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M/G/1 Queue with Vacation, Two Cases of Repair Facilities and Server Timeout

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Abstract:

The paper, we studied and designed is to show the way server faces the customer in the service system, when the system had breakdowns with two cases TRF ("types of repair facilities") also at the timeout period. At the earlier stage of a busy period, if the server finds system empty, he will stay for fixed time 'e'., If no one enters in during the length of e, server goes for vacation otherwise the commences service to the approaching customers. During vacation server finds system empty, he will take next vacation after that he will commence service for the arrived and waiting customer exhaustively. While continuous working the system may face shatter down and it is reactivated within two cases TRF(" case I is offered when it fetch to amidst of service and customer would like to stay back in the system only with a chance of 'x' to get the remaining service. Where as in case2 repair is initiated if the customer whose service is interrupted due to server failure quits the service zone and joins head of queue(with a probability (1-x)"). Also, numerical values are illustrated.

Keywords: *vacation queueing system, server failure, two case TRF and Server timeout*

1. INTRODUCTION

Now a day's the word Queuing is the most frequently problems facing every day in our life. For example, Queueing is quite common in many fields, and are frequently occur at a Bank, ATM's center, ration center, books in library, telephone exchange, in supermarket and at a petrol station, etc. Hear A.K. Erlang [1], had published "The Theory of Probabilities and Telephone Conversations", now he is considered as father of the field and also treated as first congestion problems in the beginning of 20th century.

Lot of study has been carried about VQS ("vacation queueing system") term, some of the authors work done on day to day life. The extensive study by Doshi [2], the most remarkable works done in recent and past by some researchers on vacation models include Fuhrmann and Cooper [3], Takagi [12], Tian

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and Zhang [13]. Li and Yang [7] detailed different queues with vacation. Madan and Al-Rub [8] explained optional server vacations for a single server queue.

When there are no customers in the system, then the system becomes empty, before going to vacation server stay for fixed time ‘c’ known as server timeout; Oliver C. Ibe [9] discussed the M/G/1 vacation queueing system with server timeout and elaborate the mean waiting queue time of the system and the for the same model he also developed it for N-policy. E. Ramesh Kumar and Y. Praby Loit [4] studied and derived the mean waiting time of a vacation queueing system. K. Satish Kumar, K. Chandan and Y. Saritha [6] studied and derived explicit expressions for the system length.

Server breakdown have been investigated in different frame-works in recent past. An M/Ek/1 queueing model with arrival rate dependent on server breakdown was investigated by Shogan [11]. Single server queue subjected to breakdown and repair has been studied by a number of authors. Using supplementary variable technique Kailash C. Madan [5], Sharma D.C.[10], explained a machine repairable system with spares and two repairmen where “the partial server vacation” is applied. In their system, the first repairman never takes vacations and always available for serving the failed units. Vasantakumar and Srinivasarao [14] obtained the optimal operating policy for an N–policy two-phase queueing system with server startup, breakdowns and delayed repair.

By considering the concept of vacation and server breakdowns, the present paper is an attempt to optimize the loss due to server vacation and breakdowns by introducing Timeout and TRF. Structure of the paper is as follows: Section 2 gives model description, Section 3 gives analysis of the model and also derived part of the ESL (“expected system length”), Section 4 shows numerical tables and graphs as an illustration to Analysis of single server VQS with two cases TRF and server timeout..

2. MODEL DESCRIPTION

In single server queueing system, customers are arriving in Poisson process with rate ‘λ’. The service times are assumed as general. At the earlier stage of a busy period, if the server finds system empty, he will stay for fixed time ‘e’., if no one enters in during the length of e, server goes for vacation otherwise commences service to the approaching customers. During vacation server finds system empty, he will take next vacation, after that he will commence service for arrived and waiting customer exhaustively. While continuous working the system may face a break down and it is reactivated within (TRF). (“ case I is offered when it fetch to amidst of service and customer would like to stay back in the system only with a chance of ‘x’ to get the remaining service. whereas in case2 repair is initiated if the customer whose service is interrupted due to server failure quits the service zone and joins head of queue(with a probability (1-x)”)..

3. MODEL OF THE ANALYSIS

While customers arriving with Poisson process rate ‘λ’ into waiting line. The system is of exponential service with rate ‘μ’. ‘X’, is the time and customer is to serve, also has a general distribution with c.d.f of $F_X(x)$. The LST function of X is” $D_X(s)$.

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$$D_X(s) = E(e^{-SX}) = \int_0^\infty e^{-Sx} f_X(X)dx$$ (1)

The term a vacation period is symbolized as 'V' with general distribution and c.d.f of $F_V(v)$ and the LST function of V is $D_V(s)$.

$$D_V(S) = \frac{\gamma}{S+\gamma} \quad (2)$$

If the system has shatter down with rate '**α**', then system go for two cases TRF. Let $D_{L(M/G/1)}(z)$ is p.g.f the sum of the customers in the system is given by [Ref 9]"

$$D_{L(M/G/1)}(Z) = \frac{(1-\rho)h_\alpha q_1 S^* q_2(z-1)}{h_\alpha q_1(z-S^*q_2)-z\alpha qR^*q_1(1-S^*q_2)} \quad (3)$$

Where
$q_1 = \lambda(1-Z)$ *and* $q_2 = \alpha + \lambda(1-Z)$

Let 'M' is the sum of customers in the system in the beginning of busy period,

$P_M(m)$ =P [M=m] is the p.m.f variable of M is $D_M(z)$ and it is given below:

$$D_M(Z) = E(Z^M) = \sum_{a=1}^{\infty} Z^m p_M(m)$$ (4)

Let 'N' is the sum of customers with the arbitrary exisisting customer left. The pmf of N is $P_N(n)$ is $D_N(z)$ and it is given below:

$$D_N(Z) = \frac{1-D_M(Z)}{(1-Z)E(M)}$$ (5)

Let 'L' be the length system. The p.m.f of 'L' is $P_L(l)$ is $D_L(z)$ and it is given below:

$$D_L(Z) = D_N(Z) + D_{L(M/G/1)}(Z)$$ (6)

Let W_q denotes the EWQ ("expected waiting queue") in the system, to determine ESL of the customer is [Ref 10]

$$\lambda\left(E(W_q) + E(X)\right) = \left.\frac{dG_L(Z)}{dz}\right|_{Z=1} = E(L) \quad (7)$$

"By using $D_M(z)$, we can derive mean E (M) and second movement $E(M^2)$. [Ref 8]"

$$D_M(Z) = ZR_2 + \frac{D_V(\lambda-\lambda Z)-D_V(\lambda)}{1-D_V(\lambda)}R_3 \quad (8)$$

Where $R_2 = \frac{D_V(\lambda)\{1-e^{-\lambda e}\}}{(1-e^{-\lambda e})D_V(\lambda)}$ and $R_3 = \frac{\{1-D_V(\lambda)\}}{(1-e^{-\lambda e})D_V(\lambda)}$

From these, we get $D_M'(1) = E(M) = R_2 + \frac{\lambda E(V)}{1-D_V(\lambda)}R_3$

$$= \frac{D_V(\lambda)(1-e^{-\lambda e})+\lambda}{1-e^{-\lambda e}D_V(\lambda)} \quad (9)$$

And
$$D_M''(1) = E(M^2) = \frac{\lambda E(V)+\lambda^2 E(V^2)}{1-D_V(\lambda)}R_3$$

$$= \frac{\lambda^2}{1-e^{-\lambda e}D_V(\lambda)} \quad (10)$$

From equation (2) using this expression $D_V(S) = \frac{S}{V+S} = D_V(\lambda) = \frac{S}{\lambda+S}$, we rewrite equation (9) and (10) as given below

$$E(M) = \frac{\gamma(1-e^{-\lambda e})+\lambda(\lambda+S)}{((\lambda+S)-e^{-\lambda e}S)} \quad (11)$$

By And $E(M^2) = \frac{\lambda^2(\lambda+S)}{((\lambda+S)-e^{-\lambda e}S)}$ (12)

"By substituting equation (3), (5) in equation (6), we get"

$$D_L(Z) = \frac{1-D_M(Z)}{(1-Z)E(M)} + \frac{(1-\rho)\, h_\alpha\, q_1 V^* q_2(z-1)}{h_\alpha\, q_1(z-V^*q_2)-z\alpha qR^*q_1(1-V^*}$$ (13)

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$$D_L(Z) = \frac{1-D_M(Z)}{(1-Z)E(M)} + \frac{(1-\rho)\, h_\alpha\, q_1 V^* q_2 (1-z)}{z\alpha q R^* q_1 (1-V^* q_2) - h_\alpha\, q_1 (z-V^* q_2)}$$
(14)

$$D_L(Z) = k * \frac{(1-D_M(Z))\, h_\alpha\, q_1 V^* q_2}{z\alpha q R^* q_1 (1-V^* q_2) - h_\alpha\, q_1 (z-V^* q_2)}$$
(15)

Consider $k = \frac{1-\rho}{E(M)}$

(16)

The derivative of equation (16) w.r.to z is

$$D_L'(z)\{z\alpha q R^* q_1 (1-S^* q_2) - h_\alpha\, q_1 (z-S^* q_2)\} + D_L(Z)\{\alpha q R^* q_1 (1-S^* q_2)(1-h_\alpha)\} = K * \{(-D'_M(Z))h_\alpha\, q_1 S^* q_2 + (1-D_M(Z))h'_\alpha q_1 S^* q_2) + (1-D_M(Z))h_\alpha\, q_1 s' * q_2)\}$$
(17)

In (17)substituting K, q_1, q_2, z=1 also eqn (11) and (12) , the result is

$$D_L(1) = \frac{\{(E(A)\mu(\mu+\alpha)(1-\rho)}{\mu(\mu+\alpha)(1-\rho)E(A)} \qquad (18)$$

$$D_L(1) = 1 \qquad (19)$$

The derivative of equation (17) w.r.to z is

$$D_L''(z)\{z\alpha q R^* q_1 (1-V^* q_2) - h_\alpha\, q_1 (z-V^* q_2)\} + 2D_L'(z)\{\alpha q R^* q_1 (1-V q_2)(1-h_\alpha)\} + D_L(z)(2(\alpha q * V * q_2) - 2\left(\frac{\alpha}{\mu+\alpha}\right) - \frac{\mu^2\lambda}{(\mu+\alpha)^2}) = K * \{(-D''_M(Z))h_\alpha\, q_1 V^* q_2 + 2(-D'_M(Z)'')h_\alpha\, q_1 V'^* q_2)\} \qquad (20)$$

In equation (20), substituting K, q_1, q_2, z=1 also equation (11) ,(12) and (19), then the result is

$$D_L'(1) = E(L) = \frac{(1-\rho)\mu\lambda}{(\mu+\alpha)(\alpha^2(q\mu-1)+q\mu(q\lambda-1-\lambda)} + \frac{(1-\rho)\mu[\mu+\alpha)[\lambda^2(\lambda+S)]}{\{(\mu+\alpha)^2\alpha^2((q\mu-1)+q\mu(q\lambda-1-\lambda))\}[S(1-e^{-\lambda e})+\lambda(\lambda+S)]} + \frac{(1-\rho)\alpha q\lambda\mu\left((\lambda+S)-e^{-\lambda e}S\right)}{S(1-e^{-\lambda e})+\lambda(\lambda+S)](\mu+\alpha)^2(\alpha^2(q\mu-1)+q\mu(q\lambda-1-\lambda))} + \frac{(1-\rho)\alpha[\mu+\alpha)((\lambda+S)-e^{-\lambda e}S)}{S(1-e^{-\lambda e})+\lambda(\lambda+S)](\mu+\alpha)^2(\alpha^2(q\mu-1)+q\mu(q\lambda-1-\lambda))} + \frac{(1-\rho)(\mu^2\lambda)((\lambda+S)-e^{-\lambda e}S)}{2S(1-e^{-\lambda e})+\lambda(\lambda+S)2(\mu+\alpha)^2(\alpha^2(q\mu-1)+q\mu(q\lambda-1-\lambda))}$$
(21)

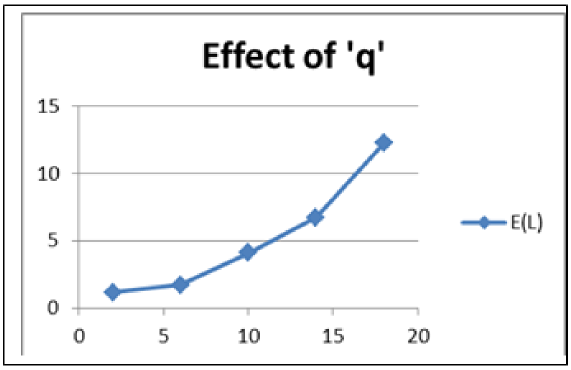
Particular case: By eliminating q, alpha, gamma values in the above expression we get M/G/1 model.

4. Numerical Values Thus by using above expression (21) and changing the values of other parameters by fixing one parameter, we get some numerical results as given below in;

<u>4.1. TABLES and GRAPHS</u> On the chance of getting differing parameters of (q, λ, μ, c, α, and γ) of ESL for fixed values of q=3, λ=1, μ=35, e=1.5, α=4 and γ =5.5

<u>TABLE 1 and GRAPH 1</u> By fixing the parameter ‘q’

PARAMETER	EFFECT OF PARAMETERS	E(L)
	3	1.1442
	6	1.6956
	10	4.1047
	14	6.7162
q	18	12.2574



<u>TABLE 2 and GRAPH 2</u> By fixing the parameter ‘λ’

PARAMETER	EFFECT OF PARAMETERS	E(L)
	1	1.1442
	3	4.8931
λ	8	8.2669
	11	19.3216
	14	68.5207

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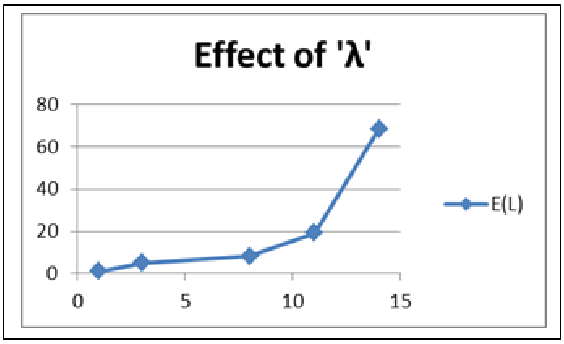


TABLE 3 and GRAPH 3 By fixing the parameter 'μ'

PARAMETER	EFFECT OF PARAMETERS	E(L)
μ	35	1.1442
	40	0.9926
	45	0.8641
	50	0.8521
	55	0.8435

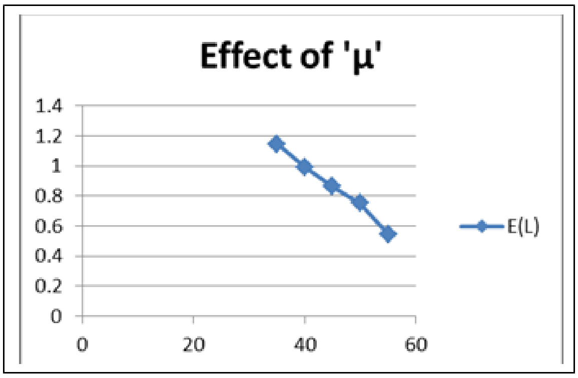


TABLE 4 and GRAPH 4 By fixing the parameter 'γ'

PARAMETER	EFFECT OF PARAMETERS	E(L)
γ	5.5	1.1442
	6	0.9912
	6.5	0.9841
	7	0.8803
	7.5	0.8625

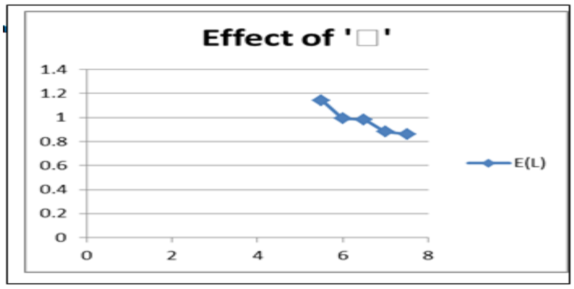


TABLE 5 and GRAPH 5 By fixing the parameter 'α'

PARAMETER	EFFECT OF PARAMETERS	E(L)
α	4	1.1442
	5	1.2787
	6	1.3356
	7	1.5953
	8	1.6562

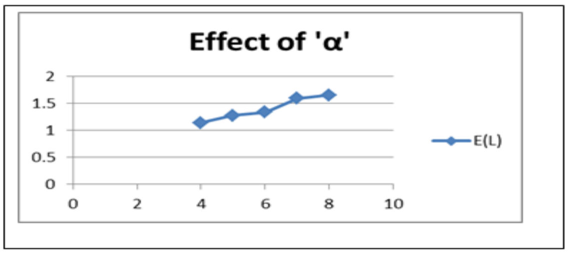
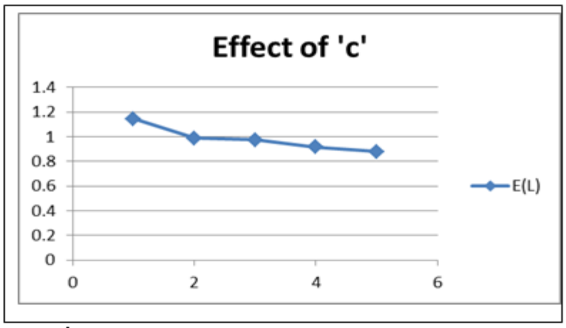


TABLE 6 and GRAPH 6 By fixing the parameter 'c'

PARAMETER	EFFECT OF PARAMETERS	E(L)
e	1.5	1.1442
	2.5	0.9867
	3.5	0.9747
	4.5	0.9172
	5.5	0.8768



Numerical results of different parameter are as follows:

1. With an increase of values q, λ and α, the E (L) is also increasing.
2. With an increase of values μ,γ and c, the E (L) is falling down.

5. CONCLUSION

We have derived expression for Expected system length of an M/G/1 queue with vacation, Two cases of RF and Server Timeout. Finally the sensitivity analysis is also performed to show impact for various parameters to the given sExpected system length.

REFERENCES

1. A.K. Erlang, (1909), A. The theory of probabilities and telephone conversations. Nyt Tidsskrift for Mathematic B 20, 33–39.
2. Cooper, R.B. and v. (1985). Stochastic decompositions in M/G/1 queue with generalized vacations. *Operations Research*, 33, 1117-1129.
3. Doshi, B.T (1986), Queueing system with vacations. A survey on queueing system: Theory and Applications. 1(1), 29-66.
4. E.Ramesh Kumar and Y. Praby Loit (2016), A Study on Vacation Bulk Queueing Model with setup time and server timeout. IJCMS, 5(12), 81-89.
5. K.C. Madan, (2003), An M/G/1 type queue with Time-Homogeneous Breakdowns and Deterministic Repair Times, Soochow Journal of Mathematics Volume 29, No. 1, pp. 103-110, January 2003.
6. K. Satish Kumar, K. Chandan and Y. Saritha. (2018), studied Optimal Strategy Analysis of N-Policy M/Ek/1 Vacation Queueing System with Server Start-Up and Time-Out, Journal of emerging Technologies and innovative Research, vol 5, issue 2, 2349-5162.
7. Li, H. and Yang, T. (1995), A single server retrial queue with server vacations and a finite number of input sources. *European Journal of Operational Research*, 85(1), 149-160.
8. Madan, K. C. and Al-Rub, A. Z. A. (2004). On a single server queue with optional phase type server vacations based on exhaustive deterministic service and a single vacation policy, *Applied Mathematics and Computation*, 149(3), 723-734.
9. Oliver C. Ibe (2015), M/G/1 Vacation Queueing Systems with Server Timeout, American Journal, 5(2), 77-88. http://www.scrip.org/journal/ajor.
10. Sharma D.C.(2012), Machine Repair Problem with Spares and N-Policy Vacation, Research Journal of Recent Sciences, Vol. 1(4), 72-78, April (2012).
11. Shogan, A. W. (1979), A single server queue with arrival rates dependent on server breakdown. *Naval Research Logistics Quarterly*, 26, 487-497.
12. Tian,N. and Zhang,Z. G. (2006), *Vacation Queueing Models –Theory and Applications*, Springer, NewYork.
13. Takagi. H (1991), Queueing analysis : a foundation of performance evaluation, Vol. I, vacation and priority systems, Part I. North-Holland, Amsterdam.
14. VVasanta Kumar and TSrinivasa Rao(2013), Optimal control of an N-policy two-phase Mx/M/1 queueing system with server startup subject to the server breakdowns and delayed repair, American International Journal of Research in Science, Technology, Engineering&Mathematics,3(1),93-102.