

Critical Path Problem for the Do- Decagonal Neutrosophic Numbers Using New Ranking Method

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Abstract

Neutrosophy is a new branch of philosophy that studies the origin, nature and scope of neutralities as well as their interactions with different ideational spectra. Neutrosophic sets have been introduced as a generalization of crisp sets, fuzzy sets, and intuitionistic fuzzy sets to represent uncertain, inconsistent and incomplete information about real world problems. Elements of neutrosophic set are characterized by a truth-membership, falsity-membership and indeterminacy membership functions. Neutrosophic set theory is applied in multi attribute decision making problems. In this paper, project network in neutrosophic environment is proposed with the introduction of a new ranking method to solve critical path problem where the activity durations are in the form of do-decagonal neutrosophic number. The critical path problem is one of the several related techniques for planning and managing of complicated projects in real world applications. Comparing to the conventional ranking methods we got a better result using the new ranking method. Further an illustrative example is provided to validate the proposed approach.

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I. INTRODUCTION

Neutrosophic sets have been introduced as the generalization of crisp sets, fuzzy sets and Intuitionistic fuzzy sets and is used to represent uncertain, inconsistent and incomplete information about a real-world problem. The concepts of fuzzy and Intuitionistic fuzzy were introduced by H. A. Zadeh[4,15] and S. Atanossosov[3] in the periods 1965 and 1986 respectively. The logic of Neutrosophic has been proposed by F. Smarandache[1,7] which is based on non-standard analysis that was given by Abraham Robinson in 1960. In neutrosophic set indeterminacy is quantified explicitly whereas the truth membership, indeterminacy membership and false membership are independent. Wang et. al[14] introduced the idea of single valued neutrosophic set in many practical problems.

In 2018, Mai Mohammed et.al[11] introduced the problem of obtaining critical path for a network using Triangular Neutrosophic Number. They obtained crisp value for each activity by using score and accuracy function. Under fuzzy environment applications on Pentagonal numbers, Octogonal numbers, Decagonal numbers, Dodecagonal numbers are done by Umamaheswari et. al.,[13], Malini et al.,[2,5,10], Felix et al [6] and Jatinger Pal singh et. al[8]. Recently S. Mullai and R.Surya[12] introduced Neutrosophic Program Evaluation Review Technique.

Project management is concerned with selecting, planning, execution and control of projects in order to meet or exceed stakeholders need or expectation from project[9]. The success any large-scale project is very much dependent upon the quality of the

planning, scheduling, and controlling of various phases of the project.

In many situations, the data obtained for decision makers are only approximate, which gives rise of Neutrosophic critical path problem. Procedure of CPM includes the development work breakdown structure of a project, estimate the resources needed and establish precedence relationship among activities, translation of activities into network and carrying out the construction of network and preparation of schedule of activities.

Here let us introduce dodecagonal neutrosophic numbers and by using this number, critical path and project completion time are obtained for a project network. The proposed method for converting dodecagonal neutrosophic number into a crisp one is the Ranking method

II. PRELIMINARIES

Here, in this section let us recall some basic definitions of Neutrosophic set and Neutrosophic number which are used in the sequel.

A. NEUTROSOPHIC SET

Let U be a universe. A neutrosophic set B in U is described by the Truth membership function TB , indeterminacy membership function IB and falsity membership function FB . These membership functions are real standard elements of $[0, 1]$ and can be written in the form as follows:

$$B = \{ \langle x, (TB(x), IB(x), FB(x)) \rangle : x \in U \text{ and } TB(x), IB(x), FB(x) \in [0, 1] \}$$

$$TB(x), IB(x), FB(x) \in [0, 1]$$

The sum of truth, indeterminacy and falsity membership values have no restrictions.

Therefore, $0 \leq TB(x) + IB(x) + FB(x) \leq 3$.

B. SINGLE VALUED NEUTROSOPHIC SET

Let U be a universe. Let B be a single valued neutrosophic set defined in the universe U , that can

be used in real life situations especially in science and engineering applications described by a Truth membership function TB , an indeterminacy membership function IB , and falsity membership function FB . $TB(x)$, $IB(x)$ and $FB(x)$ are real standard elements of $[0,1]$. It can be written in the form

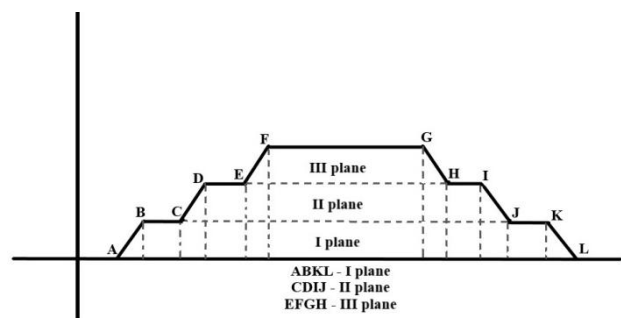
$$B = \{ \langle x; (TB(x), IB(x), FB(x)) \rangle : x \in U \text{ and where } TB(x), IB(x), FB(x) \in [0; 1] \}$$

C. SINGLE VALUED NEUTROSOPHIC NUMBER

Let $\widetilde{B}_1 = \langle T_1, I_1, F_1 \rangle$ be a single valued neutrosophic number. Then, the score function $s(\widetilde{B}_1)$, accuracy function $a(\widetilde{B}_1)$, and certainty function $c(\widetilde{B}_1)$ of a single valued neutrosophic numbers are defined as

1. $s(\widetilde{B}_1) = (T_1 + 1 - I_1 + 1 - F_1)/3$
2. $a(\widetilde{B}_1) = T_1 - F_1$
3. $c(\widetilde{B}_1) = T_1$

D. DO-DECAGONAL NEUTROSOPHIC NUMBER



The Dodecagonal Neutrosophic number is defined as

$$\bar{A} = (a_1, a_2, a_3, \dots, a_{12}; \alpha_a, \theta_a, \beta_a)$$

Where, α_a – Truth Membership Value

θ_a – Indeterminant Membership Value

β_a - False Membership Value

The Truth membership function, Indeterminacy membership function and falsity membership function are given as follows:

$T_a(x)$ – Truth membership function

$I_a(x)$ – Indeterminacy membership function

$F_a(x)$ – Falsity membership function

TRUTH MEMBERSHIP FUNCTION:

$$T_a(x) = \begin{cases} 0, & \text{for } x < a_1 \\ k_1 \left(\frac{x-a_1}{a_2-a_1} \right), & \text{for } a_1 \leq x \leq a_2 \\ k_1, & \text{for } a_2 \leq x \leq a_3 \\ k_1 + (\alpha_a - k_1) \left(\frac{x-a_3}{a_4-a_3} \right), & \text{for } a_3 \leq x \leq a_4 \\ k_2, & \text{for } a_4 \leq x \leq a_5 \\ k_2 + (\alpha_a - k_2) \left(\frac{x-a_5}{a_6-a_5} \right), & \text{for } a_5 \leq x \leq a_6 \\ \alpha_a, & \text{for } a_6 \leq x \leq a_7 \\ k_2 + (\alpha_a - k_2) \left(\frac{a_8-x}{a_8-a_7} \right), & \text{for } a_7 \leq x \leq a_8 \\ k_2, & \text{for } a_8 \leq x \leq a_9 \\ k_1 + (\alpha_a - k_1) \left(\frac{a_{10}-x}{a_{10}-a_9} \right), & \text{for } a_9 \leq x \leq a_{10} \\ k_1, & \text{for } a_{10} \leq x \leq a_{11} \\ k_1 \left(\frac{a_{12}-x}{a_{12}-a_{11}} \right), & \text{for } a_{11} \leq x \leq a_{12} \\ 0, & \text{for } a_{12} \leq x \end{cases}$$

$$F_a(x) = \begin{cases} 1, & \text{for } x < a_1 \\ 1 + (1-k_1) \left(\frac{a_1-x}{a_2-a_1} \right), & \text{for } a_1 \leq x \leq a_2 \\ k_1, & \text{for } a_2 \leq x \leq a_3 \\ k_1 + (k_1 - \beta_a) \left(\frac{a_3-x}{a_4-a_3} \right), & \text{for } a_3 \leq x \leq a_4 \\ k_2, & \text{for } a_4 \leq x \leq a_5 \\ k_2 + (k_2 - \beta_a) \left(\frac{a_5-x}{a_6-a_5} \right), & \text{for } a_5 \leq x \leq a_6 \\ 0, & \text{for } a_6 \leq x \leq a_7 \\ \beta_a + (k_2 - \beta_a) \left(\frac{x-a_7}{a_8-a_7} \right), & \text{for } a_7 \leq x \leq a_8 \\ k_2, & \text{for } a_8 \leq x \leq a_9 \\ k_2 + (\beta_a - k_2) \left(\frac{x-a_9}{a_{10}-a_9} \right), & \text{for } a_9 \leq x \leq a_{10} \\ k_1, & \text{for } a_{10} \leq x \leq a_{11} \\ k_1 + (\beta_a - k_1) \left(\frac{x-a_{11}}{a_{12}-a_{11}} \right), & \text{for } a_{11} \leq x \leq a_{12} \\ 1, & \text{for } x \geq a_{12} \end{cases}$$

$$I_a(x) = \begin{cases} 1, & \text{for } x < a_1 \\ 1 + (1-k_1) \left(\frac{a_1-x}{a_2-a_1} \right), & \text{for } a_1 \leq x \leq a_2 \\ k_1, & \text{for } a_2 \leq x \leq a_3 \\ k_1 + (k_1 - \theta_a) \left(\frac{a_3-x}{a_4-a_3} \right), & \text{for } a_3 \leq x \leq a_4 \\ k_2, & \text{for } a_4 \leq x \leq a_5 \\ k_2 + (k_2 - \theta_a) \left(\frac{a_5-x}{a_6-a_5} \right), & \text{for } a_5 \leq x \leq a_6 \\ 0, & \text{for } a_6 \leq x \leq a_7 \\ \theta_a + (k_2 - \theta_a) \left(\frac{x-a_7}{a_8-a_7} \right), & \text{for } a_7 \leq x \leq a_8 \\ k_2, & \text{for } a_8 \leq x \leq a_9 \\ k_2 + (\theta_a - k_2) \left(\frac{x-a_9}{a_{10}-a_9} \right), & \text{for } a_9 \leq x \leq a_{10} \\ k_1, & \text{for } a_{10} \leq x \leq a_{11} \\ k_1 + (\theta_a - k_1) \left(\frac{x-a_{11}}{a_{12}-a_{11}} \right), & \text{for } a_{11} \leq x \leq a_{12} \\ 1, & \text{for } a_{12} \leq x \end{cases}$$

Where $0 < k_1 < k_2 < 1$

III. DO-DECAGONAL NEUTROSOPHIC NUMBER

A. CONVENTIONAL RANKING METHOD

Definition: Let $\bar{A}$ be a normal do-decagonal neutrosophic number. The measure of $\bar{A}$ is

$$R(\bar{A}) = \frac{1}{4} [(a_1 + a_2 + a_{11} + a_{12})k_1 + (a_3 + a_4 + a_9 + a_{10})(k_2 - k_1) + (a_5 + a_6 + a_7 + a_8)(1 - k_2)] \times \left(\frac{2 + \alpha_a - \theta_a - \beta_a}{3} \right) - (1)$$

, where $0 < k_1 < k_2 < 1$

B. PROPOSED NEW RANKING METHOD

Definition: Let $\bar{A}$ be the do-decagonal neutrosophic number. The measure of $\bar{A}$ is

$$R'(\bar{A}) = \frac{1}{12} [(a_1 + a_{12}) + K_1(a_2 + a_3 + a_{10} + a_{11}) + K_2(a_4 + a_5 + a_8 + a_9) + (a_6 + a_7)] \times \left(\frac{2 + \alpha_a - \theta_a - \beta_a}{3} \right) - (2)$$

and where $0 < k_1 < k_2 < 1$

IV. ALGORITHM: CPM

CPM is the set of activities that must be executed according to precedence constraints determining which activities must establish after the achievement of stated other activities. Let us describe some terms used in sketching network diagram of CPM:

- Activity: It is part of a project that has a distinct starting and finishing.
- Event or Node: Starting and finishing points of activities denoted by circles are called nodes or events.
- Critical Path: Is known as the longest Path.

The complication of finding critical path is simple in a network with distinct crisp duration of activities. So, we convert the do-decagonal Neutrosophic duration into a crisp duration.

The four parameters of the activity are used to find the Critical path. The four parameters are

- Earliest Start (ES)- The time at where it start at the completion of precedence activities.
- Earliest Finish (EF)- Addition of the activity's earliest start and the time required for completion of the activity
- Latest Finish (LF)- time at where the completion of the activity without project delay
- Latest Start (LS)- difference of the time of latest finish and the time required for the completion of the activity

The Slack time for the activity is the time between the earliest start/finish time and latest start/finish time. The path in which the activities without slack time in the diagram of network, that is, ES=LS and EF=LF is known as Critical path.

Let us discuss the proposed algorithm as follows:

STEP 1: To work with uncertain or inconsistent information of activity time, we determined activity time as Do-decagonal Neutrosophic number.

Activity A: {1,2,4,5,7,8,11,13,14,16,19,20;0.6,0.5,0.4}

$$R_1 = \frac{1}{12} [(1+20) + 0.5(2+4+16+19) + 0.7(5+7+13+14) + (8+11)] \times \left(\frac{2+0.6-0.5-0.4}{3} \right)$$

$R_1 = 4.15$

STEP 2: Convert the Do-decagonal Neutrosophic duration activity into a crisp duration activity using the equation (1) and (2).

STEP 3: Draw CPM network diagram using the crisp duration.

STEP 4: Determine the Critical Path, the network's longest path.

STEP 5: Determine the expected project completion time.

V. ILLUSTRATIVE EXAMPLE

A. Using the proposed new ranking method we convert the Do-decagonal Neutrosophic activity time into crisp value.

Assume: $K_1 = 0.5$;

$K_2 = 0.7$

Activity	Dodecagonal Neutrosophic Activity Time	Immediate predecessor
A	{1,2,4,5,7,8,11,13,14,16,19,20;0.6,0.5,0.4}	-
B	{2,3,4,5,7,8,10,11,13,14,16,17;0.7,0.6,0.5}	-
C	{1,3,5,7,8,9,11,12,13,15,17,18;0.8,0.5,0.3}	A
D	{2,4,5,6,8,9,10,12,14,15,16,17;0.6,0.3,0.5}	B
E	{3,5,6,7,9,10,12,13,15,16,17,19;0.3,0.5,0.4}	B
F	{2,3,4,7,9,10,13,16,18,19,21,22;0.8,0.3,0.2}	C
G	{3,4,6,8,9,11,12,14,15,16,18,20;0.8,0.5,0.7}	C
H	{4,5,7,8,9,11,12,13,16,18,19,21;0.5,0.6,0.4}	D
I	{1,4,5,7,9,10,11,13,14,15,19,20;0.8,0.5,0.3}	E,F

Activity B: {2,3,4,5,7,8,10,11,13,14,16,17;0.7,0.6,0.5}

$$R_2 = \frac{1}{12} [(2+17)+0.5(3+4+14+16)+0.7(5+7+11+13)+$$

$$(8+10)] \times \left(\frac{2+0.7-0.6-0.5}{3} \right)$$

$$R_2 = 3.59$$

Activity C: {1,3,5,7,8,9,11,12,13,15,17,18;0.8,0.5,0.3}

$$R_3 = \frac{1}{12} [(1+18)+0.5(3+5+15+17)+0.7(7+8+12+13)+(9+11)] \times \left(\frac{2+0.8-0.5-0.3}{3} \right)$$

$$R_3 = 4.83$$

Activity D: {2,4,5,6,8,9,10,12,14,15,16,17;0.6,0.3,0.5}

$$R_4 = \frac{1}{12} [(2+17)+0.5(4+5+15+16)+0.7(6+8+12+14)+$$

$$(9+10)] \times \left(\frac{2+0.6-0.3-0.5}{3} \right)$$

$$R_4 = 4.3$$

Activity E: {3,5,6,7,9,10,12,13,15,16,17,19;0.3,0.5,0.4}

$$R_5 = \frac{1}{12} [(3+19)+0.5(5+6+16+17)+0.7(7+9+13+15)$$

$$+(10+12)] \times \left(\frac{2+0.3-0.5-0.4}{3} \right)$$

$$R_5 = 3.75$$

Activity F: {2,3,4,7,9,10,13,16,18,19,21,22;0.8,0.3,0.2}

$$R_6 = \frac{1}{12} [(2+22)+0.5(3+4+19+21)+0.7(7+9+16+18)$$

$$+(10+13)] \times \left(\frac{2+0.8-0.3-0.2}{3} \right)$$

$$R_6 = 6.77$$

Activity G: {3,4,6,8,9,11,12,14,15,16,18,20;0.8,0.5,0.7}

$$R_7 = \frac{1}{12} [(3+20)+0.5(4+6+16+18)+0.7(8+9+14+15)+(11+12)] \times \left(\frac{2+0.8-0.5-0.7}{3} \right)$$

$$R_7 = 4.43$$

Activity H: {4,5,7,8,9,11,12,13,16,18,19,21;0.5,0.6,0.4}

$$R_8 = \frac{1}{12} [(4+21)+0.5(5+7+18+19)+0.7(8+9+13+16)+(11+12)] \times \left(\frac{2+0.5-0.6-0.4}{3} \right)$$

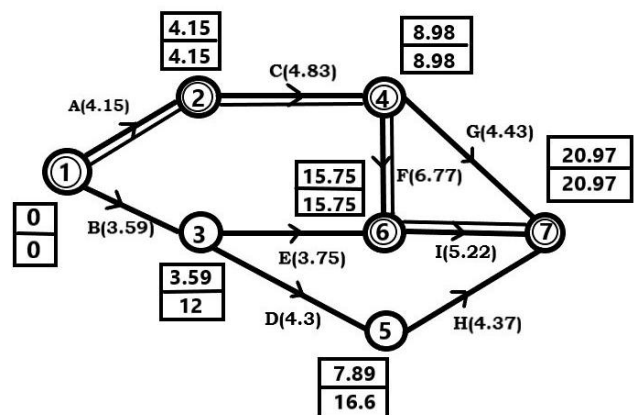
$$R_8 = 4.37$$

Activity I: {1,4,5,7,9,10,11,13,14,15,19,20;0.8,0.5,0.3}

$$R_9 = \frac{1}{12} [(1+20)+0.5(4+5+15+19)+0.7(7+9+13+14)+(10+11)] \times \left(\frac{2+0.8-0.5-0.3}{3} \right)$$

$$R_9 = 5.22$$

Activity	Activity Time	Immediate predecessors
A	4.15	-
B	3.59	-
C	4.83	A
D	4.3	B
E	3.75	B
F	6.77	C
G	4.43	C
H	4.37	D
I	5.22	E,F



$$1 \rightarrow 2 \rightarrow 4 \rightarrow 6 \rightarrow 7 = A \rightarrow C \rightarrow F \rightarrow I$$

$$\begin{aligned} \text{Project completion time} &= \text{Time of A}(4.15) + \text{Time of C}(4.83) + \text{Time of F}(6.77) + \text{Time of I}(5.22) \\ &= 4.15 + 4.83 + 6.77 + 5.22 = \mathbf{20.97} \end{aligned}$$

A. Using the conventional ranking method we convert the given Do-decagonal Neutrosophic activity time into a crisp value by same values of k_2, k_1 in new ranking method.

$$\text{Activity A: } R_1 = \frac{1}{4}[(1+2+19+20)0.5 + (4+5+14+16)(0.7-0.5) + (7+8+11+13)(1-0.7)] \times \left(\frac{2+0.6-0.5-0.4}{3}\right)$$

$$\mathbf{R_1 = 5.77}$$

Activity B:

$$R_2 = \frac{1}{4}[(2+3+16+17)0.5 + (4+5+13+14)0.2 + (7+8+10+11)0.3] \times \left(\frac{2+0.7-0.6-0.5}{3}\right)$$

$$\mathbf{R_2 = 4.9}$$

Activity C:

$$R_3 = \frac{1}{4}[(1+3+17+18)0.5 + (5+7+13+15)0.2 + (8+9+11+12)0.3] \times \left(\frac{2+0.8-0.5-0.3}{3}\right)$$

$$\mathbf{R_3 = 6.62}$$

Activity D:

$$R_4 = \frac{1}{4}[(2+4+16+17)0.5 + (5+6+14+15)0.2 + (8+9+10+12)0.3] \times \left(\frac{2+0.6-0.3-0.5}{3}\right)$$

$$\mathbf{R_4 = 5.88}$$

Activity E:

$$R_5 = \frac{1}{4}[(3+5+17+19)0.5 + (6+7+15+16)0.2 + (9+10+12+13)0.3] \times \left(\frac{2+0.3-0.5-0.4}{3}\right)$$

$$\mathbf{R_5 = 5.17}$$

Activity F:

$$R_6 = \frac{1}{4}[(2+3+21+22)0.5 + (4+7+18+19)0.2 + (9+10+13+16)0.3] \times \left(\frac{2+0.8-0.3-0.2}{3}\right)$$

$$\mathbf{R_6 = 9.24}$$

Activity G:

$$R_7 = \frac{1}{4}[(3+4+18+20)0.5 + (6+8+15+16)0.2 + (9+11+12+14)0.3] \times \left(\frac{2+0.8-0.5-0.7}{3}\right)$$

$$\mathbf{R_7 = 6}$$

Activity H:

$$R_8 = \frac{1}{4}[(4+5+19+21)0.5 + (7+8+16+18)0.2 + (9+11+12+13)0.3] \times \left(\frac{2+0.5-0.6-0.4}{3}\right)$$

$$R_8=5.98$$

Activity I:

$$R_9=\frac{1}{4}[(1+4+19+20)0.5+(5+7+14+15)0.2+(9+10+11+13)0.3] \times \left(\frac{2+0.8-0.5-0.3}{3}\right)$$

$$R_9= 7.2$$

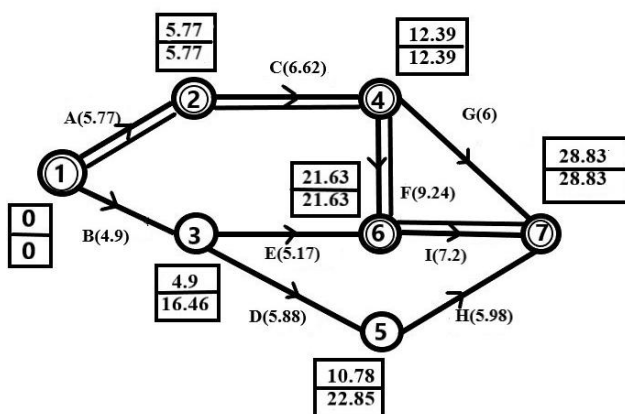
CONCLUSION

In this paper, we have taken the Do-decagonal Neutrosophic numbers and converted them into crisp value by using new ranking method and also by conventional ranking method. Further the same is illustrated to obtain CPM. Compared to the conventional ranking method we got a better result using the new ranking method.

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Activity	Activity Time	Immediate predecessors
A	5.77	-
B	4.9	-
C	6.62	A
D	5.88	B
E	5.17	B
F	9.24	C
G	6	C
H	5.98	D
I	7.2	E,F



Determine the Critical Path,

$$1 \rightarrow 2 \rightarrow 4 \rightarrow 6 \rightarrow 7 = A \rightarrow C \rightarrow F \rightarrow I$$

Project completion time=Time of A(5.77)+Time of C(6.62)+Time of F(9.24)+Time of I(7.2)

$$= 5.77+6.62+9.24+7.2= \mathbf{28.83}$$

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