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Robust Controller Design Satisfying Iso-Damping Characteristics

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Abstract

In this paper, we propose a robust controller design that satisfies the characteristics of phase margin and Iso-damping braking using a reduction model with secondary delay time.

For higher-order models, it is not easy to find a specific frequency that satisfies the so-damping and fault margins. As the model reduction method, the reduction method using a Nyquist plot shows excellent characteristics in the frequency domain and time domain. In this paper, the reduction model with the second delay time considering the improved steady-state is applied.

The PID controller design method that satisfies the characteristics of phase margin and iso-damping is as follows. First, the parameter value of the Ti controller that satisfies the characteristics of iso-damping is determined by using the reduction model with the second delay time. Second, the Kc and Td parameter values are obtained by considering only the phase margin in the control transfer function that satisfies the phase margin and gain margin. The PID controller parameters thus obtained have the characteristics of robust control by satisfying the characteristics of phase margin and iso-damping. Simulation results show that the proposed method is excellent for lower and higher models.

In this paper, we designed a PID controller that satisfies the phase margin and iso-damping using a reduced model. In order to satisfy the above characteristics, improvement of the model reduction algorithm is required. Extending this method will improve the performance of the classic PID controller.

Keywords: *Robust control, PID, iso - damping, phase-margin, gain-margin, Model reduction*

1. Introduction

Despite the recent rapid development of control theory, PID (Proportional and Integral and Derivative) controllers, which are simple in structure and excellent in control performance and relatively easy in parameter adjustment, have been widely used. Although various PID controller design methods have been studied [1-6], Ziegler-Nichols' critical vibration method is still widely used. The critical vibration method can calculate a control parameter value with a simple formula based on the measurement of the critical gain and the critical frequency of the process. In 1984, Astrom and Hagglund proposed an automatic

TEST Engineering & Management

March - April 2020
ISSN: 0193-4120 Page No. 4241 - 4248

tuning method for obtaining critical gain and critical frequency in a simple relay experiment. [7-11] At this time, the description function is used to obtain critical gain and critical frequency of the system. do. The information obtained from the relay tuning experiment can also be used to design a PID controller that satisfies the relative stability of gain and phase. The PID controller designed through the relay tuning experiment cannot achieve satisfactory control performance under external disturbance and sensor change. Therefore, a lot of research is being conducted on the controller insensitive to the effects of noise and the like. The control method that guarantees the robustness of the system is called robust control, and one of them is the method using the characteristics of the iso-damping [12,13].

This method ensures that the phase Bode plot at certain frequencies w_gof the loop transfer function maintains the phase margin that meets the performance specifications in any interval so that the response of the phase Bode is insensitive to changes in gain. As a prerequisite for the design method using the characteristics of iso-damping, the differential values for magnitude and phase are necessary at the specific frequency w_g of the frequency transfer function of the control process. For this reason, it is easy to apply to lower order systems, but there is a problem that is difficult to apply to higher-order systems. In this paper, we propose a generalized design method of a robust controller that simultaneously satisfies the characteristics of phase margin and equal braking at a specific frequency w_g using a reduction model with a second-order delay.

Simulation results show that a robust controller can be designed that satisfies the characteristics of phase margin and iso-damping for higher-order models. This paper is composed of PID controller design, simulation, consideration and conclusion which satisfy the iso-damping such as improved model reduction algorithm, phase margin, etc.

2. Robust Controller Design

This chapter describes the algorithm to obtain a reduced model with second-order with delay time using the Nyquist plot in the frequency domain and the design of the PID controller that satisfies the iso-damping such as phase margin as shown in Figure 1.

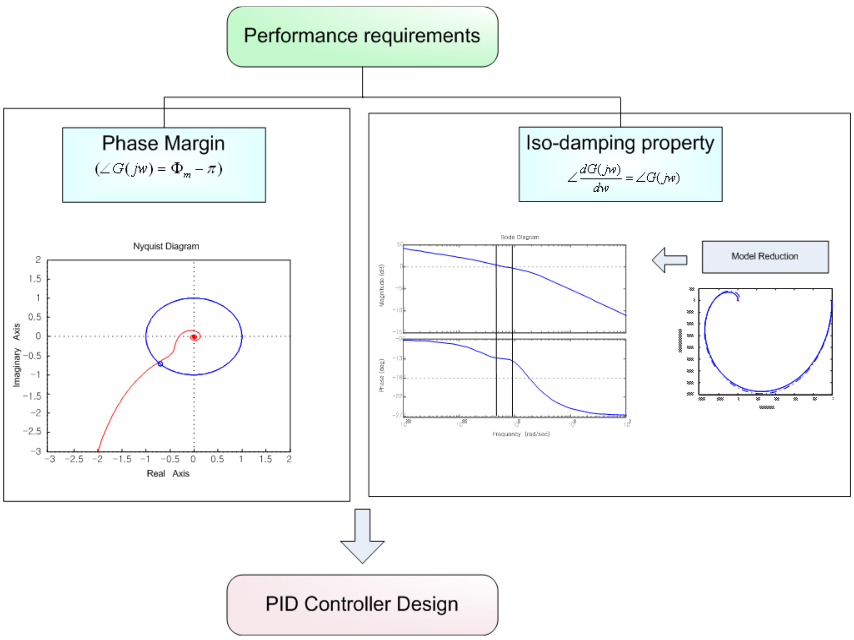


Figure 1. PID controller design satisfying the characteristics of phase margin and iso-damping

March - April 2020
ISSN: 0193-4120 Page No. 4241 - 4248

2.1. Model Reduction Algorithm

In the case of higher-order models, it is difficult to find the derivative values for phase and magnitude at a specific frequency w_g of the loop transfer function. Therefore, after the reduction model with second-order plus delay time is obtained for all models[14], this model can be used to design a controller with generalized characteristics of iso-damping.

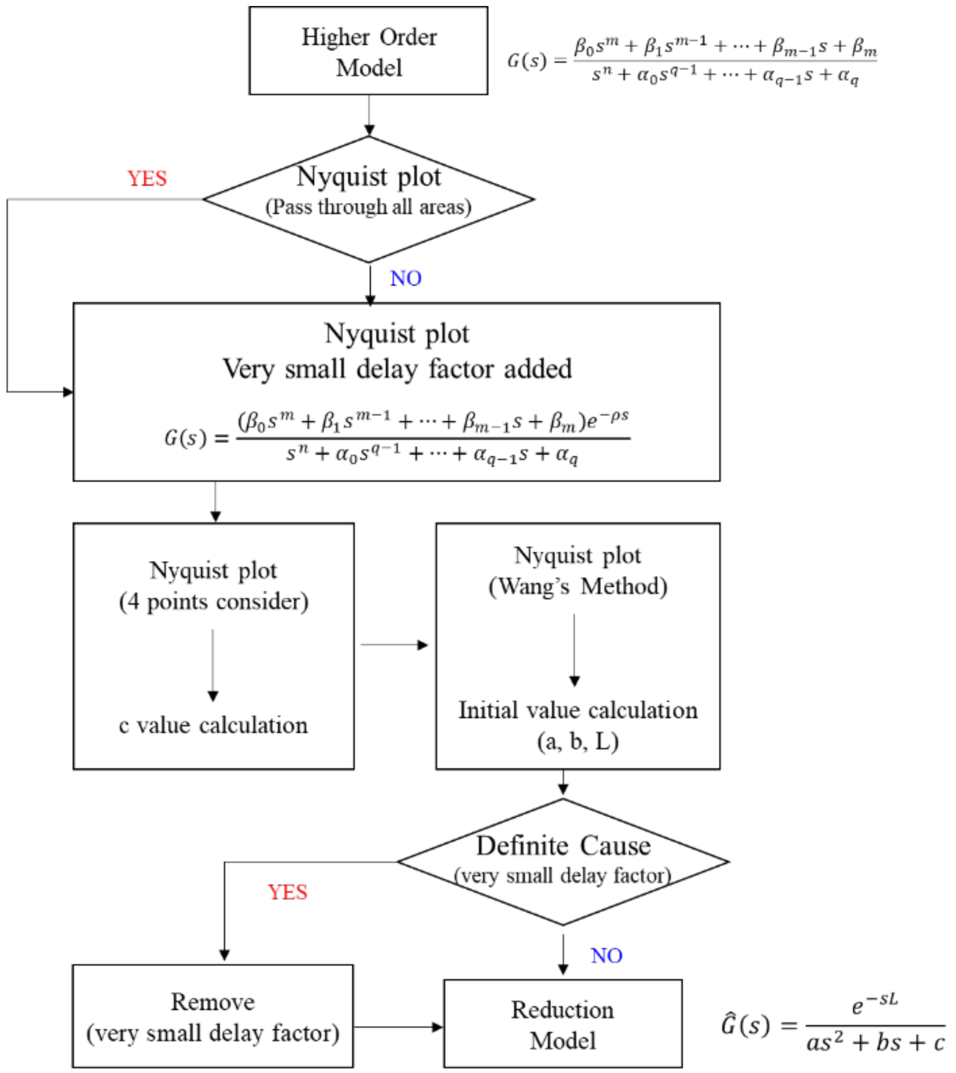


Figure 2. Flowchart of the improved reduction model algorithm

2.2. PID tuning algorithm

The PID controller tuning algorithm that simultaneously satisfies the specific phase margin and iso-damping using the reduction model with the second delay time is as follows.

The transfer function of the control process is called $G_p(s)$, the transfer function of the controller $G_c(s)$, and the specific phase margin(Φ_m) is expressed by equations (1) to (3) in terms of iso-damping characteristics.

$$|G_c(jw_g)_p G_p(jw_g)| = 1 \qquad (1)$$

$$\Phi_m = \arg\,[\,G_c(jw_g)_p G_p(jw_g)] + \pi \qquad (2)$$

$$\angle \frac{dG(s)}{ds}|_{s=jw_g} = \angle G(s)_{s=jw_g} \qquad (3)$$

Here, w_g means the frequency that meets the point where the amplitude of the Nyquist curve is 1, that is, the gain crossover frequency.

TEST Engineering & Management

March - April 2020
ISSN: 0193-4120 Page No. 4241 - 4248

The transfer function $G_c(s)$ of the PID controller is represented by equation (4), and the control process transfer function having the second delay time is expressed by equation (5).

$$G_c(s) = k_p(1 + \frac{1}{T_i s} + T_d s) = \frac{k_d s^2 + k_p s + k_i}{s} \quad (4)$$

$$G_p(s) = \frac{e^{-sL}}{as^2+bs+c} \quad (5)$$

Here, $k_i = \frac{k_p}{T_i}$, $k_d = k_p T_d$

The frequency transfer function $G_c(jw)$ of Eq. (4) is shown in Eq. (6), and the magnitude and angle values are shown in Eqs. (7) and (8).

$$G_c(jw) = k_p(1 + \frac{1}{jwT_i} + jwT_d s) \quad (6)$$

$$|G_c(jw)| = |k_p|\sqrt{1 + (\frac{w^2 T_i T_d - 1}{wT_i})^2} \quad (7)$$

$$\angle G_c(jw) = tan^{-1}(\frac{w^2 T_i T_d - 1}{wT_i}) \quad (8)$$

The derivative function $\frac{dG(jw)}{dw}$ for w of the open-loop frequency transfer function $G(jw)$ is given by Eq. (9).

$$\frac{dG(jw)}{dw} = G_p(jw)\frac{dG_c(jw)}{dw} + G_c(jw)\frac{dG_p(jw)}{dw} \quad (9)$$

The frequency transfer function $G_p(jw)$ of the control process is expressed as the size and angle as follows.

$$G_p(jw) = \left|G_p(jw)\right| \angle G_p(jw) \quad (10)$$

Taking the natural logarithm on both sides of Eq. (10) and differentiating with respect to w is equal to Eq. (11)

$$\frac{dp(w)}{dw} = G(jw)(Fa1 + jFb1) \quad (11)$$

Here, $Fa1 = \frac{d\ln|G_p(jw)|}{dw}$, $Fb1 = \frac{d\angle G_p(jw)}{dw}$

Differentiating Eq. (15) with respect to w gives Eq. (12).

$$\frac{dG_c(jw)}{dw} = jk_p\left(T_d + \frac{1}{w^2 T_i}\right) \quad (12)$$

Substituting Eq. (11) and Eq. (12) into Eq. (9) is the same as Eq. (13).

$$\frac{dG(jw)}{dw} = k_p G_p(jw)\{w^2 T_i F_a(w) - (w^2 T_i T_d - 1)\ wF_b(w)\} + j\{w^2 T_i F_a(w) - (w^2 T_i T_d - 1)\ wF_a(w) + w^2 T_i T_d + 1\} \quad (13)$$

Considering only the angle in equation (12) is the same as equation (14).

$$\angle \frac{dG(jw)}{dw} = \angle G_p(jw) + \tan^{-1}\left(\frac{wT_i Fb(w) + (w^2 T_i T_d - 1)Fa(w) + w^2 T_i T_d + 1}{Fa(w)wT_i - (w^2 T_i T_d - 1)Fb(w)}\right) \quad (14)$$

Here, $Fa(w) = wFa1(w),\ Fb(w) = wFb1(w)$

Equation (15) taking into account the characteristics of iso-damping is as follows.

$$\left(\frac{w^2 T_i T_d - 1}{wT_i}\right) = \left(\frac{wT_i Fb(w) + (w^2 T_i T_d - 1)Fa(w) + w^2 T_i T_d + 1}{Fa(w)wT_i - (w^2 T_i T_d - 1)Fb(w)}\right) \quad (15)$$

The phase margin conditional expression of Equation (2) can be expressed as Equation (16).

$$\angle G(jw) = \angle G_c(jw) + \angle G_p(jw) = \Phi_m - \pi \quad (16)$$

Equation (17) can be obtained through equations (8) and (16).

$$\left(\frac{w^2 T_i T_d - 1}{wT_i}\right) = \tan\left(\Phi_m - \angle G_p(jw)\right) \quad (17)$$

Summarizing using equations (17) and (15), the control parameter T_i value is equal to equation (18).

$$T_i = \frac{-2}{w\{Fa(w) + \tan\left(\Phi_m - \angle G_p(jw)\right) + \tan^2(\Phi_m - \angle G_p(jw))Fa(w)\}} \quad (18)$$

Using the second model of delay reduction described in the previous chapter, Fa (w) is

TEST Engineering & Management

March - April 2020
ISSN: 0193-4120 Page No. 4241 - 4248

obtained as shown in Eq. (19).

$$Fa(w) = w\,\frac{d\ln|G_p(jw)|}{dw} = -w\left\{\frac{2a^2w^3-(2ac-b^2)w}{c^2-(2ac-b^2)w^2+a^2w^4}\right\} \quad (19)$$

From the equations (4) and (5), the open-loop transfer function $G_c(s)\,G_p(s)$ is obtained as follows.

$$G_p(s)G_c(s) = \frac{e^{-sL}(k_d s^2+k_p s+k_i)}{(as^2+bs+c)s} = k_p s\,\frac{e^{-sL}}{(as^2+bs+c)s} + (k_d s^2 + k_i)\frac{e^{-sL}}{(as^2+bs+c)s} \quad (20)$$

The open-loop transfer function molecular formula exponential portion (e^{-sL}) of equation (20) is applied to Euler's formula and expressed as frequency transfer function G (jw), as shown in equation (21).

$$G(jw) = k_p rx + zry + jk_p ix + jziy \quad (21)$$

Here, $z = k_i - k_d w^2$,

$$rx = \frac{w\cos(wL)(cw-aw^3)-bw^3\sin(wL)}{b^2w^4+(cw-aw^3)^2},$$

$$ry = \frac{(-b\,w^2\cos(wL)-(cw-aw^3)\sin(wL))}{b^2w^4+(cw-aw^3)^2},$$

$$ix = \frac{(-w\sin(wL)(cw-aw^3)-bw^3\cos(wL))}{b^2w^4+(cw-aw^3)^2},$$

$$iy = \frac{(b\,w^2\sin(wL)-(cw-aw^3)\cos(wL))}{b^2w^4+(cw-aw^3)^2}$$

Looking at the properties of equation (31), First, the shape part is the sum of the product of the control parameter k_p, z and the function rx, ry, and the imaginary part is the sum of the product of the control parameter k_p, z, and the function ix, iy. Second, the components of the function (rx, ry, ix, iy) are composed of the parameter values (a, b, c, L) of the reduction model and the angular frequency w. In this paper, the following two steps algorithm is applied to design a PID controller that satisfies the characteristics of phase margin and iso-damping.

Step 1) Determine the values of rx, ry, ix, iy.
To determine the value of the function, we need to determine the parameter values (a, b, c 의, L) and w of the reduced model. The parameter value of the reduced model can be obtained through equations (9) and (10), and the w value can be obtained through iterative calculation. This value can be used to determine the rx, ry, ix, iy values.

Step 2) Determination of PID control parameter values that satisfy the characteristics of phase margin and iso-damping. Satisfying the phase margin in equation (21) is the same as equations (22) and (23).

$$k_p\,rx + \frac{k_p}{T_i}\,ry - k_p T_d w^2 ry = -\cos(Pm) \quad (22)$$

$$k_p\,ix + \frac{k_p}{T_i}\,iy - k_p T_d w^2 iy = -\sin(Pm) \quad (23)$$

According to equations (18), (22) and (23), the parameter values of the PID controller satisfying the characteristics of phase margin and iso-damping are as follows.

$$k_p = \frac{\sin(\Phi_m)ry-\cos(\Phi_m)iy}{rxiy-ixry} \quad (24)$$

$$T_i = \frac{-2}{w\{Fa(w)+\tan(\Phi_m-\angle G_p(jw))+\tan^2(\Phi_m-\angle G_p(jw))Fa(w)\}} \quad (18)$$

$$T_d = \frac{T_i k_p rx+k_p ry+\cos(\Phi_m)T_i}{k_p T_i w^2 ry} \quad (25)$$

3. Simulation and Consideration

In this chapter, we can obtain the reduced model by the proposed method for the higher-order models and design the PID controller that satisfies the characteristics of phase margin and iso-damping using the reduced model. We prove this through simulation.

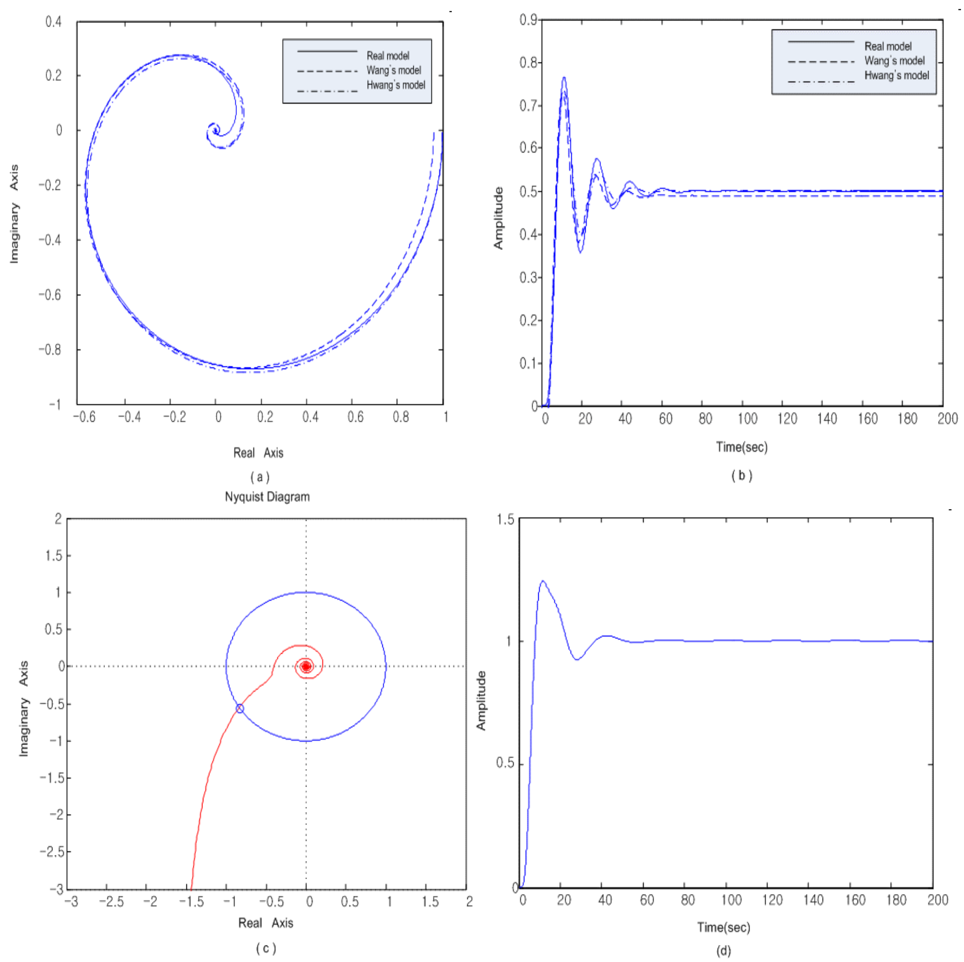
As the simulation example, an eighth-order model with no delay time and a third-order model including delay time were selected. The controller parameter value that satisfies can be obtained by equations (18), (24) and (25). The parameter values of the reduced model and PID controller are shown in Table 1

Table 1: Simulation model and control parameters

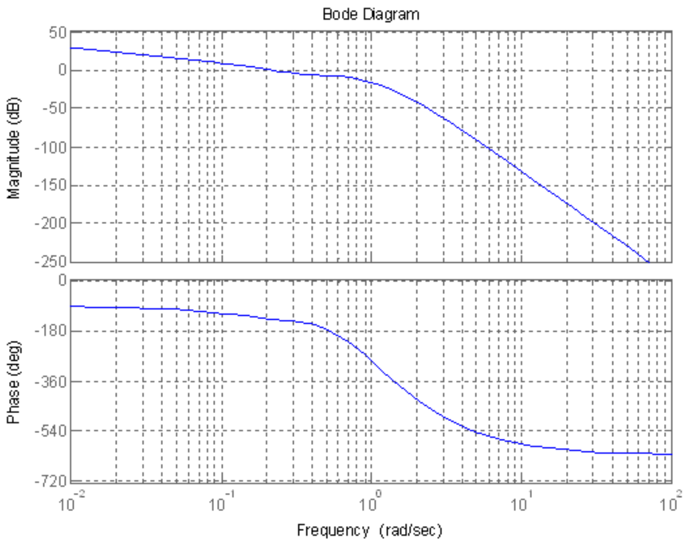
	Transfer function	Reduction Model	PID Controller	Performance specification
1	$\frac{1}{(s+1)^8}$	$\frac{e^{-3.6006s}}{8.4030s^2+4.8632s+1}$	$0.84492\left(1+\frac{1}{2.9319s}+2.9645s\right)$	Phase margin (34.5^{o}), Iso-damping

Figures (a) and (b) in Figure 3 show the frequency response and time response for the higher-order and reduced models, respectively.

Figure 3 (c) satisfies the phase margin as the Nyquist diagram response after the controller design, and Figure 3 (d) shows the unit feedback response after the controller design. Figure 3 (e) shows the bode diagram response and it can be seen that it satisfies the performance specification (phase margin, iso-damping).



March - April 2020
ISSN: 0193-4120 Page No. 4241 - 4248



(e)

Figure 3. Response characteristic for $G(s) = 1/(s+1)^8$

4. Conclusion

In this paper, we propose a robust controller design that satisfies the characteristics of phase margin and back braking using a reduction model with secondary delay time. In the lower order model, it is easy to find a specific frequency that satisfies the characteristics of iso-damping, but in the higher-order model, it is very complicated, so the reduced model is used. The reduced model algorithm used in the paper is an improved method. Through the reduction model obtained, Ti controller parameter values satisfying the characteristics of iso-damping were determined, and Kc and Td parameter values were calculated to satisfy the phase margin. It can be confirmed through simulation.

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References

[1] K.J.Astrom and T.Hagglund. Automatic tuning of simple regulators with specifications on phase and amplitude margins. Automatica. 1984 Jan;20(5): 645-651.

[2] W.K.Ho, C.C.Hang, W.Wojsznis, and Q.H.Tao. Frequency domain approach to self-tuning PID control. Contr.Eng. Practice.1996; 4(6):807-813

[3] W.K.Ho, O.P.Gan, E.B.Tay, and E.L.Ang. Performance and gain and phase margins of well-known PID tuning formulas. IEEE Trans. Contr. Syst. Technol.1996; 4: 473-477

[4] M.Zhuang and D.P.Atherton. Automatic tuning of optimum PID controllers. Proc. Inst. Elect. Eng.,1993 May; 140(3) 3: 216-224.

[5] K.J.Astrom: Automatic tuning of PID regulators: Instrument Soc. Amer; 1998. p. 49.

[6] K.Y.Kong, S.C.Goh, C.Y.Ng, H.K.Loo, K.L.Ng, W.L. Cheong, et al. Feasibility report on frequency domain adaptive controller, Dept. Elect. Eng., Nat. Univ. Singapore, Internal Rep.; 1995.

[7] Q.G.Wang, T.H.Lee, H.W.Fung, Q.Bi and Y. Zhang, PID tuning for improved performance. IEEE Trans. Contro. Syst. Technol. 1999 July; 7(4): 457-465.

[8] Y.Shamash. Model reduction using the Routh stability criterion and the Pade approximation technique. Int. J. Control, 1975; 21(3): 475-484.

[9] David E. Goldberg.Genetic Algorithms in Search, Optimization, and Machine Learning. AAddison - Wesley Publishing Company, Inc, 1989. p. 250.

March - April 2020
ISSN: 0193-4120 Page No. 4241 - 4248

[10] W.K.Ho, C.C.Hang, and L.S.Cao. Tuning of PID controllers based on gain and phase margin specifications. Automatica. 1995;31(3); 497-502.
[11] W.K.Ho, T.H.Lee, H.P.Han, and Y.Hong. Self - Tuning IMC-PID Control with Interval Gain and Phase Margins Assignment. IEEE Transactions on Control Systems Technology. 2001 May; 9(3):535-541
[12] A. Karimi, D. Garcia, and R.Longchamp, "PID controller design using Bode's integrals", IEEE Transactions on Control Systems Technology, 2003 Nov;11(6) 812 – 821. DOI : 10.1109 / TCST.2003.815541
[13] YangQuan Chen, Kevin L. Moore. " Relay Feedback Tuning of Robust PID Controllers With Iso-Damping Property." IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics), 2005 Feb, 35(1), 23-31. DOI: 10.1109/TSMCB.2004.837950
[14] Qing-Guo Wang, Chang-Chieh Hang, and Qiang Bi. "A Technique for Frequency Response Identification from Relay Feedback". IEEE Transactions on Control Systems Technology. 1999 Jan ;7(1): 122-128. DOI: 10.1109/87.736766