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A New Estimator of Value at Risk under Peaks over Threshold Framework using Optimal Loss Function

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Abstract

Value at Risk (VaR) is one of the most popular measures of risk associated with financial instruments. The generalized Pareto distribution (GPD) has been widely used to fit observations exceeding the tail threshold in the peaks over threshold (POT) framework. In this paper we propose a new estimator of GPD parameters and hence VaR& Expected Shortfall (ES) under POT framework. The procedure minimizes the differences between the empirical distribution function and the theoretical distribution function of GPD using an optimal loss function. A simulation study is carried out in presence of outliers to compare the performance of proposed estimator of VaR and ES with some of the existing estimators with respect to bias and mean square error. The study observed that the proposed estimator performs on par with some of the existing robust methods considered in the study in terms of mean square error for certain values of shape parameter($k<0$) and moderately large sample size. The efficiency of proposed estimator is more than existing robust estimators considered in this study. In addition, the study includes comparison of these estimators using real dataset.

***Keywords:** Generalized Pareto Distribution, Extreme Value Theory, Value at Risk, M-estimation, Peaks over threshold*

1. Introduction

1.1 Background

Value at risk (VaR) is used widely in financial industry by all stake holders, like investors, portfolio managers, rating agencies and regulators. It indicates the maximum amount that an investor may loose over a given time horizon and with a given probability. It is commonly used since it is easy to understandand it is reported as a single number that represents potential losses with some confidence level. There are several methods available in literature (*Jorion, 2001, Kuester et al., 2006)* for estimating VaR and one among them is based on extreme value theory (*McNeil, 2000*). The field of extreme values has attracted the attention of Statisticians, Engineers, and Economists in the lastfew decades and there are two widely used approaches to analyze extreme data *(Pickands, 1975; Galambos, 1981)*, namely, the block-maxima approach *(Beirlant et al. 1996)* and the peaks-over-threshold (PoT) approach *(Davison and Smith, 1990).* The first approach considers the distribution of the maximum order statistic. A generalized extreme value GEV distribution is then fitted to the series of extremal observations. But this block-maxima approach is wasteful of data as only one data point in each block is

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taken, see:*Fisher and Tippett (1928)* and *Gnedenko (1943)*.The second approach extracts the peak values which exceed a certain threshold and in this method, the excess values over high threshold are modeled with generalized Pareto distribution (GPD); See *McNeil and Saladin (1997)*. More on the two approaches can be found in *Caires (2009*) and *Ferreira and de Haan (2015).*

1.2 Generalized Pareto Distribution and Peaks over Threshold (PoT) framework

Pickands(1975) and *Balkemaand de Haan (1974)* proved that the limiting distribution of exceedances (or peaks) of a random variable X over a sufficiently high threshold u is generalized Pareto distribution (GPD) with distribution function F(x) and probability density function f(x) are given below

$$F(X|\mu,\sigma,k) = \begin{cases} 1-\left(1-k\frac{x-\mu}{\sigma}\right)^{1/k} & k \neq 0 \\ 1-exp\left(-\frac{x-\mu}{\sigma}\right) & k = 0 \end{cases}$$

$$f(X|\mu,\sigma,k) = \begin{cases} \frac{1}{\sigma}\left(1-k\frac{x-\mu}{\sigma}\right)^{1/k-1} & k \neq 0 \\ \frac{1}{\sigma}\ exp\left(-\frac{x-\mu}{\sigma}\right) & k = 0 \end{cases} \quad \text{- (1.2.1)}$$

range of x: for $k \leq 0,\ \mu \leq x \leq \infty$
and for $k > 0,\ \mu \leq x \leq \ \mu + \frac{\sigma}{k}$.

Where,$\mu\ (\mu \in R),\ \sigma\ (\sigma > 0)$ and $k\ (k \in R)$, are the location, scale and shape parameters respectively.

The distribution can be classified into three types depending on the shape parameter k; as heavy-tailed, medium-tailed and short-tailed, according as k< 0, k = 0 and k > 0 respectively.

Under PoT framework, we can estimate extremes for arbitrary distributions, if threshold value is sufficiently high. But the choice of threshold is critical, as high threshold leads to high variance due to few exceedences, but not biased, and a low threshold would necessitate using samples that are no longer considered as being in the tails which leads to increased bias. Hence one has to balance between bias and precision in selecting threshold value u. In literature several threshold selection methods have been suggested; See *Embrechts et al. 1999b* and *Caeiro and Gomes(2016)*. Most of them are based on graphical approaches and include iterative algorithms.

It is often seen that the number of exceedances is small in peaks-over-threshold approach and thereby, even a single abnormally large value may distort the estimates. The objective of the study is to estimate VaR in presence outliers under POT framework. The study proposes a new robust estimators of parameters of GPD and hence of VaR and expected shortfall, under POT framework.

The remainder of this paper is organized as follows. Section 2 covers various estimation procedures for parameters of GPD and estimation of VaR that are in the literature and in section 3 we propose a new parameter estimator for estimating parameters of GPD using minimization method. In Section 4, we compare the performances of proposed method with existing methods for estimating parameters of GPD and VaR through simulation study. Section 5 covers results of empirical study. Section 6 concludes the paper.

2. Estimation of parameters, Value at Risk and Expected shortfall

2.1 Estimation methods for parameters of GPD

Various parameter estimation methods have been studied for generalized Pareto distribution in literature and several methods have been compared under various conditions for estimating the GPD parameters; See *P.Z Bermudeza& S. Kotz (2010, Part I & II)*. However, there are no universally accepted methods for estimating GPD parameters. Even if few methods are better than others over certain range of shape parameter k, they suffer from various constraints and convergence problems. Among these, maximum likelihood method (MLE) is preferred due to its asymptotic optimality properties and has been studied by *Davison (1984), Smith(1984,1985), Grimshaw(1993). Hosking and Wallis (1987)* compared maximum likelihood estimates with method of moments (MOM) and probability weighted moment (PWM) estimates over small ranges of k, |k|≤ ½as it is common to observe k between -1 and ½ (Zhang and Stephen 2009) and found thatprobability weighted method performs well for 0≤ k ≤1 and very good for k ≤ ½.*Castillo and Hadi (1997)* introduced elemental percentile method (EPM) and compared it with the MOM and the PWM methods, using root mean square error criterion when |k| ≤ 2, through simulation study and showed that the PWM estimator performs well in small samples for k ≤ ½.

Zhang and Stephens (2009) and *Zhang (2010)* developed empirical Bayes method (EBM) based on the likelihood and which uses a data-driven prior to estimate parameters of GPD. This prior is chosen in such a way that the estimates always exist and can be expressed as explicit functions of the observations which enable the estimates to be computed very efficiently. They showed that EBM performs better than MLE, MOM, PWM and Likelihood Moment Estimator (LME) with respect to bias and mean square error when – ½ < k < ½. *Piao, Chen et.al., (2017)* proposed estimator by minimizing differences between empirical and model distribution function through TukeyBiweight function (Yohai&Zamar, 1998) where they used EBM estimates as initial estimates.

Luceño (2006) proposed estimators based on minimum distance approach by minimising the squared differences between empirical and model distribution functions, given in terms of various goodness- of-fit statistics, including the Cramer–von Mises statistic (CM), the Anderson– Darling

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statistic (AD) and the right-tail weighted Anderson–Darling statistic (ADR) and compared few maximum goodness-of-fit (MGF) estimators with Quasi Maximum Likelihood (QML), MLE, MOM, PWM and EPM over k = -2, -1, 0, 1, 2. It is seen that many simulation studies for comparing different methods of estimation of GPD parameters, have been conducted by many authors but these simulation studies are somewhat difficult to compare, as they have been performed under different conditions. Moreover, some of the methods have never been compared via a simulation study. See: *Bermudeza& S. Kotz (2010) part I & II*.

2.2Value at Risk and Expected Shortfall

Generally returns on investments at stock markets follow normal distribution and as a result, under typical conditions, VaR is thought to be almost as effective as expected shortfall (ES) at capturing risk. However, the financial crisis in 2007 highlights the importance of measuring the risk associated with non-normal returns. In this connection we are using generalized Pareto distribution to model returns exceeding a sufficiently high threshold value in case of violation of normality assumption and for heavy tailed distribution. The procedure is to estimate parameters of GPD and use it for estimation of VaR and ES.

If X denotes the return random variable with distribution function $F_X(x)$, then for each $\alpha \epsilon (0, 1)$, the VaR with 100x (1–α)% confidence coefficient is defined as:

$VaR_{(\alpha)}(X) = \inf(x: P(X \leq x) > \alpha) = \sup(x: P(X < x) \leq \alpha)$

If $X \sim GPD\ (\mu, \sigma, k)$, then α^{th} quantile is given by

$VaR_{(\alpha)}(X) = \mu + \sigma\, F_{k,\sigma}^{-1}(\alpha)$.

$$VaR(\alpha) = u + \frac{\sigma}{k}\left[\left[\frac{n}{N_u}(1-\alpha)\right]^{-k} - 1\right]$$

If $X \sim GPD(\mu = 0, \sigma, k)$ then, for a threshold u, the random variable Y = X – u| X > u has GPD (σ – ku, k) (Zhang and Stephens, 2009), which reflects the fact that the excess over threshold operation does not affect the shape parameter of the GPD.

As value at risk is not a linear function of parameters of GPD, there is a need for studying estimation of VaR, especially in the presence of outliers.The main drawback with the use of VaR as a risk measure is that, it does not respond to losses exceeding the confidence level, as a result it cannot capture the risk associated with the shape of the distribution beyond the confidence level. *Artzner et al. (1997)* propose the use of expected shortfall as an improvement on VaR.

$$ES_{(\alpha)}(X) = E\left(X \mid X > VaR_{(\alpha)}(X)\right)$$

$$ES(\alpha) = \frac{VaR(\alpha)}{1-k} + \frac{\sigma - ku}{1-k}$$

This is expected value of return random variable beyond value at risk

3. Proposed Method

3.1 Motivation

P. Chen et. al (2017) recently proposed two new estimators for the GPD parameters using the minimum distance estimation, where the Tukeybiweight function (Rey, 2012) is used as the distance measure which minimizes the distance between the empirical distribution function and the theoretical distribution function of GPD. The two estimators were shown to be consistent and it is claimed that as distance measure is borrowed from robust estimation, these estimators are robust to outlier contamination with breakdown point as high as 50% and 95% efficiency under gaussian errors.

As we are interested in estimation of VaR in presence of outliers we propose a robust method for estimating parameters of GPD similar to methods proposed by P.Chen et.al (2017) using an optimal loss function (Yohai, V.J., and Zamar, R.H. (1988), & M.Salibian-Barrera et. al, 2008) which has high breakdown point 50% and efficiency of 97.5% under Gaussian errors.

3.2 Introduction

In this section, two robust estimators for the GPD parameters are proposed based on the M-estimation (Huber, 1973). It can be shown that a simple regression model can be established for estimating GPD parameters and then the M-estimation procedure of Huber can be applied. The proposed estimators are M-estimators of GPD parameters obtained under anoptimal loss function.

3.3 Proposed Method

Consider an i.i.d. sample$x_1, x_1, \ldots\ldots x_n$from $F_\theta(x_i)$and let θ= (σ, k). Let $F_n(x)$ be the corresponding empirical distribution function.If we define residuals similar to those in a linear regression, i.e.,

$$r_i(\theta) = F_n(x_i) - F_\theta(x_i); \quad i = 1, 2, \ldots\ldots n;$$

Where,

$$F_\theta(x_i) = \begin{cases} 1 - \left(1 - k\frac{x}{\sigma}\right)^{1/k} & k \neq 0 \\ 1 - exp\left(-\frac{x}{\sigma}\right) & k = 0 \end{cases}$$

range of (σ, k) and x as in 1.2.1

$$F_n(x_i) = \frac{i - 0.5}{n};$$

We may be able to obtain an estimator of θ by minimizing a function ρ(.) (distance measure) of $r_i(\theta)$. For the GPD, it is found that the estimators obtained by minimizing $\frac{1}{n}\sum_{i=1}^{n}[r_i(\theta)]^2$ are sensitive to the shape

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parameter k (Song and Song 2012). When sample size is small, some outlying observations in the sample may have decisive impact in minimizing $\frac{1}{n}\sum_{i=1}^{n}[r_i(\theta)]^2$ (least squares method) at some values of shape parameter. Borrowing the idea from M-estimation, we may reduce the influence of the outlying observations in $r_i(\theta)$ by using an appropriate distance function $\rho(.)$.

The M-estimator for θ is defined as

$$\hat{\theta}_n = \operatorname{argmin}\frac{1}{n}\sum_{i=1}^{n}\rho(r_i(\theta)) \qquad - \quad (3.3.1)$$

In the literature, many ρ functions have been proposed (Rey 2012). We have considered following ρ function (Yohai and Zamar 1988)

$$\rho_c(u) = \begin{cases} 1.38\left(\frac{u}{c}\right)^2 & \left|\frac{u}{c}\right| \le \frac{2}{3} \\ 0.55 - 2.69\left(\frac{u}{c}\right)^2 + 10.76\left(\frac{u}{c}\right)^4 - 11.66\left(\frac{u}{c}\right)^6 + 4.04\left(\frac{u}{c}\right)^8; & \frac{2}{3} < \left|\frac{u}{c}\right| \le 1 \\ 1, & \left|\frac{u}{c}\right| > 1 \end{cases}$$

Where, c is called the tuning parameter. By setting c = 1.214, the M-estimator with the above optimal function has an efficiency of 97.7% under independent Gaussian errors. Theconsistency and asymptotic normality of an M-estimator for the linear regression with i.i.d. errors are well established in Huber (2011, chap.11). Observing that $r_i(\theta)$'s are asymptotically normal and have different asymptotic variances(Van der Vaart 1998, chap.19), i.e.,

$$\sqrt{n}r_i(\theta_0) = \sqrt{n}\left[F_n(x_i) - F_{\theta_0}(x_i)\right] \xrightarrow{d} N\left(0, F_{\theta_0}(x_i)\left(1 - F_{\theta_0}(x_i)\right)\right), \quad -(3.3.2)$$

for every θ_0 in parametric space$\{(\sigma, k), \sigma > 0, k \in R\}$

We can construct a weighted M-estimator as

$$\widehat{\theta_n^*} = \operatorname{argmin}\frac{1}{n}\sum_{i=1}^{n}\rho(r_i^*(\theta)) \qquad -(3.3.3)$$

Where, $r_i^*(\theta) = \frac{r_i(\theta)}{w_i(\theta)}$; with weights$w_i(\theta) = \sqrt{F_\theta(x_i)\left(1 - F_\theta(x_i)\right)}$.

However, optimizing (3.3.3) with this weight is difficult, as $\rho(.)$a complicated function of the parameters θ, direct optimization of 3.3.3 is difficult, as such we use the weight$w_i(\theta) = \sqrt{F_{\widehat{\theta_n}}(x_i)\left(1 - F_{\widehat{\theta_n}}(x_i)\right)}$ where $\widehat{\theta_n}$ is unweighted M-estimator obtained from (3.3.1). We have employed iterative reweighed least squares (IRLS) algorithm and used estimates obtained from EBM (ZJ) method for initial values, as they exist for all values of shape parameter.

4. Simulation Study

4.1 Introduction

As our objective is to estimate VaR in presence of outliers, we compared performance of proposed method with some robust methods and few traditional methods which are available in R-Environment through a simulation study using bias and root mean square error criteria. We considered six robust methods among which, four are based on minimum distance approach and six non-robust methods (which are available at R-environment) for comparison in estimating parameters of GPD, VaR and ES in presence of two additive outliers under PoT framework.

Table 1: List of Estimation methods considered in the study

Robust Methods	Non Robust Methods
1. Proposed Method-1 (Unweighted) 2. Proposed Method-2 (Weighted) 3. P.Chen method-1 (Unweighted, PZ method), proposed by *P.Chenet.al* (2017), 4. P.Chen method-2 (Weighted, WPZ method) proposed by *P.Chen et.al* (2017), 5. Median Estimator, (MED), *Peng and Welsh, (2001)* 6. Minimum Density Power Divergence, (MDPD),*Juárez and Schucany (2004)*	7. Empirical Bayes Method, (*EBM or* ZJ), *Zhang (2010),* 8. PICKANDS (Pickands) *Pickands, J. (1975)* 9. Maximum Likelihood Estimator, (MLE), *Smith (1984).* 10. Maximum Penalized Likelihood, (MPLE), *Coles and Dixon (1999)* 11. Probability Weighted Moments Unbiased,*(*PWMU), *Hosking and Wallis (1987)* 12. Probability Weighted Moments Biased,*(* PWMB), *Hosking and Wallis (1987)*

Throughout the study, location parameter is set at 0 and scale parameter is set at 1, as simulation results are invariant of scale parameter*(Hosking & Wallis, 1987)*. In practice the value of shape is commonly observed between -1 and ½(*Zhang and Stephens, 2009*) and alsoit is not uncommon to observe shape parameter k ≥ ½ (infinite variance)*(Castillo et. al 2005)*, due to violation of normality assumption (heavy tailed distribution), thereforewe restrict our attention to the case of shape (k) values between -1 and +1.

Bias and RMSE are computed in estimating VaR and ES using all methods considered in this study, at different confidence levels $(1-\alpha)$ = 0.95, 0.98 and 0.99) under PoTsetup. Under this setup the sizes of exceedances are

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usually small due to critical choice of threshold, therefore, in order to have some exceedances, a sample of 1,000 random observations are generated from GPD and exceedances of size n = 20, 40 and 80 are obtained at p = 0.02, 0.04 and 0.08 respectively. For each combination of sample size (n), shape (k) and confidence level ($1 - \alpha$), bias and mean squared error of the estimators are obtained based on 10,000 Monte Carlo replications using R-software.

$$Bias = E(\hat{\theta} - \theta) \ ; \quad RMSE = \sqrt{E\left[\left(\hat{\theta} - \theta\right)^2\right]};$$

Performanceof the above methods of estimators in estimating parameters of GPD, VaR and ES are compared in the absence of outliers and in the presence of two additive outliers.

4.2 Algorithm

Step 1: Generated a random sample of size 1,000 observations from GPD at location (μ) = 0, scale σ = 1 and shape = k

Step 2: Select a threshold value u, taken to be $(1 - p)^{th}$ sample quantile

Step 3: Compute true Value at Risk (VaR) and true Expected Shortfall (ES) at α

$$VaR(\alpha) = u + \frac{\sigma}{k}\left[\left[\frac{n}{N_u}(1-\alpha)\right]^{-k} - 1\right]$$

$$ES(\alpha) = \frac{VaR(\alpha)}{1-k} + \frac{\sigma - ku}{1-k}$$

Step 4: Compute mean, standard deviation of observations obtained in step 1 and replace two random observations by additive outliers (mean+(5*StdDev) and mean+(5.5*StdDev)) to obtain contaminated data

Step 5: Obtain observations above threshold u and generate exceedances (X – u)

Step 6: Fit GPD for exceedances (for both non-contaminated and contaminated data) using all methods (Table 1) and estimateσ (scale), k (shape),VaR and ES

Step 7: Compute bias and mean square error for scale, shape, VaR and ES for non-contaminated data and contaminated data

Step 8: Above steps are repeated (step 1 to 7), 10,000 times and compute average bias and square root ofaverage of squared error (RMSE) for scale, shape, VaR and ES for both contaminated and non-contaminated data.

4.3 Results

It is observed that the performance of different methods in estimation of parameters of GPD depends on the sample size and the shape of the sampling distribution. Also we found that there is no one estimator, which stands out as being the best in all situations. Following are some of major findings of simulation study

- ➢ RMSE in estimation of VaR increases considerably with increase in confidence level (1 – α) for all methods.
- ➢ Distribution of RMSE in estimation of VaRis found to be asymmetric over range of $-1<k<1$ for a given shape and confidence level for all methods.
- ➢ Two additive outliers affected estimation of Value at Risk and Expected shortfall even when sample size is moderately large.

For brevity RMSE values in estimating shape, VaR and ES in presence of outliers at 95% confidence and n = 40 are reported in table 2, table 4 and table 5 respectively. As we are interested in estimation of Value at Risk, we have also reported BIAS in estimating VaR in presence of outliers at n = 40 and 95% confidence level in table 3. However results for n = 20 and 80 in estimation of shape, VaR and ES are summarized and reported in table 6, 7& 8 respectively.

Table 2: RMSE in estimating shape in presence of outliers when n = 40(p= 0.04, N = 1,000) at 95% confidence level

k	**New1**	**New2**	**ZJ**	**PZ**	**WPZ**	**MLE**	**PWMU**	**PWMB**	**PICK**	**MED**	**MDPD**	**MPLE**	**MGF**
-1	1.02	1.02	**0.40**	1.02	1.01	1.58	1.72	1.70	0.55	0.95	1.31	1.48	10.54
-0.9	0.93	0.93	**0.36**	0.93	0.92	1.42	1.54	1.52	0.54	0.84	1.25	1.34	14.19
-0.8	0.84	0.83	**0.30**	0.84	0.82	1.27	1.36	1.34	0.52	0.74	1.13	1.20	20.18
-0.7	0.75	0.73	**0.26**	0.75	0.73	1.11	1.16	1.14	0.56	0.63	1.00	1.05	27.99
-0.6	0.67	0.64	**0.22**	0.67	0.65	0.95	0.95	0.93	0.54	0.52	0.86	0.90	40.32
-0.5	0.57	0.54	**0.19**	0.57	0.54	0.78	0.75	0.73	0.56	0.45	0.72	0.74	41.34
-0.4	0.49	0.44	**0.17**	0.49	0.34	0.59	0.55	0.53	0.55	0.38	0.56	0.56	11.76
-0.3	0.41	0.27	**0.16**	0.41	0.31	0.40	0.37	0.35	0.52	0.33	0.39	0.37	0.54
-0.2	0.31	0.25	**0.15**	0.31	0.29	0.23	0.26	0.24	0.58	0.39	0.24	0.20	0.29
-0.1	0.28	0.27	0.15	0.28	0.31	0.14	0.17	0.17	0.58	0.39	0.17	**0.12**	0.15
0	0.31	0.36	0.18	0.31	0.40	0.22	**0.17**	0.17	0.56	0.41	0.21	0.21	4.30

0.1	0.35	0.42	**0.17**	0.35	0.48	0.35	0.21	0.22	0.56	0.40	0.32	0.35	0.34
0.2	**0.15**	0.21	0.18	0.15	0.28	0.33	0.50	0.51	0.86	0.49	0.34	0.33	0.23
0.3	0.18	0.27	0.49	0.18	0.48	**0.17**	0.36	0.37	1.20	0.50	0.22	0.19	41.04
0.4	0.31	0.35	0.69	0.31	0.43	**0.16**	0.27	0.29	1.37	0.54	0.23	0.19	58.82
0.5	0.42	0.45	0.85	0.42	0.49	**0.19**	0.25	0.27	1.49	0.62	0.28	0.23	47.25
0.6	0.53	0.55	0.99	0.53	0.57	**0.25**	0.27	0.29	1.62	0.71	0.35	0.29	35.29
0.7	0.64	0.67	1.12	0.64	0.71	0.32	**0.31**	0.33	1.70	0.80	0.43	0.37	26.81
0.8	0.74	0.76	1.23	0.74	0.79	0.40	**0.38**	0.39	1.80	0.89	0.52	0.45	20.64
0.9	0.85	0.86	1.34	0.85	0.88	0.49	**0.45**	0.47	1.93	0.99	0.62	0.54	16.91
1	0.95	0.97	1.46	0.95	0.99	0.58	**0.53**	0.55	2.01	1.09	0.71	0.63	13.67

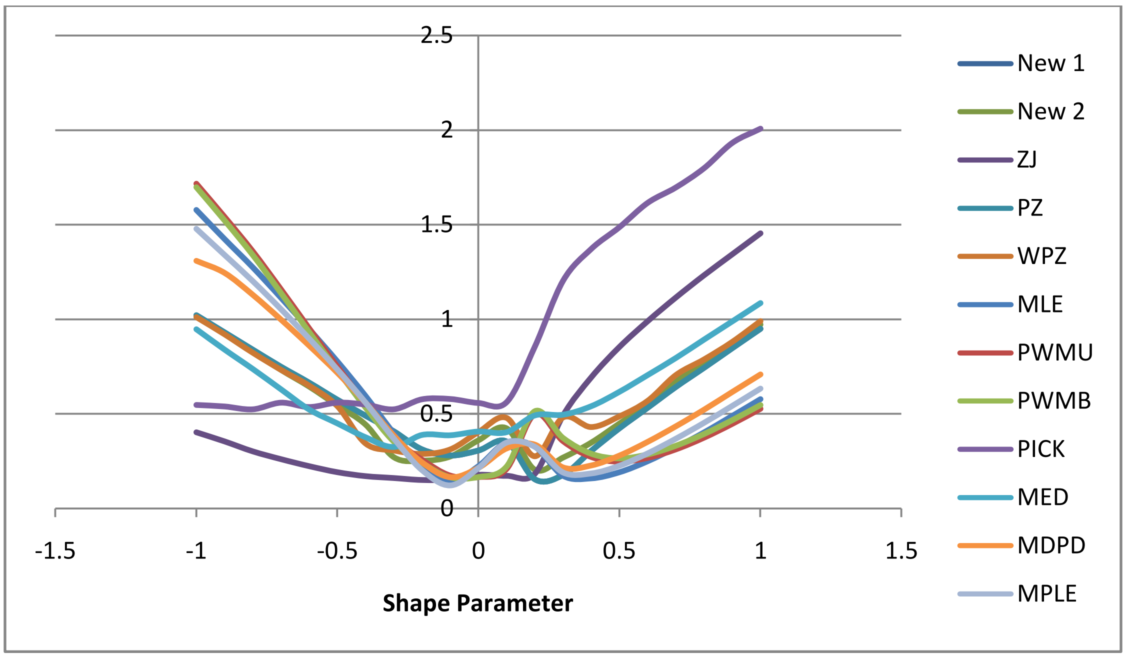

Figure 1: RMSE in estimation of shape parameter in presence of outliers when n = 40

Table 3: BIAS in estimating Value at Risk in presence of outliers when n = 40
(p= 0.04, N = 1,000) at 95% confidence level

k	**New1**	**New2**	**ZJ**	**PZ**	**WPZ**	**MLE**	**PWMU**	**PWMB**	**PICK**	**MED**	**MDPD**	**MPLE**	**MGF**
-1	0.24	0.24	0.25	0.24	0.24	0.25	0.25	0.25	0.24	0.24	0.25	0.25	-0.91
-0.9	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.23	0.24	0.24	0.24	-1.07
-0.8	0.23	0.23	0.23	0.23	0.23	0.24	0.24	0.23	0.22	0.23	0.23	0.23	-1.19
-0.7	0.22	0.22	0.23	0.22	0.22	0.23	0.23	0.23	0.21	0.22	0.23	0.23	-1.19
-0.6	0.21	0.21	0.22	0.21	0.21	0.22	0.22	0.22	0.20	0.21	0.22	0.22	-1.13
-0.5	0.20	0.20	0.21	0.20	0.20	0.20	0.20	0.20	0.19	0.19	0.20	0.20	-0.93
-0.4	0.18	0.18	0.19	0.18	0.18	0.19	0.18	0.18	0.16	0.17	0.18	0.18	-0.04
-0.3	0.15	0.16	0.16	0.15	0.16	0.16	0.15	0.15	0.14	0.14	0.15	0.16	0.17

-0.2	0.12	0.12	0.12	0.12	0.12	0.11	0.11	0.11	0.10	0.11	0.11	0.11	0.12
-0.1	0.07	0.07	0.07	0.07	0.07	0.04	0.06	0.05	0.06	0.06	0.05	0.04	0.04
0.1	-0.09	-0.08	- 0.11	- 0.09	-0.08	-0.19	-0.12	-0.13	-0.10	-0.10	-0.17	-0.19	-0.20
0.2	0.03	0.04	0.05	0.03	0.04	0.03	0.00	0.00	-0.03	0.00	0.03	0.03	0.05
0.3	0.13	0.13	0.15	0.13	0.14	0.14	0.13	0.13	0.08	0.12	0.14	0.14	-0.08
0.4	0.17	0.17	0.17	0.17	0.17	0.17	0.17	0.17	0.13	0.16	0.17	0.17	-0.73
0.5	0.18	0.18	0.19	0.18	0.18	0.19	0.18	0.18	0.16	0.17	0.18	0.18	-0.94
0.6	0.19	0.19	0.19	0.19	0.19	0.19	0.19	0.19	0.17	0.18	0.19	0.19	-1.10
0.7	0.19	0.19	0.19	0.19	0.19	0.19	0.19	0.19	0.17	0.19	0.19	0.19	-1.25
0.8	0.19	0.19	0.19	0.19	0.19	0.19	0.19	0.19	0.18	0.19	0.19	0.19	-1.38
0.9	0.19	0.19	0.19	0.19	0.19	0.19	0.19	0.19	0.18	0.19	0.19	0.19	-1.49
1	0.19	0.19	0.19	0.19	0.19	0.19	0.19	0.19	0.18	0.19	0.19	0.19	-1.57

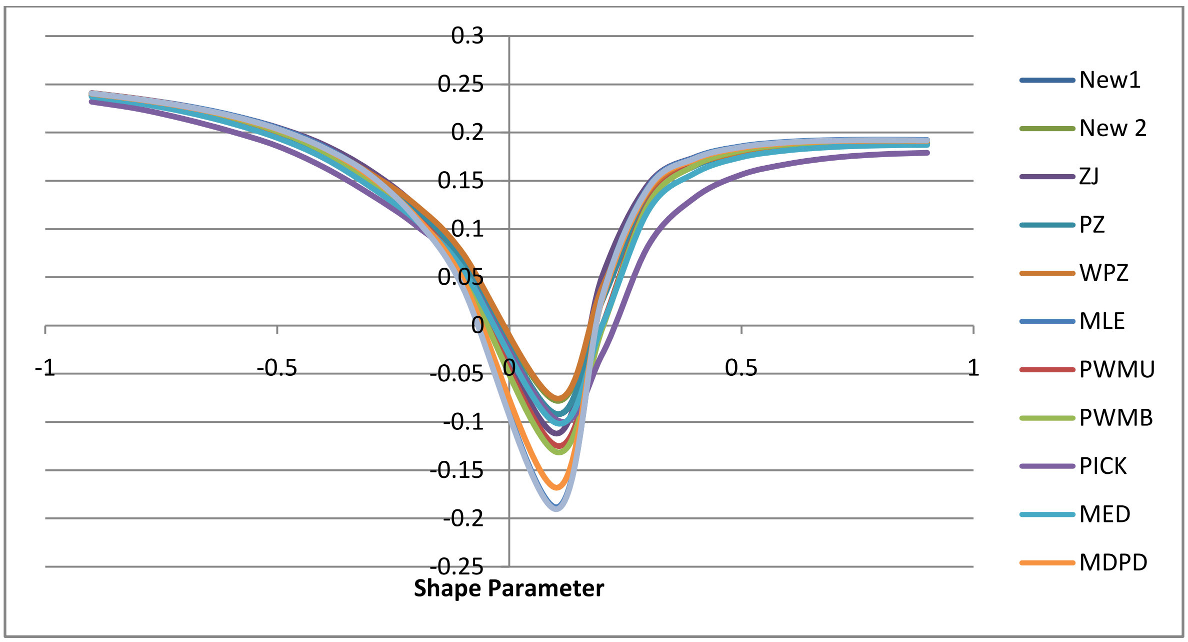

Figure 2: BIAS in estimation of Value at Risk in presence of outliers when n = 40

Table 4:RMSE in estimating Value at Risk in presence of outliers when n = 40
(p= 0.04, N = 1,000) at 95% confidence level

k	**New1**	**New2**	**ZJ**	**PZ**	**WPZ**	**MLE**	**PWM U**	**PWM B**	**PICK**	**MED**	**MDP D**	**MPL E**	**MGF**
-1	0.24	0.24	0.25	0.24	0.24	0.25	0.25	0.25	**0.24**	0.24	0.25	0.25	1.14
-0.9	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	**0.23**	0.24	0.24	0.24	1.19
-0.8	0.23	0.23	0.23	0.23	0.23	0.24	0.24	0.23	**0.22**	0.23	0.23	0.23	1.22
-0.7	0.22	0.22	0.23	0.22	0.22	0.23	0.23	0.23	**0.21**	0.22	0.23	0.23	1.20
-0.6	0.21	0.21	0.22	0.21	0.21	0.22	0.22	0.22	**0.20**	0.21	0.22	0.22	1.13
-0.5	0.20	0.20	0.21	0.20	0.20	0.20	0.20	0.20	**0.19**	0.19	0.20	0.20	0.97

-0.4	0.18	0.18	0.19	0.18	0.18	0.19	0.18	0.18	**0.17**	0.17	0.18	0.18	0.41
-0.3	0.15	0.16	0.16	0.15	0.16	0.16	0.15	0.15	**0.14**	0.15	0.15	0.16	0.17
-0.2	0.12	0.13	0.12	0.12	0.13	0.11	0.12	0.11	0.12	0.11	0.11	**0.11**	0.12
-0.1	0.08	0.08	0.07	0.08	0.08	0.06	0.07	0.07	0.09	0.08	0.06	**0.06**	0.06
0.1	0.12	**0.11**	0.13	0.12	0.11	0.22	0.15	0.16	0.17	0.15	0.20	0.22	0.23
0.2	**0.04**	0.05	0.06	0.04	0.05	0.05	0.06	0.06	0.11	0.05	0.04	0.05	0.06
0.3	0.13	0.14	0.15	0.13	0.14	0.14	0.13	0.13	**0.10**	**0.12**	0.14	0.14	0.35
0.4	0.17	0.17	0.17	0.17	0.17	0.17	0.17	0.17	**0.14**	**0.16**	0.17	0.17	0.75
0.5	0.18	0.18	0.19	0.18	0.18	0.19	0.18	0.18	**0.16**	**0.17**	0.18	0.19	0.94
0.6	0.19	0.19	0.19	0.19	0.19	0.19	0.19	0.19	**0.17**	**0.18**	0.19	0.19	1.11
0.7	0.19	0.19	0.19	0.19	0.19	0.19	0.19	0.19	**0.17**	**0.19**	0.19	0.19	1.26
0.8	0.19	0.19	0.19	0.19	0.19	0.19	0.19	0.19	**0.18**	**0.19**	0.19	0.19	1.39
0.9	0.19	0.19	0.19	0.19	0.19	0.19	0.19	0.19	**0.18**	**0.19**	0.19	0.19	1.52
1	0.19	0.19	0.19	0.19	0.19	0.19	0.19	0.19	**0.18**	**0.19**	0.19	0.19	1.62

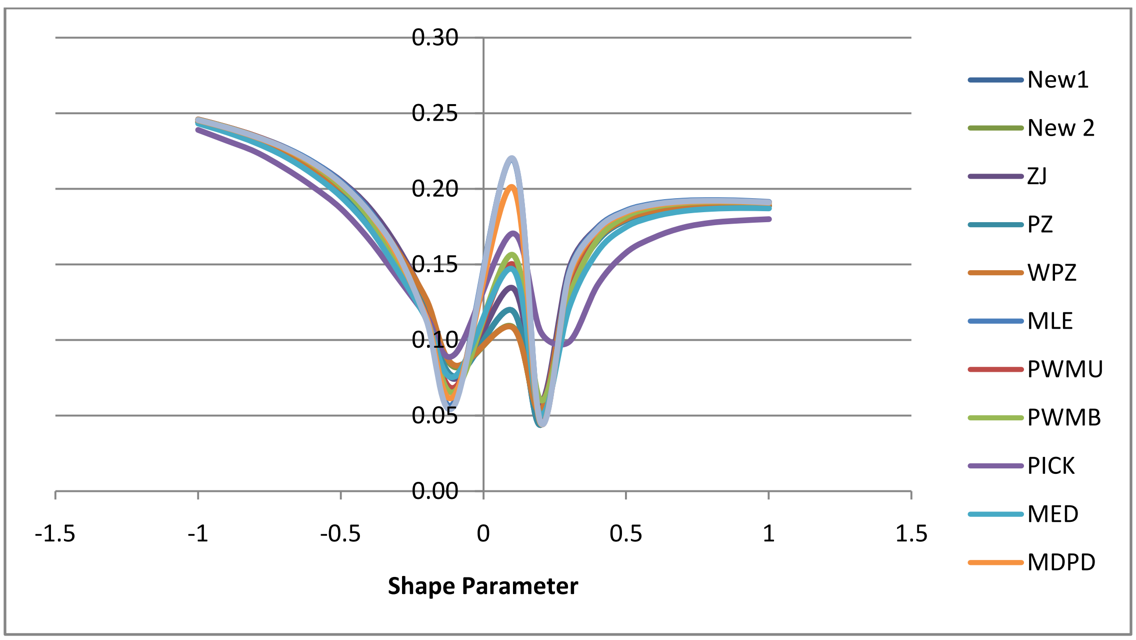

Figure 3: RMSE in estimation of Value at Risk in presence of outliers when n = 40

Table 5: RMSE in estimating Expected Shortfall in presence of outliers when n = 40
(p= 0.04, N = 1,000) at 95% confidence level

k	**New1**	**New2**	**ZJ**	**PZ**	**WPZ**	**MLE**	**PWMU**	**PWMB**	**PICK**	**MED**	**MDPD**	**MPLE**	**MGF**
-1	0.35	0.35	0.37	0.35	0.35	0.34	**0.32**	0.32	0.36	0.35	0.35	0.34	21.79
-0.9	0.37	0.37	0.38	0.37	0.37	0.35	**0.33**	0.33	0.37	0.36	0.36	0.35	2.07
-0.8	0.37	0.38	0.40	0.37	0.38	0.35	**0.34**	0.34	0.38	0.38	0.37	0.36	1.97
-0.7	0.38	0.38	0.41	0.38	0.38	0.36	**0.35**	0.35	0.40	0.38	0.37	0.37	1.92
-0.6	0.38	0.38	0.43	0.38	0.38	0.36	**0.35**	0.35	0.41	0.38	0.37	0.37	1.85

-0.5	0.37	0.37	0.43	0.37	0.38	0.35	**0.35**	0.35	0.48	0.40	0.36	0.36	1.70
-0.4	0.34	0.34	0.42	0.34	0.40	0.34	**0.34**	0.34	0.39	0.85	0.34	0.34	0.83
-0.3	0.29	0.29	0.39	**0.29**	0.37	0.30	0.30	0.30	0.92	6.01	0.30	0.30	0.29
-0.2	0.29	0.30	0.32	0.29	0.34	0.24	0.24	0.24	3.96	1.60	0.24	0.24	**0.24**
-0.1	0.32	0.32	0.24	0.32	0.31	0.15	0.15	0.15	20.13	5.27	0.14	0.15	**0.13**
0.1	0.60	0.58	0.70	0.60	0.51	0.32	0.31	**0.31**	19.01	13.92	0.32	0.32	0.36
0.2	0.30	**0.29**	0.39	0.30	0.37	0.41	0.41	0.41	0.48	0.48	0.41	0.41	0.41
0.3	**0.77**	0.78	0.92	0.77	0.88	0.83	0.83	0.83	0.89	0.85	0.83	0.83	1.26
0.4	1.12	1.13	1.21	1.12	1.14	1.12	**1.12**	1.12	1.18	1.15	1.13	1.13	2.25
0.5	1.46	1.46	1.51	1.46	1.46	1.44	**1.43**	1.43	1.49	1.47	1.45	1.45	2.73
0.6	1.89	1.89	1.93	1.89	1.89	1.87	**1.86**	1.86	1.91	1.89	1.88	1.88	3.31
0.7	2.57	2.58	2.61	2.57	2.58	2.56	**2.55**	2.55	2.59	2.58	2.57	2.56	4.15
0.8	3.92	3.92	3.95	3.92	3.92	3.91	**3.90**	3.90	3.94	3.92	3.91	3.91	5.65
0.9	7.93	7.93	7.95	7.93	7.93	7.92	**7.91**	7.91	7.94	7.93	7.92	7.92	9.82

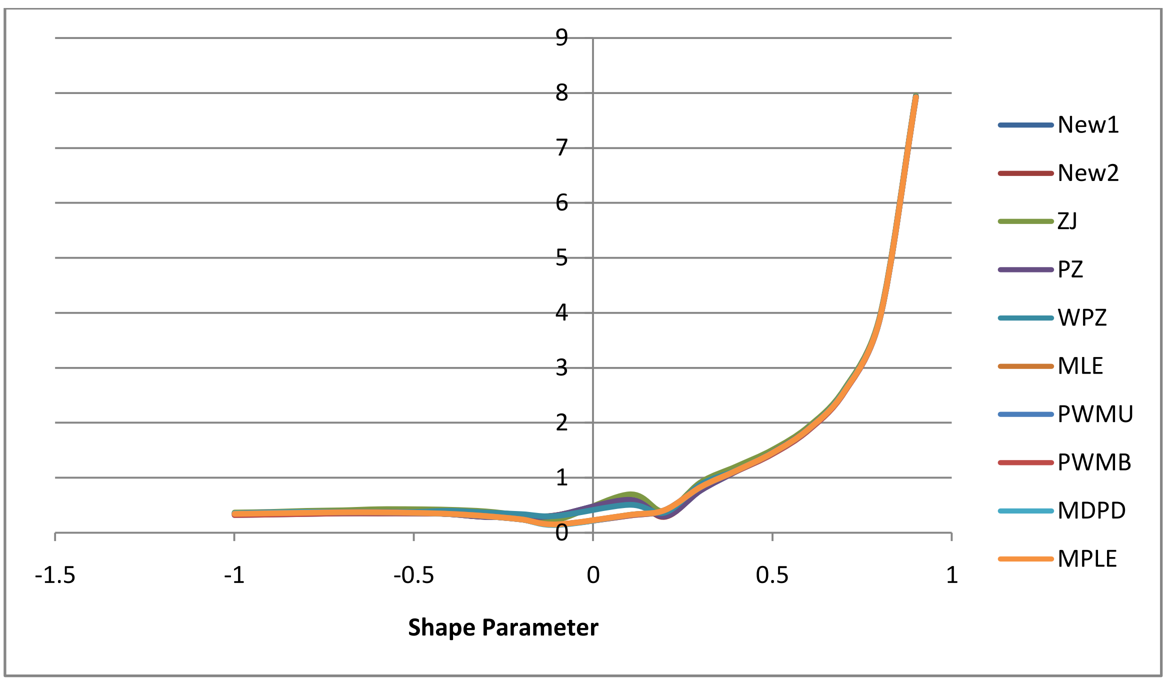

Figure 4: RMSE in estimation of Expected Shortfall in presence of outliers when n = 40

Detailed summary of simulation study in absence of outlier is given below

Shape Parameter

- PWM methods are consistent in estimating shape parameter, but however bias increases sharply when k > ½ and PWM underestimates shape over the range of -1 < k < 1.
- As sample size increases MPLE& MDPD were performing on par with PWM method but RMSE of MDPDincreases sharply, when k>½, in estimating shape parameter.

Scale Parameter

- As sample size increases all methods were performing equally well in estimation of scale.
- rate of change in RMSE for scale is more when k > 0, as compared to k < 0 for all estimators.

Value at Risk

- RMSE in estimating VaR is increasing as confidence level ($1 – \alpha$) increases for all sample sizes.
- When sample size is small (n = 20, 40) all methods overestimates VaR, when k<0, and underestimates VaR when k > 0.
- But as sample size increases (n=80), all methods underestimates VaR when k < 0, and overestimates VaR when k > 0.
- RMSE in estimating VaR for all methods increases rapidly when k > 0.2.
- Proposed methods are performing on par with robust methods (PZ and WPZ) proposed by Chen.et.al(2017), which are are performing well in estimation of VaR, when k ≤ 0.2 and k>0.1 respectively for large sample size (n=80).

Expected Shortfall

- As sample size increases proposed methods and methods proposed by Chen et.al (PZ, WPZ) methods are performing equally well in estimating expected shortfall
- But however, EBM is better than others in estimating ES based on RMSE when -0.7 < k < 0 for large sample size. (n=80)

We finally conclude that no estimators are performing better than others in estimating shape, VaR and ES over the range of -1 < k < 1 in absence of outlier.

4.4 Summary of Simulation study

Table 6: Best Methods for shape parameter based on least RMSE

Sample size	Non Contaminated	Contaminated
n=20	MDPD (-1<k<-0.5), PWMU (0.1<k<1)	ZJ (-1<k<-0.2), PWMU (0.1<k<1)
n = 40	MGF(-0.5<k<-0.1), PWMU (0.1<k<1)	ZJ(-0.9<k<-0.4), PWMU (0<k<1)
n = 80	MGF(k=-1,-0.9) MDPD(k= -0.8,-0.7) MGF (-0.6 ≤ k ≤ -0.2) MPLE(k= -0.1) ZJ(k=0); PWMU (k=0.1) WPZ=(k=0.2,0.3) ZJ(0.4 ≤ k ≤ 1)	PICKANDS(-1≤ k ≤ -0.7), MED(k= -0.5, -0.4) PWMB (k= -0.3) MPLE (k= -0.2, -0.1, 0) PWMU (k=0.1); ZJ (k=0.2) NEW1 &PZ(0.3 ≤ k ≤ 0.6), MLE (0.7 ≤ k ≤ 1)

Table 7: Best Methods for Value at Risk at 95% based on least RMSE

Sample size	Non Contaminated	Contaminated
n=20	PICKANDS (k ≤ - 0.6) MPLE (-0.5 ≤ k ≤ -0.2) New1 &PZ (k = -0.1) ZJ(0< k ≤ 0.3) PWMU (k=0.4, 0.5)	MGF (k ≤ - 0.4) MED (k = -0.3) MPLE (k = -0.2) ZJ(0< k ≤ 0.3) PWMU (k=0.4)

n = 40	PICKANDS (-1≤ k ≤ - 0.7) MLE (-0.6 ≤ k ≤ -0.1) WPZ (k=0.1) ZJ(k=0.2) MGF (k = 0.3,0.4,0.5) PICKANDS (0.6≤ k ≤ 1)	PICKANDS (-1≤ k ≤ -0.3) MPLE (k= -0.2, -0.1) NEW2(k=0.1) NEW1& PZ (k=0.2) PICKANDS((0.3≤ k ≤ 1)
n=80	NEW1 &PZ (-1≤ k ≤ -0.3) ZJ (k=-0.2) MPLE(k= -0.1) WPZ(k=0.1) PWMU(k=0.2) ZJ(k=0.3) NEW1 &PZ (0.4≤ k ≤ 1)	MED(-1≤ k ≤ -0.6) NEW1 &PZ (-0.5≤ k ≤ -0.2) MPLE(k= -0.1) WPZ(k=0.1) MPLE(k=0.2) NEW1& PZ (k=0.3,0.4) MED (0.5≤ k ≤ 1)

Table 8: Best Methods for Expected Shortfall at 95% based on least RMSE

Sample size	Non Contaminated	Contaminated
n=20	PICKANDS (k =-1,-0.9) PWM (-0.9 ≤ k ≤ -0.6) MLE (k = -0.5,-0.4,-0.3) PWM (k= 0.1) ZJ(0.2 ≤ k ≤ 0.9)	PICKANDS (-1 ≤ k ≤ -0.8) ZJ (-0.7 ≤ k ≤ -0.3) PWMU (k= 0.1) ZJ(0.2≤ k ≤ 0.9)
n = 40	PICKANDS(k=-1,-0.9) MED(k=-0.8) ZJ(-0.7 ≤ k ≤ -0.2) MDPD(k= -0.1) PWMB(k= 0.1) MLE(k=0.2) PICKANDS(k=0.3,0.4) MED(k=0.5,0.6,0.7,0.8) ZJ (k=-0.9)	PWMU (-1 ≤ k ≤ -0.4) PZ (k=-0.3) MDPD(-0.2,-0.1) PWMB(k=0.31) NEW2 (k=0.2) NEW1(k=0.3) PWMU(k=0.4 ≤ k ≤ 0.9)
n=80	MED(k = -1, -0.9,-0.8) ZJ (-0.7 ≤ k ≤ -0.2) PWMU(k=-0.1) MPLE(k=0.1) MED(k=0.2) PICKANDS (0.3 ≤ k ≤ 0.9)	PWMB(k = -1) PWMU(k = -0.9,-0.8) NEW1 &PZ (-0.7 ≤ k ≤ -0.2) MDPD (k=-0.1) MPLE(k=0.1) MLE(k=0.2) NEW1 & PZ (0.3 ≤ k ≤ 0.9)

Following are the observations from above tables

- It is clear that estimators that perform well for VaR are different from those of shape parameter, which is evident due to nonlinear relationship between VaR and shape parameter.
- It is observed that proposed methods are performing on par with PZ method proposed by Chen.et.al (2017) as loss function selected in our study behaves similar to the one used by Chen.et.al(2017).
- It is also observed that proposed method (NEW1) performs well for certain values of shape parameter in presence of outliers when sample size is sufficiently large.

Though RMSE values are not reported for 98% and 99% confidence levels, in the study we have found that proposed method and PZ method performs identically in estimating VaR when sample size is moderately large.

5. Empirical Study

5.1 S&P 500 Market Index

We have considered S&P 500 stock market index daily returns (log-differences) of the closing values between December 31, 2009 and February 15, 2018 from National Stock Exchange (NSE) which was downloaded from http://www.finance.yahoo.com. As our interest is in comparison of estimation methods in presence of outliers, we covered the period of financial crisis and summary of descriptive statistics are reported below

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Table 9: Descriptive Statistics

Mean	1761.3
Standard Error	10.021
Median	1830.2
Mode	1178.1
Standard Deviation	453.29
Kurtosis	-1.0652
Skewness	0.1637
Count	2046

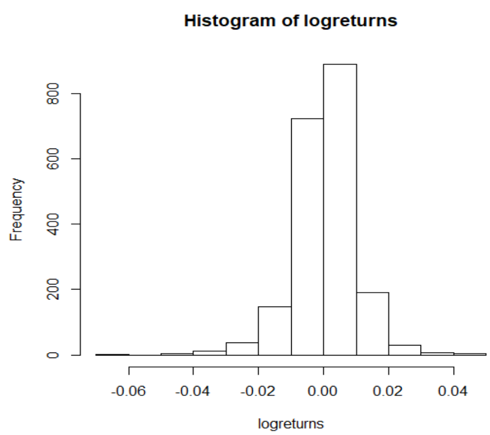


Figure 5: S&P 500Density of daily log returns

5.2 Outlier Detection

In order to study the effect of outliers we applied outlier detection procedures using methods proposed by Chen & Liu (2012) and found that there are thirteen additive outliers and three among them occurs to the right end of the distribution, which confirms the presence of additive outliers among exeedances in our study. The summary of identified additive outliers is reported below along with graphical representation of outlier effects.

Table 10: Summary of additive outliers foundin S&P 500 stock market index

Outliers	log returns	Time	coefhat	t-stat
1	**0.043**	**2009:99**	**0.04244**	**4.887**
2	-0.04	2009:107	-0.0404	-4.65
3	-0.035	2009:117	-0.0356	-4.1
4	-0.049	2010:47	-0.0496	-5.71
5	-0.069	2010:49	-0.0696	-8.01
6	-0.046	2010:57	-0.0462	-5.32
7	-0.037	2010:115	-0.038	-4.37
8	**0.0424**	**2010:129**	**0.04181**	**4.814**
9	-0.04	2012:336	-0.0408	-4.7
10	**0.0383**	**2012:338**	**0.0377**	**4.34**
11	-0.037	2013:182	-0.0372	-4.28
12	-0.042	2014:223	-0.0424	-4.89
13	-0.038	2014:226	-0.0389	-4.47

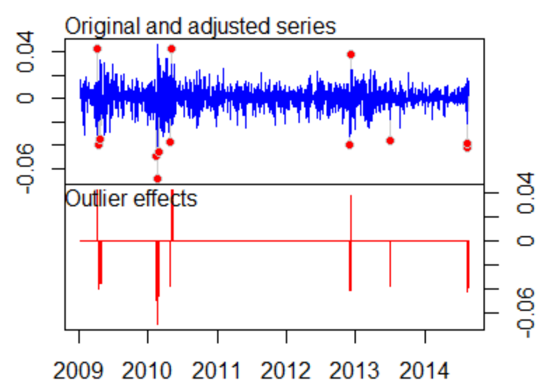
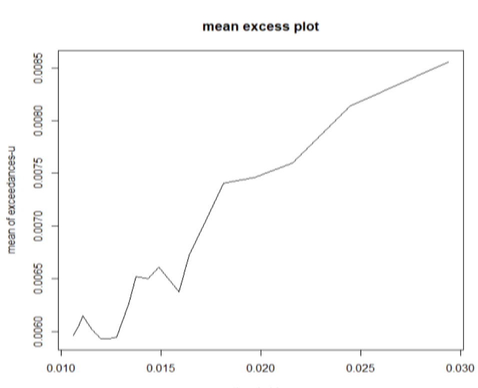


Figure 6: Plot of outlier effects

5.3 Choice of threshold value

It has been observed that the distribution is nearly symmetric and in order to generate exceedances, we applied mean excess plot and zipf plot to approximate threshold value



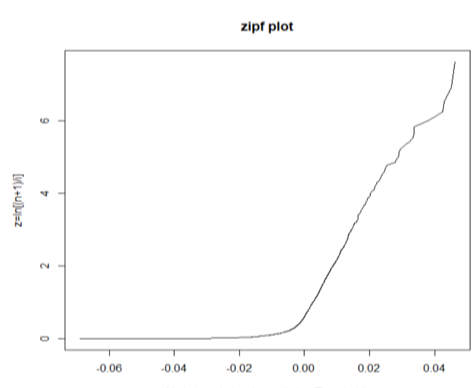


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Figure 7: Threshold Selection Plots

From above plots, we have selected threshold at 98% percentile at u = 0.01967 and summary of exceedances (log returns – threshold) are generated are given below

Table 11: Descriptive Statistics S&P 500 Exceedances

Mean	0.00746
Standard Error	0.00114
Median	0.00479
Standard Deviation	0.00728
Kurtosis	0.95
Skewness	1.324
Count	41

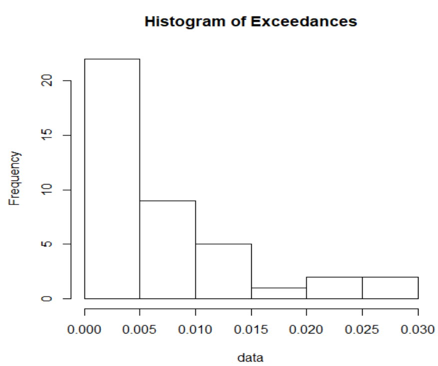

Figure 8. Histogram of Exceedances

5.4 Goodness of fit

We have considered Anderson-Darling statistic (A^2) and Cramer-von Mises statistic (W^2) to measure the goodness of fit

$$W^2 = \sum_{i=1}^{n} \left\{ F(x_{(i)}) - \frac{(2i-1)}{(2n)} \right\}^2 + \frac{1}{12n}$$

$$A^2 = -n - \left(\frac{1}{n}\right) \sum_{i=1}^{n} (2i-1) \left[log \{F(x_{(i)})\} + log \{1 - F(x_{(n+1-i)})\} \right]$$

Goodness of fit statistics and p-values are estimated using parametric bootstrap techniques to compare different methods of estimation in fitting GPD for exceedances and summary of the results are reported in table 5.

Table 12: Goodness of fit statistics

Methods	Robust (Yes/ No)	Parameters		Cramer Von statistics		Anderson Darling statistics	
		Shape	Scale	W^2	P-value	A^2	P-value
New1(R)	Yes	-0.1312	0.0071	0.032	0.953	0.2421	0.831
New 2 (R)	Yes	-0.1311	0.0071	0.033	0.946	0.256	0.95
EBM (NR)	No	-0.0217	0.0073	0.03	0.942	0.252	0.95
PZ (R)	Yes	-0.1545	0.0068	0.0248	0.955	0.2393	**0.966**
WPZ (R)	Yes	-0.1279	0.0069	0.0251	0.93	0.2342	**0.966**
MLE (NR)	No	0	0.0075	13.6667	0	Inf	0
PWMU (NR)	No	0.0608	0.007	0.0528	0.73	0.5168	0.684
PWMB (NR)	No	0.0408	0.0072	0.0403	0.802	0.3816	0.792
PICK (NR)	No	0.0413	0.0068	0.0587	0.9517	0.5487	0.9336
MED (R)	Yes	0.1185	0.0066	0.1171	0.6709	1.2296	0.4815
MDPD (R)	Yes	0.01	0.0075	0.033	**0.9907**	0.2709	**0.9942**
MPLE (NR)	No	0	0.0075	13.6667	0	Inf	0

As p-values are high for all methods (Except for MLE and MPLE), it is clear that Generalized Pareto Distribution is good choice for modelling exceedances generated at u = 0.01967 (m=41, 98th percentile). Among six robust methods five are having p-values more than 0.8 and among six non robust methods

Table 13: Estimates of VaR and Expected shortfall along with in-sample failure rate

Methods	95%			98%			99%		
	VaR	ES	Failure Rate	VaR	ES	Failure Rate	VaR	ES	Failure Rate
NEW1	0.0108	0.0178	0.0919	0.0178	0.0239	0.0347	0.0224	0.0298	0.0196
NEW2	0.0108	0.0178	0.0919	0.0178	0.0239	0.0347	0.0224	0.0298	0.0196
ZJ	0.0125	0.0201	0.0714	0.0195	0.0271	0.0284	0.0248	0.0324	0.0156
PZ	0.0124	0.0196	0.0743	0.0197	0.0256	0.0274	0.0243	0.0294	0.0156
WPZ	0.0124	0.0197	0.0729	0.0196	0.0258	0.0274	0.0244	0.0299	0.0156
MLE	0.0216	0.0293	0.0210	0.0196	0.0273	0.0210	0.0216	0.0293	0.0210
PWMU	0.0132	0.0204	0.0660	0.0196	0.0271	0.0279	0.0247	0.0325	0.0156
PWMB	0.0128	0.0202	0.0675	0.0195	0.0271	0.0284	0.0247	0.0325	0.0156
PICK	0.0100	0.0193	0.0993	0.0190	0.0275	0.0293	0.0253	0.0332	0.0132
MED	0.0131	0.0213	0.0660	0.0194	0.0298	0.0284	0.0252	0.0376	0.0142
MDPD	0.0107	0.0195	0.0924	0.0192	0.0271	0.0284	0.0251	0.0324	0.0142
MPLE	0.0216	0.0293	0.0210	0.0200	0.0277	0.0210	0.0216	0.0293	0.0210

From above table as confidence level $(1-\alpha)$ increases, in-sample failure rates for proposed methods are converging towards expected in-sample failure rates, which indicates proposed methods are performing on par with other methods in estimating Value at Risk for higher values of confidence levels, especially in presence of outliers.

6. Concluding Remarks

The main objective of this study was the investigation of the performances of some estimators of Value at Risk under PoT framework in the presence of outliers when sample size is small and moderately large. With this aim, we proposed a new robust method for estimating VaR& ES and compared its performance with few robust methods which are widely used in estimation of parameters of GPD through simulation and also with non robust methods. Weobservethat all methods considered in simulation study are affected by additive outliers and further it is found that PWM based methods are consistent in estimating shape parameter when sample size is small (n=20) which supports finding reported in Hosking and Wallis (1987) and Castillo and Hadi (1997). Finally in absence of outlier, we conclude that no estimator is performing better than others in estimating shapeparameter over the range of $-1 < k < 1$.

We note that our proposed method is performing on par with PZ method, proposed by P. Chen et.al, (2017) based on minimum distance approach and M-estimation. These methods are performing uniformly better than others in estimation of VaR in presence and absence of outliers when sample size is moderately large, for certain values of $k < 0$. Hence, we conclude that one can use proposed methods for estimating Value at Risk(extreme quantiles) and Expected Shortfall without looking for outliers when the size of exceedances is large under peaks over threshold framework. We believe that above findings and conclusion facilitate investors/practitioners in selecting appropriate method in calculating risk measures wherever outliers are likely to occur.

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