

Based on the social behavior and hunting procedure the mathematical model for GWO was developed. Hunting includes encircling prey, positioning wolves, updating positions.

Encircling behavior is modeled by (1) and (2). Positions updating is represented by (5) – (7).

$$E = |D \bullet Y_p(t) - Y(t)| \quad (1)$$

$$Y(t+1) = Y_p(t) - B \bullet E \quad (2)$$

$$B = 2b \bullet s_1 - b \quad (3)$$

$$D = 2 \bullet s_2 \quad (4)$$

$$E_\alpha = |D \bullet Y_\alpha - Y|, E_\beta = |D \bullet Y_\beta - Y|, \quad (5)$$

$$E_\delta = |D \bullet Y_\delta - Y| \quad (5)$$

$$Y_1 = Y_\alpha - B_1 \bullet E_\alpha, \quad Y_2 = Y_\beta - B_2 \bullet E_\beta, \quad (6)$$

$$Y_3 = Y_\delta - B_3 \bullet E_\delta \quad (6)$$

$$Y(t+1) = (Y_1 + Y_2 + Y_3) / 3 \quad (7)$$

Where,

B and D are coefficient vectors,

s_1 and s_2 are random vectors with values in [0,1],

t is current iteration number,

b decreases linearly starting from 2 to 0,

Y is position of a wolf,

Y_p is position of prey,

Y_α , Y_β , Y_δ are best, better and good hunters respectively,

E_α , E_β , E_δ are respective distances of a wolf from α , β , δ wolf.

III. DIVERGENCE-BASED GREY WOLF OPTIMIZER

In Grey wolf optimizer α is considered to be best

solution, followed by β and third best solution is δ . Position update is done by averaging these three best solutions. In divergence-based grey wolf optimizer [12], along with three best solutions, three worst solutions are also considered. Accordingly (5), (6), and (7) are modified as (8), (9) and (10) respectively.

$$\begin{aligned} E_\alpha &= |D \bullet Y_\alpha - Y|, E_\beta = |D \bullet Y_\beta - Y|, \\ E_\delta &= |D \bullet Y_\delta - Y|, E_{\alpha'} = |D \bullet Y_{\alpha'} - Y|, \\ E_{\beta'} &= |D \bullet Y_{\beta'} - Y|, \\ E_{\delta'} &= |D \bullet Y_{\delta'} - Y| \end{aligned} \quad (8)$$

$$\begin{aligned} Y_1 &= Y_\alpha - B_1 \bullet E_\alpha, & Y_2 &= Y_\beta - B_2 \bullet E_\beta, \\ Y_3 &= Y_\delta - B_3 \bullet E_\delta, & Y_{1'} &= Y_{\alpha'} - B_{1'} \bullet E_{\alpha'}, \\ Y_{2'} &= Y_{\beta'} - B_{2'} \bullet E_{\beta'}, & Y_{3'} &= Y_{\delta'} - B_{3'} \bullet E_{\delta'} \end{aligned} \quad (9)$$

$$Y(t+1) = (Y_1 + Y_2 + Y_3) / 3 + (Y_{1'} + Y_{2'} + Y_{3'}) / 3 \quad (10)$$

$Y_{\alpha'}$, $Y_{\beta'}$, $Y_{\delta'}$ are three best hunters respectively,

$E_{\alpha'}$, $E_{\beta'}$, $E_{\delta'}$ are respective distances of a wolf from α' , β' , δ' wolf. α' is worst cases, β' is second worst case and δ' is third worst case.

Proposed DGWO proceeds as follows:

Initialization of the population Y_i

Initialize b, B and D

Set max = Max number of iterations

Determine fitness of every search agent

Identify Y_α , Y_β , Y_δ , $Y_{\alpha'}$, $Y_{\beta'}$, $Y_{\delta'}$

while (t < max)

for every search agent

update position of present search agent using eq. (10)

end for

return best wolf - Y_α

Update b , B , D

Determine fitness of

IV. TEST FUNCTIONS

each search agent

Identify Y_α , Y_β , Y_δ , $Y_{\alpha'}$, $Y_{\beta'}$,

Y_δ ,

$t = t+1$

end while

The proposed DGWO algorithm is tested with the well-known benchmark functions. Table 1 lists the functions used for testing the performance of DGWO. Population size considered is 1000. Each function is iterated for 50 generations.

Table 1. Test Functions

Function name	Function definition
F1	$f(x) = \sum_{i=1}^n x_i^2$
F2	$f(x) = \sum_{i=1}^n x_i + \prod_{i=1}^n x_i $
F3	$f(x) = \sum_{i=1}^n \left(\sum_{j=1}^i x_j \right)^2$
F4	$f(x) = \max_i \{ x_i , 1 \leq i \leq n \}$
F5	$f(x) = \sum_{i=1}^{n-1} \left[100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2 \right]$
F6	$f(x) = \sum_{i=1}^n (x_i + 0.5)^2$
F7	$f(x) = \sum_{i=1}^n i x_i^4 + \text{random}(0,1)$
F8	$f(x) = \sum_{i=1}^n -x_i \sin(\sqrt{ x_i })$
F9	$f(x) = \sum_{i=1}^n [x_i^2 - 10 \cos(2\pi x_i) + 10]$

$$F10 \quad f(x) = -20 \exp\left(-0.2 \sqrt{\frac{1}{n} \sum_{i=1}^n x_i^2}\right) - \exp\left(\frac{1}{n} \sum_{i=1}^n \cos(2\pi x_i)\right) + 20 + e$$

$$F11 \quad f(x) = \frac{1}{4000} \sum_{i=1}^n x_i^2 - \prod_{i=1}^n \cos\left(\frac{x_i}{\sqrt{i}}\right) + 1$$

$$F12 \quad f(x) = \frac{\pi}{4} \left\{ 10 \sin(\pi y_1) + \sum_{i=1}^{n-1} (y_i - 1)^2 [1 + 10 \sin^2(\pi y_{i+1})] + (y_n - 1)^2 \right\} + \sum_{i=1}^n u(x_i, 10, 100, 4)$$

$$\text{Where } y_i = 1 + \frac{x_i + 1}{4} u(x_i, 10, 100, 4) = \begin{cases} k(x_i - a)^m, x_i > a \\ 0, -a < x_i < a \\ k(-x_i - a)^m, x_i < -a \end{cases}$$

$$F13 \quad f(x) = 0.1 \left\{ \sin^2(3\pi x_1) + \sum_{i=1}^n (x_i - 1)^2 [1 + \sin^2(3\pi x_i + 1)] + (x_n - 1)^2 [1 + \sin^2(2\pi x_n)] \right\} + \sum_{i=1}^n u(x_i, 5, 100, 4)$$

$$F14 \quad f(x) = \left(\frac{1}{500} + \sum_{j=1}^{25} \frac{1}{j + \sum_{i=1}^2 (x_i - a_{ij})^6} \right)^{-1}$$

$$F15 \quad f(x) = \sum_{i=1}^{11} \left[a_i - \frac{x_1(b_i^2 + b_i x_2)}{b_i^2 + b_i x_3 + x_4} \right]^2$$

$$F16 \quad f(x) = 4 * x_1^2 - 2.1 * x_1^4 + \frac{1}{3} * x_1^6 + x_1 * x_2 - 4 * x_2^2 + 4 * x_2^4$$

$$F17 \quad f(x) = \left(x_2 - \frac{5.1}{4\pi^2} x_1^2 + \frac{5}{\pi} x_1 - 6 \right)^2 + 10 \left(1 - \frac{1}{8\pi} \right) \cos x_1 + 10$$

$$F18 \quad f(x) = [1 + (x_1 + x_2 + 1)^2 (19 - 14x_1 + 3x_1^2 - 14x_2 + 6x_1x_2 + 3x_2^2)] * [30 + (2x_1 - 3x_2)^2 * (18 - 32x_1 + 12x_1^2 + 48x_2 - 36x_1x_2 + 27x_2^2)]$$

$$F19 \quad f(x) = -\sum_{i=1}^4 c_i \exp\left(-\sum_{j=1}^3 a_{ij} (x_j - p_{ij})^2\right)$$

$$F20 \quad f(x) = -\sum_{i=1}^4 c_i \exp\left(-\sum_{j=1}^6 a_{ij} (x_j - p_{ij})^2\right)$$

$$F21 \quad f(x) = -\sum_{i=1}^5 \left[(X - a_i)(X - a_i)^T + c_i \right]^{-1}$$

$$F22 \quad f(x) = -\sum_{i=1}^7 \left[(X - a_i)(X - a_i)^T + c_i \right]^{-1}$$

$$F23 \quad f(x) = -\sum_{i=1}^{10} \left[(X - a_i)(X - a_i)^T + c_i \right]^{-1}$$

All these 23 functions are tested for minimization. Functions F1 to F7 are unimodal functions and expected minimum value for each function is 0. Functions F8 to F13 are multimodal functions and have minimum value as 0 except for F8 for which the minimum is $-418.9828 \cdot d$, where d is dimension. For all these functions F1 to F13, considered dimension is 30. Functions F14 to F23 are fixed-dimension multimodal functions; with dimension as 2, 4, 2, 2, 2, 3, 6, 4, 4, and 4 respectively. The minimum values for these F14 to F23 functions are 1, 0.00030, -1.0316, 0.398, 3, -3.86, -3.32, -10.1532, -10.4028, and -10.5363 respectively. All the 23 functions are evaluated in their standard ranges.

V. RESULTS

Table 2 presents the results for benchmark functions obtained. Each function is evaluated six times. The best of results is taken and used for comparison here. Averaged results for GWO and PSO are borrowed from [11]. All the functions are tested for minimization mode of optimization.

Table 2. Results for test functions

Function	DGWO	GWO[11]	PSO[11]
F1	9.83E-5	6.59E-28	0.000136
F2	0.004806	7.18E-17	0.42144
F3	17.85152	3.29E-06	70.12562
F4	0.09199	5.61E-07	1.086481
F5	27.23783	26.81258	96.71832
F6	0.169444	0.816579	0.000102
F7	0.001091	0.002213	0.122854
F8	-8127.52	-6123.1	-4841.29
F9	10.65953	0.310521	46.70423

F10	0.004251	1.06E-13	0.276015
F11	0.029536	0.004485	0.009215
F12	0.040636	0.053438	0.006917
F13	0.257672	0.654464	0.006675
F14	0.998004	4.042493	3.627168
F15	0.00031	0.000337	0.000577
F16	-1.03163	-1.03163	-1.03163
F17	0.397905	0.397889	0.397887
F18	3.000001	3.000028	3
F19	-3.86268	-3.86263	-3.86278
F20	-3.32062	-3.28654	-3.26634
F21	-9.98366	-10.1514	-6.8651
F22	-9.86287	-10.4015	-8.45653
F23	-10.3863	-10.5343	-9.95291

From the table it can be observed that the proposed algorithm performs quite well against the benchmark functions. For unimodal functions F1, F2, F3, F4, and F5 results of proposed DGWO are better than PSO. DGWO gives better results for unimodal functions F6 than GWO. For function F1, DGWO performs better than GWO and PSO.

For multimodal function F8, DGWO performs better than both GWO and PSO. DGWO gives better results for F9 and F10 than PSO. It also does better than GWO for functions F12 and F13.

For fixed-dimensional multimodal functions F16 to F19, DGWO, GWO and PSO give almost equivalent results and very close to standard minimum values of the functions. DGWO performs better than GWO and PSO for functions F14, F15 and F20. It also performs better than PSO against functions F21, F22

and F23 and the results are very close to standard values.

As it is the nature of optimization algorithms, the algorithm initially starts with solutions which are quite farther than being optimal and through the successive generations it starts converging to optimal values. Convergence graphs are generally used to represent the speed of convergence of the algorithm. Following figures from Fig. 3 to Fig. 25 represent the convergence of functions F1 to F23 respectively for DGWO algorithm. The convergence graphs (CG) represent how the solution keeps on improving from first generation to fiftieth generation.

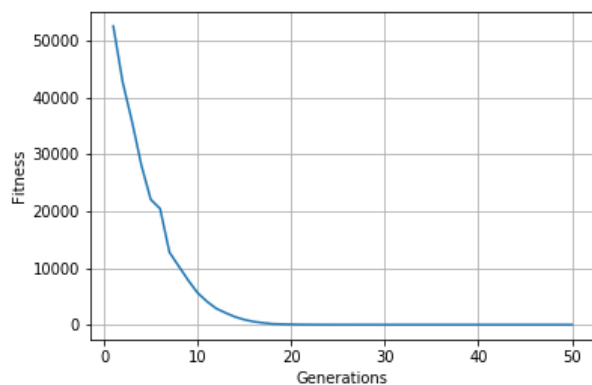


Fig. 3 CG of DGWO for F1

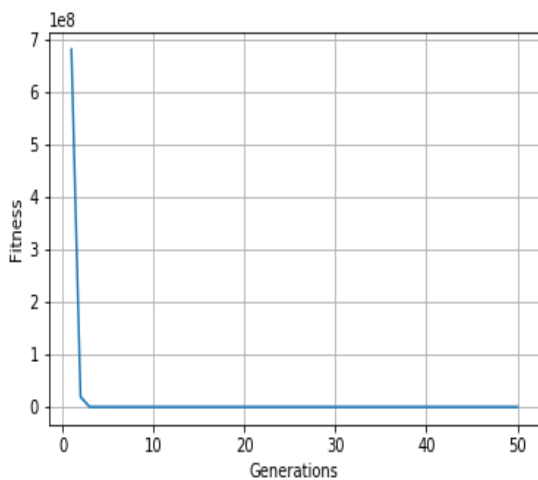


Fig. 4 CG of DGWO for F2

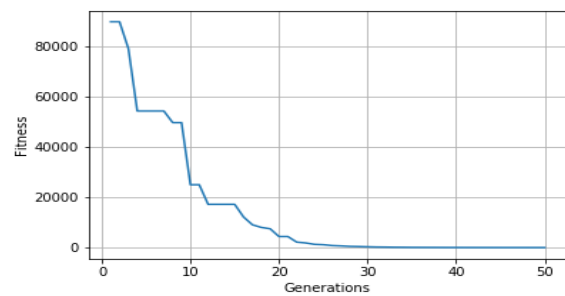


Fig. 5 CG of DGWO for F3

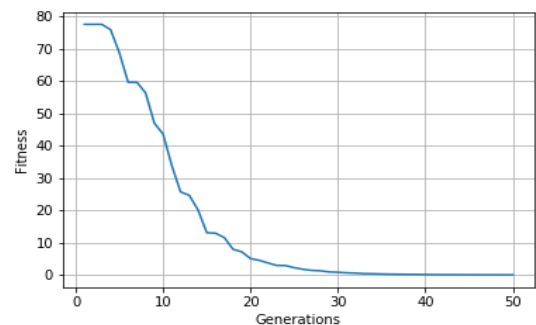


Fig. 6 CG of DGWO for F4

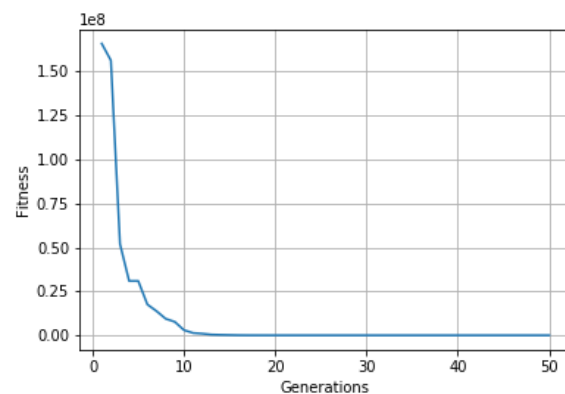


Fig. 7 CG of DGWO for F5

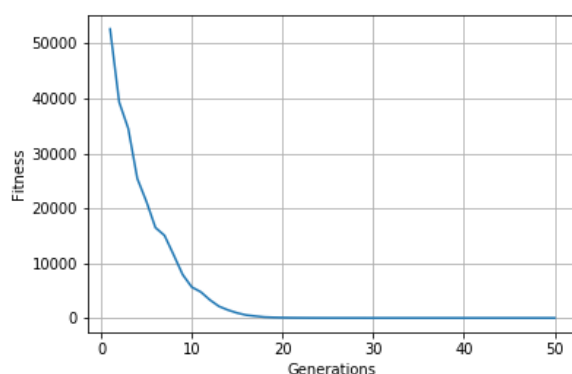


Fig. 8 CG of DGWO for F6

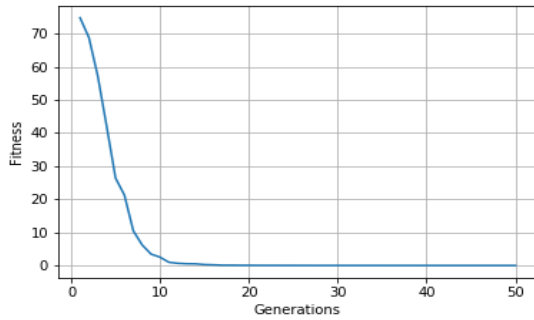


Fig. 9 CG of DGWO for F7

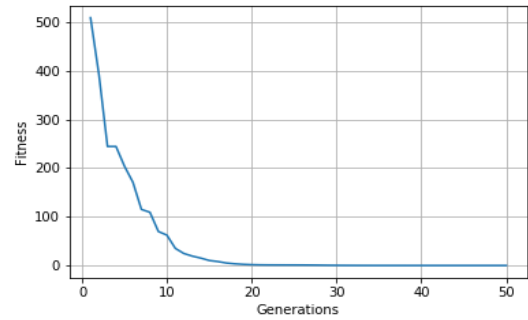


Fig. 13 CG of DGWO for F11

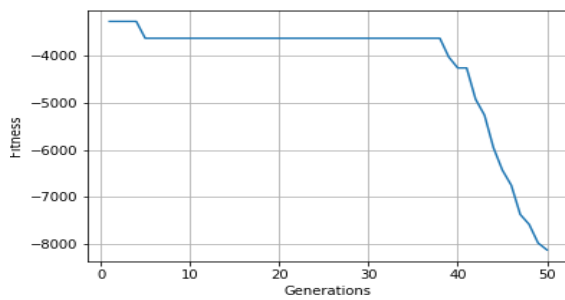


Fig. 10 CG of DGWO for F8

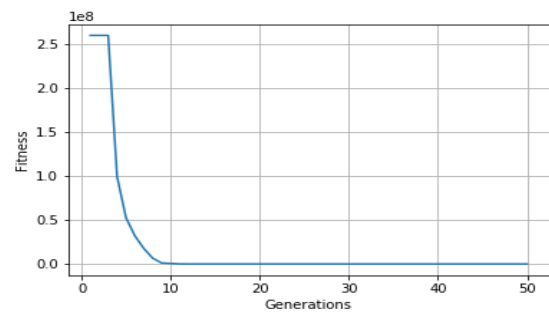


Fig. 14 CG of DGWO for F12

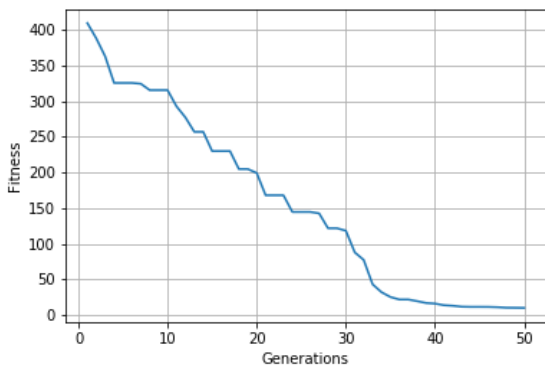


Fig. 11 CG of DGWO for F9

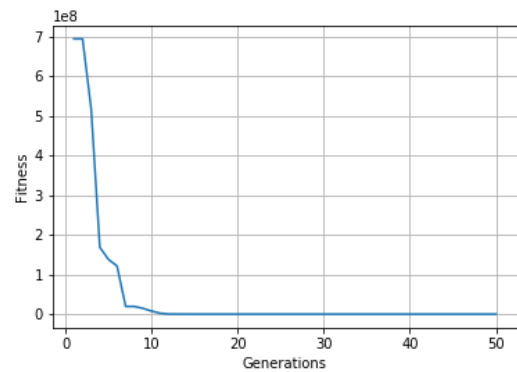


Fig. 15 CG of DGWO for F13

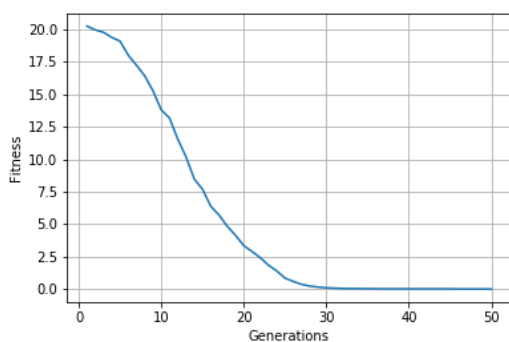


Fig. 12 CG of DGWO for F10

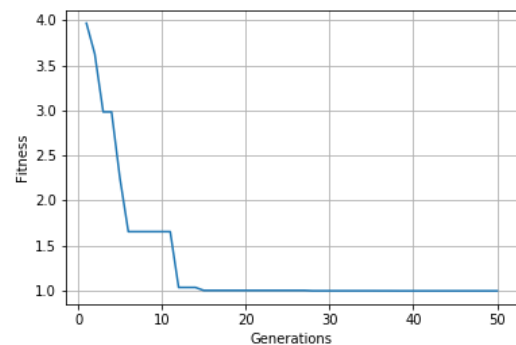


Fig. 16 CG of DGWO for F14

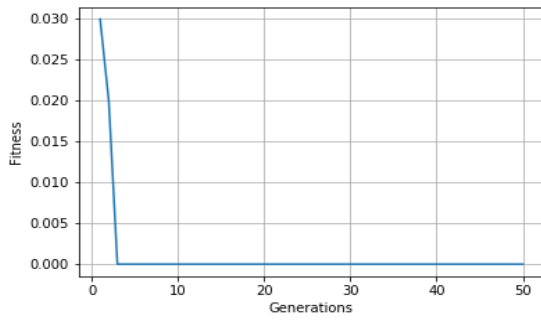


Fig. 17 CG of DGWO for F15

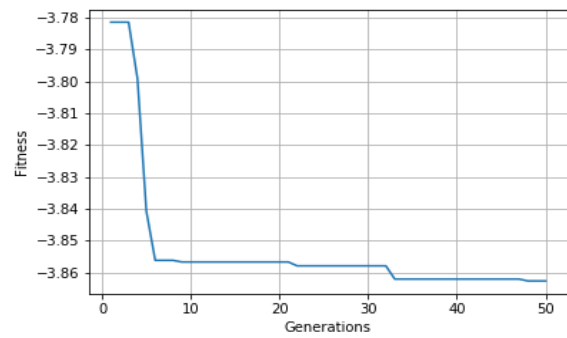


Fig. 21 CG of DGWO for F19

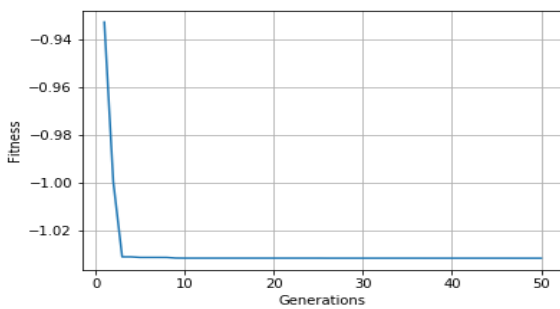


Fig. 18 CG of DGWO for F16

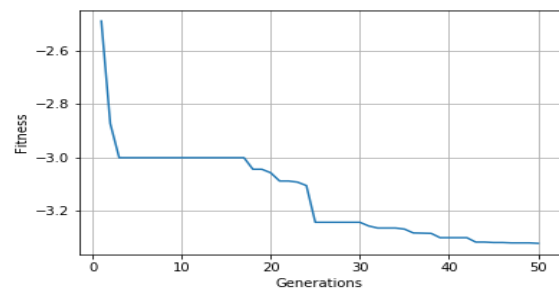


Fig. 22 CG of DGWO for F20

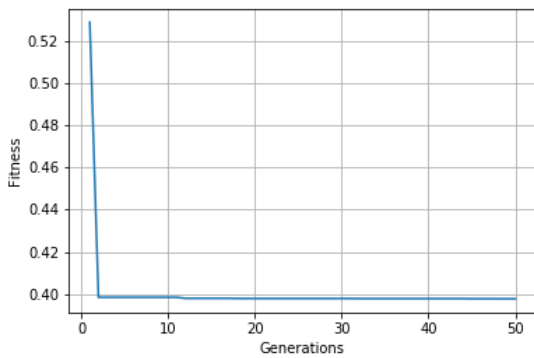


Fig. 19 CG of DGWO for F17

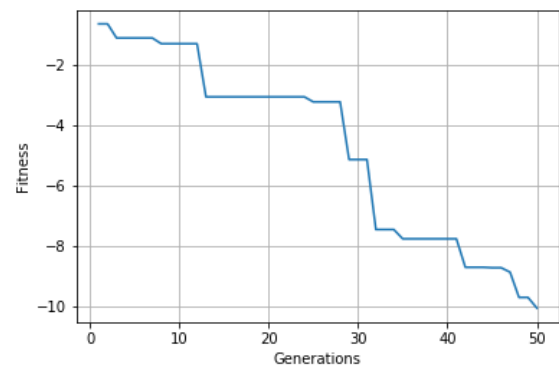


Fig. 23 CG of DGWO for F21

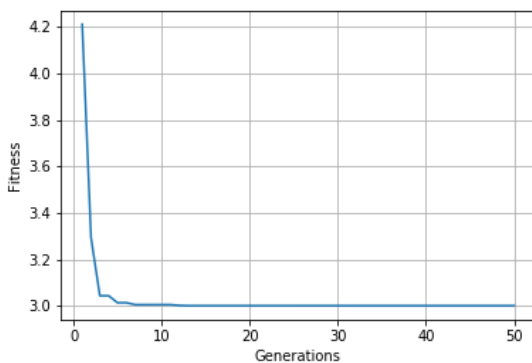


Fig. 20 CG of DGWO for F18

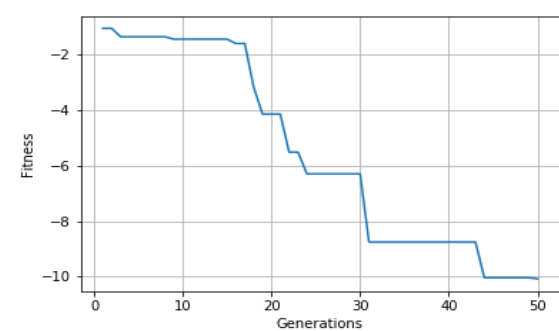


Fig. 24 CG of DGWO for F22

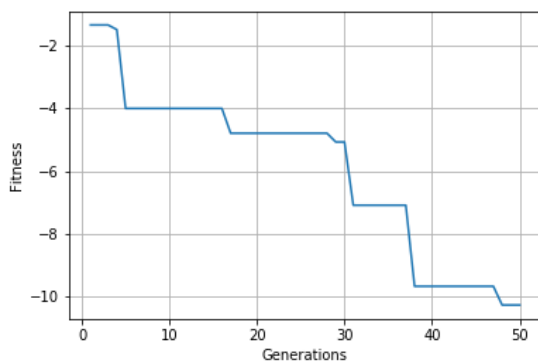


Fig. 25 CG of DGWO for F23

VI. CONCLUSIONS

The paper presented the mathematical formulation of GWO and DGWO followed by the definitions of standard benchmark functions. Total 23 test functions consisting unimodal and multimodal functions, which are specifically used for testing global minimum optimization algorithms are used. The results of the test functions for DGWO are compared with GWO and PSO. It was observed that performance of DGWO was better in many cases than the GWO and PSO. Even for fixed-dimension multimodal functions DGWO gives performance better than PSO and GWO and very close to the standard values of the functions. In future, more variations of DGWO will be developed and tested against these functions as well as will be applied to solve some real-world problems like classification of diseases in health applications.

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