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Dark Energy Anisotropic Cosmological Models" with Time Dependent Bulk Viscosity, G and Λ

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Abstract

Assuming the existence of generalized Chaplygin gas with equation of state $p = \frac{-\beta}{\rho^{\alpha}}$ with $0 \le \alpha \le 1$, we obtained Bianchi type III cosmological model with variable bulk viscosity and time varying gravitational and cosmological constants. For homogeneous and anisotropic Bianchi type III cosmological model exact solutions of Einstein equations have been obtained. In this paper with the help of various graphs physical and dynamical behaviors of the model have been discussed.

Keywords; *Anisotropic Bianchi type III, Gravitational Constant, Cosmological Constant, Bulk viscosity, Generalized Chaplygin Gas.*

I. INTRODUCTION

"Friedmann-Robertson-Walker" (FRW) model is the simplest model of expanding universe. In the early stage the universe was anisotropic which turns into isotropic phase. This fact is proved by recent experimental data. It is extensively believed that FRW model does explain correctly the early stage of universe. Whereas anisotropic models plays significant role in explanation of the universe. Bianchi model I to IX present a mid way between FRW model (homogeneous and isotropic) and inhomogeneous and anisotropic universe. Singh et.al. [1] have investigated Bianchi type III cosmological models including time varying G and Lambda. Chakraborty and Ray [2] have presented Bianchi type I, Bianchi type III and Kantowski-Sachs space-time models with bulk viscosity, where G and are the gravitational "constant" and cosmological "constant" respectively diverge with respect to time.

In order to obtain static solutions of Einstein's original equations, he customized the field equations by adding a new term know as cosmological constant. Cosmological constant plays the role of repulsive force which can counter balance attractive force of gravity. During early stages of universe the value of was large and strongly influenced the expansion of the universe. A wide range of observations favours small positive value of effective cosmological "constant" at present era. Cosmological constant problem and its consequences on cosmology with time varying have been studied by some researchers [3-7].

Dirac's thought of time dependent G attracts the scientific community's attention [8]. The thought of time dependent G supports the theory of expanding universe. G varying cosmology is steady with observations revealed by Canuto and Narlikar [9]. Several authors [10-19] have studied the variation of Gravitational and Cosmological constant, in

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different context within the frame work of general relativity.

Dissipative effect including bulk viscosity take part a very important role during evolution of the universe, hence is the subject of growing importance. In the literature [20] Cosmological models with perfect fluid have been measured as a source of matter in the context of higher derivative theories. Although the perfect fluid provides satisfactory matter distribution in the universe.A effect of bulk viscosity on the evolution of FRW model is investigated by Kalyani [21]. Bali and Dave [22] discussed Bianchi type III string cosmological model with constant bulk viscosity. Singh and Kotambkar [23] obtained the dissipative effect in higher dimensional model for flat universe with time varying gravitational and cosmological "constant".

The most outstanding discovery of the past decade is that the universe is expanding. The source of this accelerated expansion was dubbed as "dark energy". To date the character of this dark energy several possible candidates have been extensively discussed in the literature. For recent review on candidate for dark energy refer [24-25]. Recently it has been shown that to overcome the problems associated with and quintessence models, Chaplygin gas may be useful. The Chaplygin gas first introduced by Chaplygin [26] Due to negative pressure of Chaplygin gas it had recently gained new attention in cosmology. The Chaplygin gas can be expressed in the form of equation of state as:

$$p = \frac{-B}{\rho} ,$$

where B is a positive constant.

Since Chaplygin gas is effective to explain the development with time in the universe therefore a number of generalizations of Chaplygin gas have been projected in the literature. The observational evidence based on the equation of state (EoS) for generalized Chaplygin gas is also very encouraging [27]. Chaplygin gas can be referred as generalized Chaplygin gas [28] by the addition of an random constant as supporter over the mass density [29-30]. The generalized Chaplygin gas is expressed as EoS given by

$$p = \frac{-B}{\rho^{\alpha}}$$ with $0 \le \alpha \le 1$.

Recently Panigrahi [31] has investigated FRW type of space-time with generalized Chaplygin gas. Several researchers [32-38] have considered EoS of generalized Chaplygin gas to construct viable Cosmological model. Very recently Kotambkaret. al. [39] have investigated Bianchi type I model (anisotropic cosmological model) with generalized Chaplygin gas and dynamical G and Lambda. The generalized Chaplygin gas model is modified as Modified generalized Chaplygin gas model with EoS as $p = A\rho - \frac{B}{\rho^{\alpha}}$ where A is positive constant [40].

II. FIELD EQUATION:

We consider the Bianchi type III metric in the form

$$ds^2 = -dt^2 + R_1^2\, dx^2 + R_2^2\, e^{-2\alpha x}\, dy^2 + R_3^2\, dz^2 \qquad (1)$$

R_1, R_2, R_3 are function of t alone.

Einstein's field equations including G and Lambda (Λ) for perfect fluid distribution may be expressed as

$$R_{ij} - \frac{1}{2} R\, g_{ij} = -8\pi G T_{ij} + \Lambda\, g_{ij} \quad (2)$$

where G is time dependent constant of gravity and Lambda (Λ) is time dependent cosmological constants.

For viscous fluid distribution the energy momentum tensor T_{ij} can be expressed as

$$T_{ij} = (\rho + P) u_i u_j + P\, g_{ij}, \qquad (3)$$

where $P = p + \Pi$. (4)

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where Π is bulk viscous stress , p is equilibrium pressure together with $u_i u^j = 1$.

Field equation of Einstein's (2) with metric (1) indicates the following equations

$$\frac{\ddot{R}_2}{R_2}+\frac{\ddot{R}_3}{R_3}+\frac{\dot{R}_2\,\dot{R}_3}{R_2\,R_3}=-8\pi\,G(p+\Pi)+\Lambda, \quad (5)$$

$$\frac{\ddot{R}_1}{R_1}+\frac{\ddot{R}_3}{R_3}+\frac{\dot{R}_1\,\dot{R}_3}{R_1\,R_3}=-8\pi\,G(p+\Pi)+\Lambda, \quad (6)$$

$$\frac{\ddot{R}_1}{R_1}+\frac{\ddot{R}_2}{R_2}+\frac{\dot{R}_1\,\dot{R}_2}{R_1\,R_2}=-8\pi\,G(p+\Pi)+\Lambda, \quad (7)$$

$$\frac{\dot{R}_1\,\dot{R}_2}{R_1R_2}+\frac{\dot{R}_2\dot{R}_3}{R_2R_3}+\frac{\dot{R}_1\dot{R}_3}{R_1R_3}-\frac{1}{R_1^2}=8\pi\,G\rho+\Lambda, \quad (8)$$

$$\frac{\dot{R}_1}{R_1}-\frac{\dot{R}_2}{R_2}=0. \quad (9)$$

In the above equations the dots on $R_1, R_2 \,\&\, R_3$ denotes derivatives with respect to cosmic time. Through divergence of the Einstein tensor i. e. $\left(R_{ij}-\frac{1}{2}R\,g_{ij}\right)_{;j}=0$ an extra equation for the time variant G and Λ is obtained which directed to $\left(8\pi\,GT_{ij}-\Lambda g_{ij}\right)_{;j}=0$, yielding

$$8\pi\,\dot{G}\rho+\dot{\Lambda}+8\pi\,G\left[\dot{\rho}+(\rho+p+\Pi)\left(\frac{\dot{R}_1}{R_1}+\frac{\dot{R}_2}{R_2}+\frac{\dot{R}_3}{R_3}\right)\right]=0. \quad (10)$$

From equation (10), we get two equations as

$$\dot{\rho}+(\rho+p)\left(\frac{\dot{R}_1}{R_1}+\frac{\dot{R}_2}{R_2}+\frac{\dot{R}_3}{R_3}\right)=0, \quad (11)$$

$$\dot{\Lambda}+8\pi\,\dot{G}\rho=-8\pi\,G\Pi\left(\frac{\dot{R}_1}{R_1}+\frac{\dot{R}_2}{R_2}+\frac{\dot{R}_3}{R_3}\right). \quad (12)$$

The causal evolution equation for bulk viscosity in full causal non- equilibrium thermodynamics is expressed as

$$\Pi+\tau\,\dot{\Pi}=-\eta\left(\frac{\dot{R}_1}{R_1}+\frac{\dot{R}_2}{R_2}+\frac{\dot{R}_3}{R_3}\right)-\frac{\varepsilon\,\tau\,\Pi}{2}\left(\frac{\dot{R}_1}{R_1}+\frac{\dot{R}_2}{R_2}+\frac{\dot{R}_3}{R_3}+\frac{\dot{\tau}}{\tau}-\frac{\dot{\eta}}{\eta}-\frac{\dot{T}}{T}\right) \quad (13)$$

Where $T \geq 0$ absolute temperature, is always positive, τ indicate the relaxation time for transient bulk viscous effects. For causal solution $\tau > 0$. For $\varepsilon = 0$ and $\varepsilon = 1$ equation (13) trim down to evolution equation for truncated theory and full causal theory respectively. For $\tau = 0$ equation (13) trim down to evolution equation for non-causal theory (Eckart's theory).

III. COSMOLOGICAL SOLUTIONS:

For solving the set of equations three more physically plausible relations will required as we have only five essential equations (5) to (9) and eight unknowns viz. $R_1, R_2, R_3, p, \rho, G, \Lambda$ and Π

Case I: Cosmological Solution in Non- Causal case

In non causal solution $\tau = 0$, so the evolution equation (13) can be expressed as

$$\Pi=-\eta\left(\frac{\dot{R}_1}{R_1}+\frac{\dot{R}_2}{R_2}+\frac{\dot{R}_3}{R_3}\right)=-3\eta H. \quad (14)$$

In order to get absolute solution for the suggested system, some physically reasonable relations are considered.

The relation between bulk viscosity and energy density is taken as

$$\eta=\eta_0\,\rho^r, \quad (15)$$

$\eta_0 > 0$ and r is random constant.

The EoS for Chaplygin gas is given by

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$$p = \frac{-B}{\rho^{\alpha}}, \quad 0 < \alpha \le 1, \; B > 0 \quad (16)$$

The solution which is suggested for the present model as

$$\frac{\dot{R}_1}{R_1} = \frac{\dot{R}_2}{R_2} = \frac{\dot{R}_3}{R_3} = \frac{m}{t^n} \quad . \quad (17)$$

where n (Constant) is less than one.

On integrating equation (17), we get

$$R_1 = R_2 = R_3 = k \exp\left\{\frac{m t^{1-n}}{1-n}\right\} \quad (18)$$

where k is integrating constant.

By use of equations (16)-(17) in (11), we obtain

$$\dot{\rho} + \rho \frac{3m}{t^n} = \frac{3Bm}{t^n \rho^{\alpha}} \quad , \quad (19)$$

which on solving yields

$$\rho = \left[B + C \exp\left\{\frac{-3m(\alpha+1)}{1-n} t^{1-n}\right\}\right]^{\frac{1}{\alpha+1}}. \quad (20)$$

where C is integrating constant..

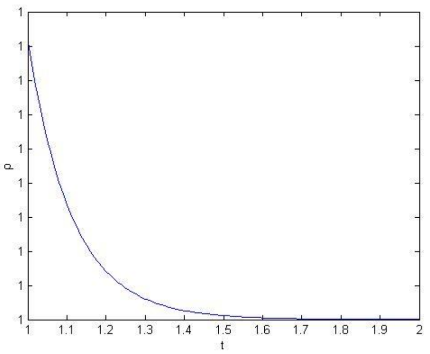

Fig. 1: Shows the nature of energy density ρ with respect to cosmic time t. Here we consider B = 1, C = 1, n = 0.25, m = 2 and $\alpha = 0.5$

From the above equation and fig. 1 it is clear that energy density is indirectly proportional to cosmic time t which is in fair agreement.

By use of equation (20), one can get

$$\dot{\rho} = \frac{-3mC}{t^n} \exp\{D t^{1-n}\}[B + C \exp\{D t^{1-n}\}]^{\frac{-\alpha}{\alpha+1}} \quad (21)$$

where $D = -\frac{3m(\alpha+1)}{1-n}$.

Now using equations (17) and (18), equation (8) becomes

$$\frac{3m^2}{t^{2n}} - \frac{1}{k^2 \exp\left(\frac{2m}{1-n} t^{1-n}\right)} = 8\pi G \rho + \Lambda \quad , \quad (22)$$

which on differentiation leads to

$$\frac{-6nm^2}{t^{2n+1}} + \frac{2m}{k^2 t^n} \exp\left(\frac{-2m}{1-n} t^{1-n}\right) = 8\pi \dot{G} \rho + 8\pi G \dot{\rho} + \dot{\Lambda} \quad (23)$$

By use of equations (12), (14), (17) and (23), we obtain

$$\frac{-6nm^2}{t^{2n+1}} + \frac{2m}{k^2 t^n} \exp\left(\frac{-2m}{1-n} t^{1-n}\right) = 8\pi G \dot{\rho} - 8\pi G \Pi \left(\frac{3m}{t^n}\right) \quad (24)$$

Equation (24) with the help of (15) and (21) produces

$$G = \frac{\left[-6nm^2 t^{-(2n+1)} + 2mk^{-2} t^{-n} \exp\left(D_1 t^{1-n}\right)\right] t^{2n}}{8\pi k^2 t \left[9m^2 \eta_0 \left(B + C \exp\left(D t^{1-n}\right)\right)^{\frac{r}{\alpha+1}} - 3m t^n \exp\left(D t^{1-n}\right)\left(B + C \exp\left(D t^{1-n}\right)\right)^{\frac{-\alpha}{\alpha+1}}\right]}, \quad (25)$$

where $D_1 = \frac{-2m}{1-n}$.

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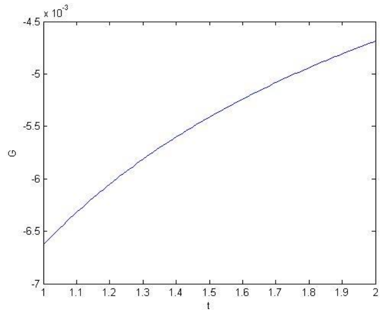

Fig. 2: shows the nature of Gravitational constant with respect to cosmic time t. Here we consider B = 1, C = 1, n = 0.25, m = 2, $\alpha = 0.5$, r = 1.5, and $\eta_0 = 1$

By use of equations (20) & (25), equation (22) gives

$$\Lambda = 3m^2 t^{-2n} - k^{-2}\exp\left(D_1 t^{1-n}\right) - \frac{\left[-6nm^2 t^{-(2n+1)} + 2mk^{-2}t^{-n}\exp\left(D_1 t^{1-n}\right)\right]t^{2n}}{k^2 t\left[9m^2\eta_0\left(B + C\exp\left(Dt^{1-n}\right)\right)^{\frac{r-1}{\alpha+1}} - 3mt^n\exp\left(Dt^{1-n}\right)\left(B + C\exp\left(Dt^{1-n}\right)\right)^{-1}\right]},$$

(26)

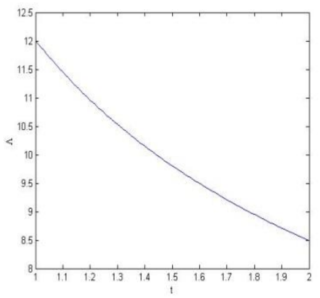

Fig. 3: shows deviation of cosmological constant with respect to cosmic time t. Here we consider B = 1, C = 1, n = 0.25, m = 2, $\alpha = 0.5$, r = 1.5, a = 1 and $\eta_0 = 1$

From equation (26) and fig. 3 shows that the deviation of G with respect to time, which goes with the observations.

Now from equations (15) and (20), we have

$$\eta = \eta_0\left[B + C\exp\left(Dt^{1-n}\right)\right]^{\frac{r}{\alpha+1}} \qquad (27)$$

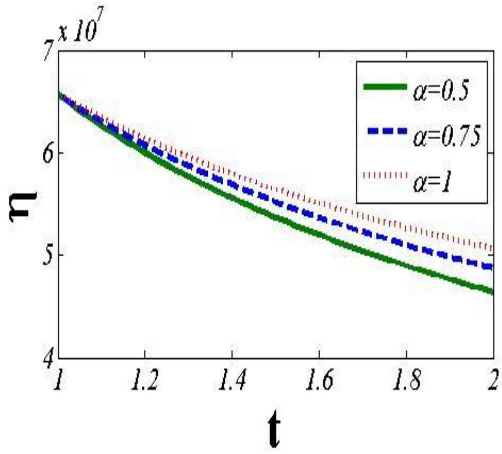

Fig. 4: shows nature of bulk viscosity coefficient with respect to cosmic time t. Here we consider B = 1, C = 1, n = 0.25, m = 2, $\alpha = 0.5, 0.75, 1$, r = 1.5.

Fig. 4 represents that the bulk viscosity coefficient is inversely proportional to cosmic time t which goes with observations.

With the help of equations (14) and (17), bulk viscous stress expression is given by

$$\Pi = \frac{-3\eta_0 m}{t^n}\left[B + C\exp\left(Dt^{1-n}\right)\right]^{\frac{r}{\alpha+1}} \qquad (28)$$

Equation (1) and (18) gives

$$ds^2 = -dt^2 + k^2\exp\left\{\frac{-2mt^{1-n}}{1-n}\right\}\left(dx^2 + e^{2x}dy^2 + dz^2\right) \qquad (29)$$

The shear scalar is expressed as

$$\sigma^2 = \frac{1}{12}\left[\left(\frac{\dot{g}_{11}}{g_{11}} - \frac{\dot{g}_{22}}{g_{22}}\right)^2 + \left(\frac{\dot{g}_{22}}{g_{22}} - \frac{\dot{g}_{33}}{g_{33}}\right)^2 + \left(\frac{\dot{g}_{33}}{g_{33}} - \frac{\dot{g}_{11}}{g_{11}}\right)^2\right] \qquad (30)$$

The Shear scalar for our model is given by

$$\sigma^2 = 0 \qquad (31)$$

The expansion scalar is defined by

$$\Theta = 3H, \quad H = \frac{\dot{R}}{R} \qquad (32)$$

expansion scalar For this model is given by

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$$\Theta = \frac{3m}{t^n} \tag{33}$$

The relation between deceleration parameter and expansion scalar is given by

$$q = -\frac{3\dot{\Theta} + \Theta^2}{\Theta^2}\ , \tag{34}$$

For this model using (33) and (34) the deceleration parameter takes the form

$$q = \frac{n}{m} t^{n-1} - 1 \tag{35}$$

Since universe has accelerating expansion, q (deceleration parameter) should be negative.

From equation (35) it is clear that if n < 1

$q \to -1 \ as \ t \to \infty$

Relative anisotropy = $\dfrac{\sigma^2}{\rho} = 0$ (36)

$\dfrac{\sigma}{\Theta} = 0 =$ Constant (37)

The expression for critical energy density is

$\rho_c = \dfrac{3H^2}{8\pi G}$ and

The expression for critical vacuum energy density is

$\rho_v = \dfrac{\Lambda}{8\pi G}$,

For this model, can be respectively expressed as

$$\rho_c = \frac{3m^2k^2\, t\left[\ 9m^2\eta_0\left(B + C\exp\left(Dt^{1-n}\right)\right)^{\frac{r}{\alpha+1}} - 3mt^n \exp\left(Dt^{1-n}\right)\left(B + C\exp\left(Dt^{1-n}\right)\right)^{\frac{-\alpha}{\alpha+1}}\right]}{\left[-6nm^2t^{-(2n+1)} + 2mk^{-2}t^{-n}\exp\left(D_1t^{1-n}\right)\right]t^{2n}}$$

938)

(38)

$$\rho_v = \frac{\left[3m^2t^{-2n} - k^{-2}\exp\left(D_1t^{1-n}\right)\right]\left[\ 9m^2\eta_0\left(B + C\exp\left(Dt^{1-n}\right)\right)^{\frac{r}{\alpha+1}} - 3mt^n \exp\left(Dt^{1-n}\right)\left(B + C\exp\left(Dt^{1-n}\right)\right)^{\frac{-\alpha}{\alpha+1}}\right]}{\left[-6mn^2t^{-(2n+1)} + 2mk^{-2}t^{-n}\exp\left(D_1t^{1-n}\right)\right]t^{2n}}$$

$$- \left\{B + C\exp\left(Dt^{1-n}\right)\right\}^{1/\alpha+1}$$

39) (39)

The expression (40) for Mass density parameter given by $\Omega_M = \dfrac{\rho}{\rho_c}$, and expression (41) represents density parameter for vacuum given by $\Omega_\Lambda = \dfrac{\rho_v}{\rho_c}$ for the present model

$$\Omega_M = \frac{t^{2n}\left\{B + C\exp\left(Dt^{1-n}\right)\right\}^{1/1+\alpha}\left[-6mn^2t^{-(2n+1)} + 2mk^{-2}t^{-n}\exp\left(D_1t^{1-n}\right)\right]}{3m^2k^2\, t\left[\ 9m^2\eta_0\left(B + C\exp\left(Dt^{1-n}\right)\right)^{\frac{r}{\alpha+1}} - 3mt^n \exp\left(Dt^{1-n}\right)\left(B + C\exp\left(Dt^{1-n}\right)\right)^{\frac{-\alpha}{\alpha+1}}\right]}$$

(

(40)

$$\Omega_\Lambda = \left[1 - \frac{t^{2n}k^{-2}}{3m^2}\exp\left(D_1t^{1-n}\right)\right] - \frac{\left[-6nm^2t^{-(2n+1)} + 2mk^{-2}t^{-n}\exp\left(D_1t^{1-n}\right)\right]t^{4n}}{3m^2k^2\, t\left[\ 9m^2\eta_0\left(B + C\exp\left(Dt^{1-n}\right)\right)^{\frac{r-1}{\alpha+1}} - 3mt^n \exp\left(Dt^{1-n}\right)\left(B + C\exp\left(Dt^{1-n}\right)\right)^{-1}\right]}$$

(41)

Case II: Causal Cosmological Solution

In this case we assume one more relation

$$\Lambda = \beta H^2, \tag{42}$$

where H is Hubble parameter expressed as

$$H = \frac{\dot{R}}{R} \text{ and } R = \left(R_1R_2R_3\right)^{1/3}, \tag{43}$$

The Hubble parameter can be expressed by using equation (17) and (43) as

$$H = \frac{m}{t^n} \tag{44}$$

Using equations (17)-(18), (42) and (44) in equation (8), we get

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$$8\pi G\rho = \frac{M}{t^{2n}} - \frac{1}{k^2}\exp\{D_1 t^{1-n}\}, \qquad (45)$$

where $M = (3-\beta)m^2$, $D_1 = \frac{-2m}{1-n}$.

From equations (20) and (45), one can

$$G = \frac{1}{8\pi}\left[B + C\exp\left(Dt^{1-n}\right)\right]^{\frac{-1}{\alpha+1}}\left[\frac{M}{t^{2n}} - \frac{1}{k^2}\exp\left(D_1 t^{1-n}\right)\right], \qquad (46)$$

By use of equations (17), (20), (42) and (46) in equation (5), we get

$$\Pi = \left[\frac{M}{t^{2n}} - \frac{2mn}{t^{n+1}}\right]\frac{-1}{8\pi G} + \frac{B}{\rho^{\alpha}}, \qquad (47)$$

By use of equation (20), equation (47) yields

$$\Pi = \frac{-U_1(t)\left[B + C\exp\left(Dt^{1-n}\right)\right]^{\frac{1}{\alpha+1}}}{U_2(t)} + \frac{B}{\left[B + C\exp\left(Dt^{1-n}\right)\right]^{\frac{\alpha}{\alpha+1}}}, \qquad (48)$$

where

$$U_1(t) = \left[\frac{M}{t^{2n}} - \frac{2mn}{t^{n+1}}\right], \quad U_2(t) = \left[\frac{M}{t^{2n}} - \frac{1}{k^2}\exp\left(D_1 t^{1-n}\right)\right]$$

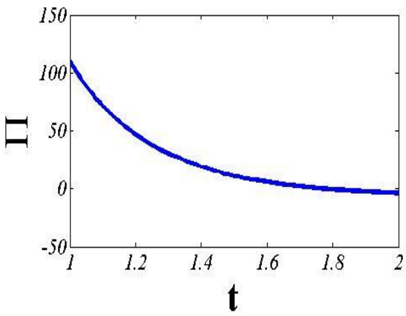

Fig.5: shows nature of bulk viscous stress with respect to cosmic time t. Here we consider B = 1, C = 1, n = 0.25, m = 2, $\alpha = 0.5, 0.75, 1$, r = 1.5.

(i) E (i) In Truncated Causal Theory the Assessment of Bulk viscosity

Now we study with respect to cosmic time t, the variation of relaxation time and bulk viscosity coefficient . We know that for the theory converted to truncated theory and therefore (13) gives the following relation

$$\Pi + \tau\dot{\Pi} = -3\eta H \qquad (49)$$

Here one more physically plausible relation is necessary to get exact solution of the system.

Thus, we think about the well known relation

$$\tau = \frac{\eta}{\rho} \qquad (50)$$

With the help of equations (17), (20), (48), (50) and (49) yields

$$\eta = \frac{U_1(t)(U_2(t))^{-1}\left[B + C\exp\left(Dt^{1-n}\right)\right]^{\frac{1}{\alpha+1}} - B\left[B + C\exp\left(Dt^{1-n}\right)\right]^{\frac{-\alpha}{\alpha+1}}}{\begin{array}{r}3BCm\alpha t^{-n}\exp\left(Dt^{1-n}\right)\left[B + C\exp\left(Dt^{1-n}\right)\right]^{-2} - U_1'(t)(U_2(t))^{-1} + U_1(t)U_2'(t)(U_2(t))^{-2} \\ -3mCt^{-n}U_1(t)(U_2(t))^{-1}\exp\left(Dt^{1-n}\right)\left[B + C\exp\left(Dt^{1-n}\right)\right]^{-1} + \frac{3m}{t^n}\end{array}} \qquad (51)$$

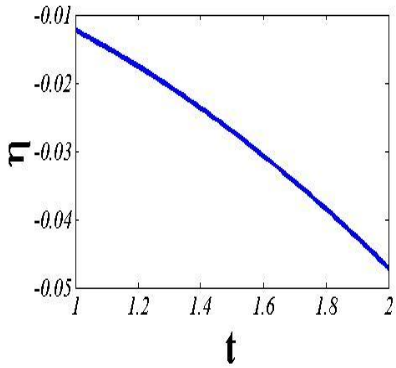

Fig. 6: shows the nature of bulk viscosity coefficient with respect to cosmic time t. Here we consider B = 1, C = 1, n = 0.25, m= 2, $\alpha = 0.5, 0.75, 1$, r = 1.5.

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From fig. 6 one can see the variation of bulk viscosity coefficient with respect to evolution of the universe.

(ii) In Full Causal Theory the Assessment of Bulk viscosity

for the theory converted to full causal theory and therefore (13) gives the following relation

$$\Pi+\tau\dot{\Pi}=-3\eta H-\frac{\tau\Pi}{2}\left(3H-\frac{\dot{\tau}}{\tau}-\frac{\dot{\eta}}{\eta}-\frac{\dot{T}}{T}\right)\quad(52)$$

On the basis of Gibb's integrability condition, the following equation of state for temperature suggested by Maartens as

$$T\propto\exp\int\frac{dp}{\rho+p},\qquad(53)$$

With the help of equation (16) and equation (53), we get

$$T=T_0\left[1-B\rho^{-(\alpha+1)}\right]^{\frac{\alpha}{\alpha+1}}\qquad(54)$$

By use of equations (20), (44), (50) and (54) in equation (52) one can get

$$\Pi+\frac{\eta}{\rho}\dot{\Pi}=-\eta\frac{3m}{t^n}-\frac{\eta\Pi}{2\rho}\left[\frac{3m}{t^n}-\frac{\dot{\rho}}{\rho}-\frac{\dot{T}}{T}\right],$$

Simplify the above equation; we get the expression for coefficient of bulk viscosity

$$\eta=\frac{U_1(t)(U_2(t))^{-1}\left[B+C\exp\left(Dt^{1-n}\right)\right]^{\frac{1}{\alpha+1}}-B\left[B+C\exp\left(Dt^{1-n}\right)\right]^{\frac{-\alpha}{\alpha+1}}}{\frac{\dot{\Pi}}{\rho}+\frac{3m}{t^n}+\left(\frac{-U_1(t)}{2U_2(t)}+\frac{B}{2}\left[B+C\exp\left(Dt^{1-n}\right)\right]^{-1}\right)\left(\frac{3m}{t^n}-\left(1+\frac{B\alpha}{\rho^{\alpha+1}-B}\right)\frac{\dot{\rho}}{\rho}\right)}$$
(55)

where $\frac{\dot{\Pi}}{\rho}=3BCm\alpha t^{-n}\exp\left(Dt^{1-n}\right)\left[B+C\exp\left(Dt^{1-n}\right)\right]^{-2}-U_1'(t)(U_2(t))^{-1}+U_1(t)U_2'(t)(U_2(t))^{-2}$
$-3mCt^{-n}U_1(t)(U_2(t))^{-1}\exp\left(Dt^{1-n}\right)\left[B+C\exp\left(Dt^{1-n}\right)\right]^{-1}$

and $\frac{\dot{\rho}}{\rho}=\frac{-3mC}{t^n}\exp\left\{Dt^{1-n}\right\}\left[B+C\exp\left\{Dt^{1-n}\right\}\right]^{-1}$

For this model critical energy density and energy density for vacuum is given by

$$\rho_c=\frac{3m^2}{t^{2n}\left[B+C\exp\left(Dt^{1-n}\right)\right]^{-1/\alpha+1}\left[\frac{M}{t^{2n}}-\frac{1}{k^2}\exp\left(D_1t^{1-n}\right)\right]}$$
(56) (56)

$$\rho_v=\frac{\beta m^2}{t^{2n}\left[B+C\exp\left(Dt^{1-n}\right)\right]^{-1/\alpha+1}\left[\frac{M}{t^{2n}}-\frac{1}{k^2}\exp\left(D_1t^{1-n}\right)\right]}$$
(57)

$$\Omega_\Lambda=\frac{\beta}{3}\qquad(58)$$

$$\Omega_M=\frac{1}{3m^2}\left[M-\frac{t^{2n}}{k^2}\exp\left(D_1t^{1-n}\right)\right]\qquad(59)$$

State finder parameters are described as $r=\frac{\dddot{R}}{RH^2}$ and

$$s=\frac{r-1}{3\left(q-\frac{1}{2}\right)}\qquad(60)$$

For the present model, state finder parameters {r, s} is expressed as

$$r=-\frac{3n}{t}+\frac{m}{t^n}+\frac{n(n+1)}{mt^{2-n}}\qquad(61)$$

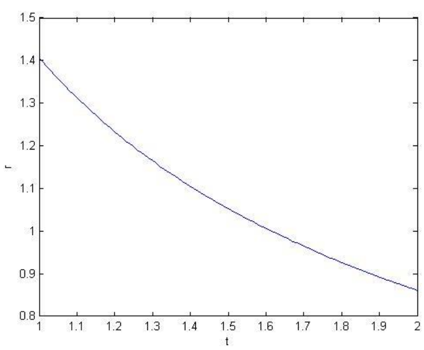

Fig 7: shows variation of state finder parameter r with respect to cosmic time t. Here we consider n = 0.25, m = 2,

$$s = \frac{2\left[3nt^{1-n} + m^2 t^{2-2n} + n(n+1) - mt^{2-n}\right]}{3\left(2nt^{n-1} - 3m\right)} \quad (62)$$

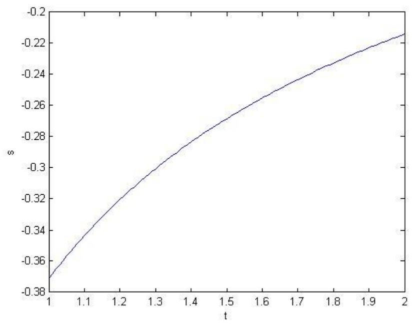

Fig.8: shows variation of state finder parameter s with respect to cosmic time t. Here we consider n = 0.25, m = 2,

From the literature, it is known that to discriminate the different dark energy models state finder parameters is useful. The pair {r, s} = {1, 1} and {r, s} = {1, 0} represent the SCDM and model respectively.

IV. CONCLUSION

In the present paper we have explored Bianchi type III anisotropic cosmological models of the universe filled with bulk viscosity in presence of generalized Chaplygin gas and time dependent gravitational and cosmological "constants". A new set of exact solution of Einstein's field equations have been obtained by considering . In all cases energy density, bulk viscosity coefficient , Cosmological constant are decreases with cosmic time t whereas gravitational constant G increases with time. For this model .Also constant, means the model become isotropy. In case II when n = 1, we get which is most fundamental result match with observations. All graphs of cosmological parameters behave properly.

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