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Mitigation Polynomials for Stress Concentration Factor around Circular Hole in Rectangular Plate under In-Plane Loading

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Abstract:
Minimizing stress concentration in structures arising due to irregularities is a major engineering concern. Various techniques are available for mitigation of stress concentration. Area reduction method by introduction of auxiliary holes in the vicinity is one of the established method. Present work provides formulation in the form of mitigation polynomials for size of auxiliary hole and its position for maximum mitigation of stress concentration factor. Formulation in the form of polynomials is arrived at by carrying out parametric finite element analysis of plate with circular hole and auxiliary holes of different size and at different positions. Different cases are analyzed to arrive at the order of mitigation polynomial. Results obtained have been validated experimentally by using two dimensional photo elasticity method. The proposed mitigation polynomials can be used to place optimum size of auxiliary hole at optimum position in the vicinity of circular hole for a particular value of stress concentration factor.

Keywords: *Stress Concentration Factor; Circular Hole; Mitigation; Auxiliary Holes ; Mitigation Polynomial.*

I. INTRODUCTION

In general all engineering structures consist of circular or other forms of openings. Presence of these openings or discontinuities alters the stress distribution around this discontinuity. Discontinuities can cause stress concentrations to occur. Stresses around discontinuities are larger than the average stresses calculated far from these critical locations. A dimensionless stress concentration factor (SCF) K, is used to quantify how the stress is concentrated. It is defined as the ratio of the maximum stress in the element to the reference stress or nominal stress.Stress concentration in the vicinity of singularity can be reduced by making the stress flow lines smooth around the singularity. Some of the approaches used for mitigation of stress concentration factor are removal of material from the vicinity by introduction of auxiliary/relief/defence holes, reinforcement of the hole by adding material, hole shape optimization etc.

2. LITERATURE REVIEW

According to Heywood [1], stress concentration can be reduced by introducing smaller auxiliary holes on either side of the original hole, to smoothen the flow of the stress trajectories past the original hole. Rajaiah [2] proposed hole shape optimization to mitigate SCF. Auxiliary hole with optimum size and shape around the central hole has been proposed to minimize SCF up to 30%. Meguid [3] presented a technique for reduction of SCF in a uni-axially loaded plate with two coaxial holes by introducing defense hole system i.e. material removal in the form of circular holes. Three systems for defense holes were described for FEM analysis. Reduction in SCF ranging from 7.5% to 11 % were achieved. Giare et al. [4] reinforced the hole using composite material for the reduction of stress concentration around the hole in an isotropic plate under in-plane loading.Sanyaland Yadav[5], proposed optimum

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sized relief holes at optimum center distance between the main hole and relief hole for reduction of stress concentration. They [6] have also extended the work by introducing multiple relief holes.Mittal and Jain [7] proposed optimization of square plate with circular hole and reported around 30% reduction in SCF. A further modification of work was carried out by optimization of hole shape & optimization of auxiliary hole shape by giving trapezoidal shape to the auxiliary holes.Optimal hole shape for minimum stress concentration in two dimensional finite plates using parametric geometry models are also given by Zhixue Wu [8]. Authors [9] have also compiled and reviewed the work of many researchers regarding stress concentration and its mitigation.It can be concluded from the above literature survey that area reduction method can be applied to mitigate SCF. It also can be identified that mitigation of SCF is dependent on various geometric parameters like, the size of relief holes, its location and a parametric ratio i.e. main hole diameter to width of the plate (D/A). In view of the above, an attempt has been carried out to formulate a mitigation polynomial dependent on the above parameters.To carry out the above objective, parametric finite element analysis of a rectangular finite isotropic plate with circular hole under in-plane loading has been carried out by varying the various geometric parameters for maximum mitigation of SCF.Polynomial equations of different orders for SCF have been derived from the above analysis to give the optimum size and location of auxiliary holes for maximum mitigation of SCF. The results have been validated for few cases experimentally using two dimensional photo elastic method.

3. ANALYSISAND RESULTS

It has been reported earlier that stress concentration effect can be mitigated by introducing auxiliary holes in the vicinity of a main hole. The mitigation is achieved due to interaction effect between two individual holes - main and auxiliary, each of which have positive and negative stress fields. Effect of mitigation is further analysed by varying different geometrical parameters, like D/A - hole to width ratio, δ - peripheral distance and R_1 - auxiliary hole radius. Analysis is carried out using finite element analysis software ANSYS and experimental validation has been carried out by two dimensional photoelasticity method.

3.1 Stress Concentration Factor

Two models are selected to incorporate various geometrical configurations to be studied. A rectangular plate of 200 mm*100 mm and of 1 mm thickness with one circular hole under in plane loading Fig.1and with one main hole and two auxiliary holes, Fig.2 are taken as Model1 and Model 2. The isotropic material of the plate is selected as alloy steel with material properties, Poisson's ratio, ν = 0.3 and Young's modulus, E = 39 GPa. Uniform in-plane tensile load (σ) is applied to the plate in both the models.

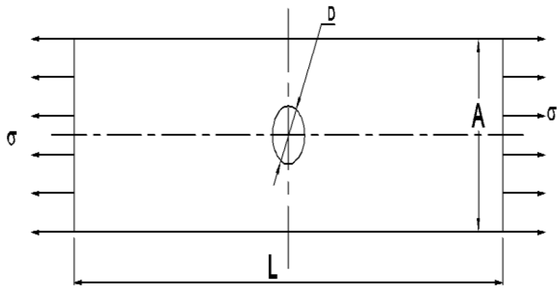

Fig.1. Plate with circular hole under in-plane loading, Model 1

An 8 nodded structural plane 82 element type, Fig.2, from element library of ANSYS with 1 mm element size has been selected for meshing on the basis of convergence test to obtain the values of stress singularity. Each node has two degrees of freedom, making a total 16 degrees of freedom per element. Due to symmetry only quarter of the model for each case is discretized and analyzed.

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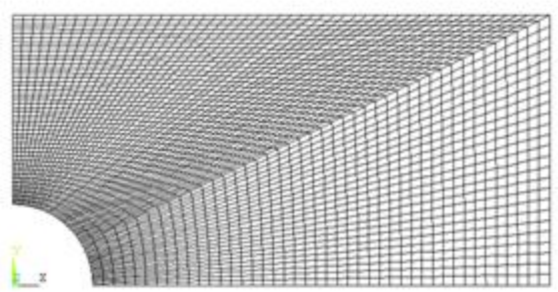

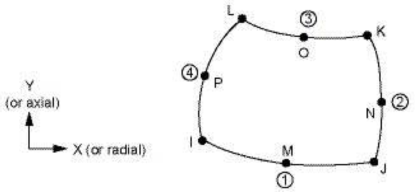

Fig.2. Model 1- Meshed one quarterplate model with Plane 82 element

D/A ratio has been selected as 0.2, 0.3, 0.4, 0.5 and 0.6 for analysis. Stress concentration factors for different D/A ratio for model 1 are determined and tabulated, Table 1.

Table1 SCF in plate with circular hole

D/A	0.2	0.3	0.4	0.5	0.6
SCF	2.5128	2.3576	2.2452	2.1715	2.1256

3.2 Mitigation of Stress Concentration by Auxiliary HolesAs is already established by earlier works, introduction of auxiliary holes in the vicinity of main hole in the direction of loading mitigates the stress concentration. The plate with same dimension and diameter is selected for analysis of mitigation effect by introducing different sized auxiliary holes at different peripheral distances, Model 2, Fig.3.

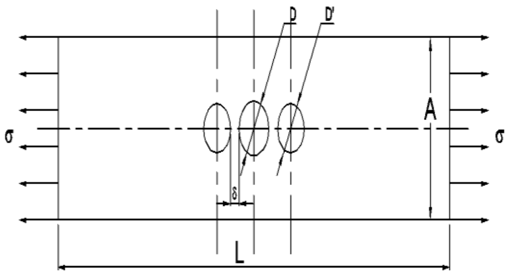

Fig.3. Plate with one main hole and one set of auxiliary holes under in plane loading, Model23.2.1. Parameters effecting mitigation of SCF

By varying the size of auxiliary holes and its position from main hole, mitigation reported is found to be varying. Thus a detailed parametric study is carried out to assess the effect of parameters, δ and R_1, peripheral distance between holes and auxiliary hole radius on the mitigation of SCF in different plates, i.e. for different D/A ratios.

3.2.1.1. Peripheral Distance, δ

To determine the optimum peripheral distance $\delta$for maximum mitigation of SCFaround the main hole, set of smaller auxiliary holes are proposed at different positions from main hole. The peripheral distance is considered in terms of radius of main hole from 0.25 to 4.5 times of radius of main hole for different D/A ratio. It is found that for all the D/A ratios considered, optimum peripheral distance for maximum mitigation in SCF is 25% of radius of main hole, i.e. $\delta = 0.25R_1$.

3.2.1.2. Radius of Auxiliary Holes, R_1To determine optimum radius of auxiliary holes for maximum mitigation of SCF for same radius of main hole, the plate has been analyzed by considering different size of auxiliary holes at different positions. For each D/A ratio radius of auxiliary holes are varied from 10% to 100% of the radius of main hole and SCF values are determined. Optimum radius is found to be varying for different D/A ratio as 85% to 93% of radius of main hole.

3.3. Mitigation Polynomials

From the above analysis, it is established that mitigated SCF values depends on D/A ratio, radius of auxiliary hole, R_1 and peripheral distance δ between the main hole and auxiliary hole. Mathematical models in the form of polynomials are proposed for SCF as a function of all the above parameters.Polynomials of different orders are developed for the same analysis data using VariReg [10]for each D/A ratio. Variation in SCF value is found for different order of polynomials.

3.3.1. Mitigation Polynomials for D/A ratio

Polynomial equations up to seventh order for SCF values are generated for all D/A ratios from 0.2 to 0.6. For each case of D/A ratio, the polynomial equationsgenerated are different. These polynomial equations have been optimized for two relative variables in support of each order of polynomial.To represent the study, polynomial equations from 1st to 7th order for minimum SCF for D/A=0.3 are annexed and shown in appendix enclosed, however to attain a particular accuracy level, polynomials up to 3rd or 4th order can be used to determine the optimum values of R_1 and δ for maximum mitigation. Polynomials upto 4th order are shown for D/A ratio 0.3 for maximum mitigation of SCF.

First order polynomial equation:

$$SCF = 2.329 - 0.023 * R_1 + 0.003 * \delta$$

Second order polynomial equation:

$$SCF = 2.484 - 0.04 * R_1 - 0.001 * \delta + 0.0003 * (R_1)^2 + 0.0005 * (R_1)^2 * \delta$$

Third order polynomial equation:

$$SCF = 2.281 + 0.056 * R_1 - 0.012 * (R_1)^2 + 0.0005 * (R_1)^3 + 0.0003 * (R_1)^2 * \delta$$

Fourth order polynomial equation:

$$SCF = 2.368 + 0.0149 * (R_1) - 0.0056 * (\delta) + 0.0036 * (R_1)^2 + 0.0004 * (\delta)^2 - 0.0028 * (R_1) * (\delta) - 0.0014 * (R_1)^2 - 0.0085 * (\delta)^3 + 0.0001 R_1 * (\delta)^2$$

In case of all the D/A ratio the optimum peripheral distance δ, for maximum mitigation in SCF is 25% of radius of main hole. Further, minimum SCF has been reported at different radius of auxiliary hole for different D/A ratio. For D/A ratios 0.2, 0.3, 0.4, 0.5 and 0.6 it is respectively found to be as 93.69%, 91.77%, 90.42% and 90.41% of radius of main hole.

4. Results

The data obtained from analysis of all the cases considered has been plotted in the form of surface plots for D/A equals to 0.2 to 0.6.

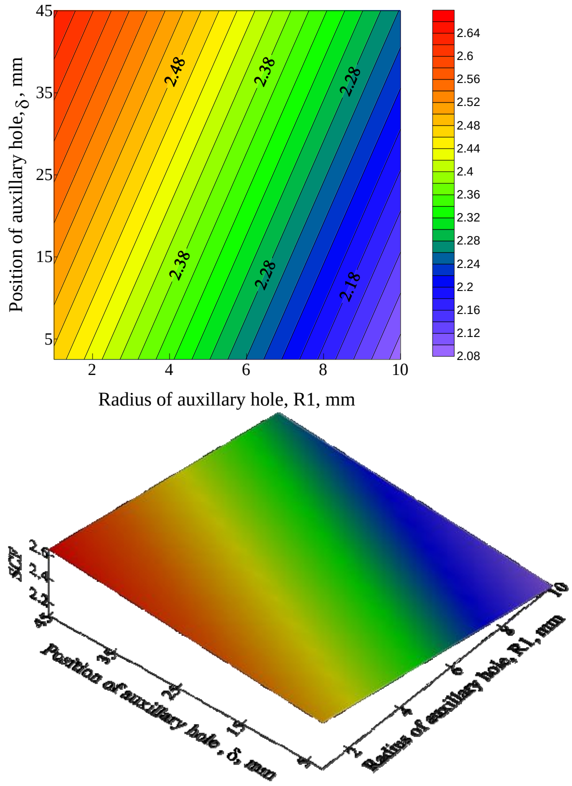

Fig. 4. Surface Plot for D/A = 0.2

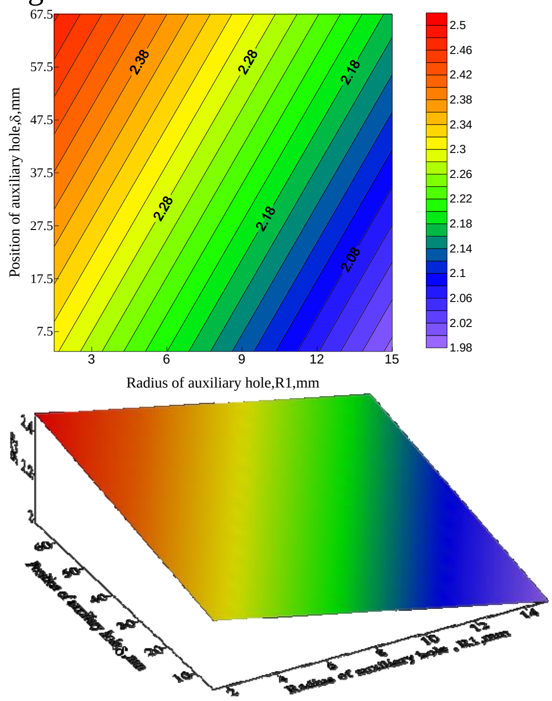

Fig. 5. Surface Plot for D/A = 0.3

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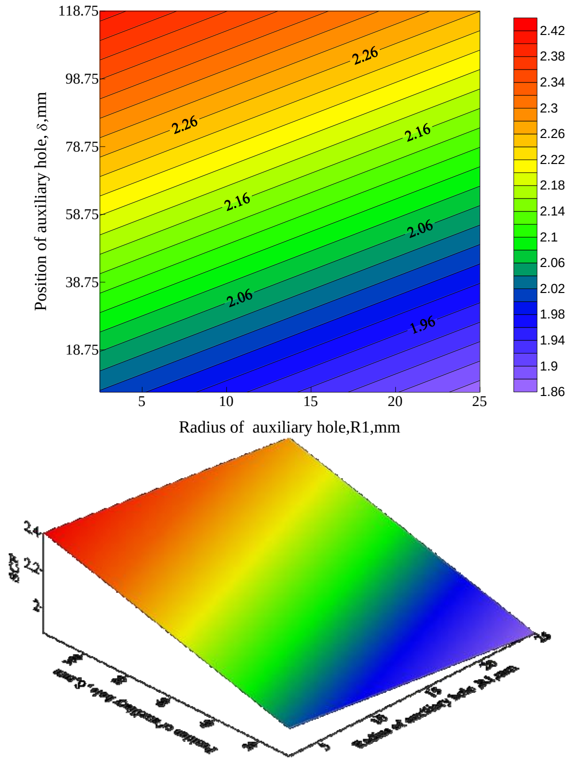

Fig. 6. Surface Plot for D/A = 0.5

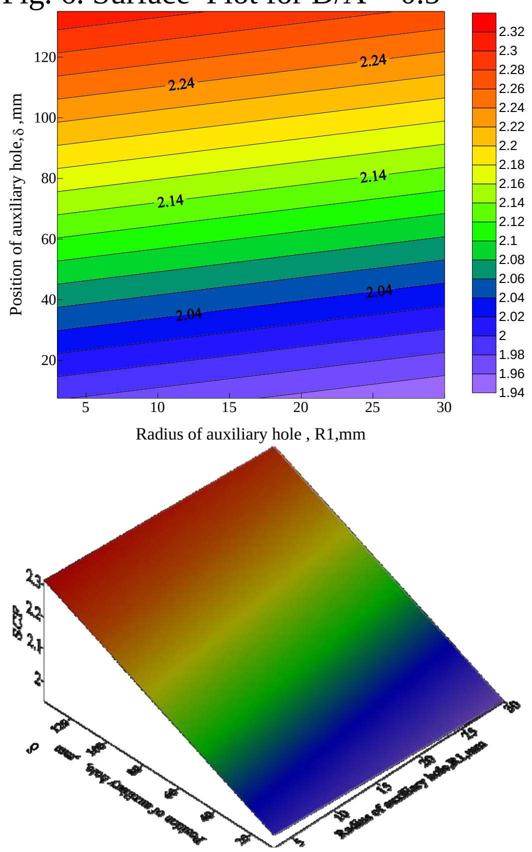

Fig. 7. Surface Plot for D/A = 0.6

Blue region depicts lower values of SCF while the orange region represents higher values of SCF. The slope of lines representing SCF values are different for different D/A values. These surface plots can be directly used to decide upon the location and size of auxiliary holes for a minimum value of SCF.

5. EXPERIMENTAL ANALYSIS

Analysis results have been validated by experimental method. Two dimensional photo elasticity method based on stress optic law has been used to validate the results. The homogeneous mixture of araldite CY 230-1 1N and hardener Aradur HY-951 has been used in 12:1 ratio to cast the photo elastic epoxy circular calibration discs and rectangular plates. Casted plates have been cut in required size and the holes are drilled to get plate models for different D/A ratios. One example for D/A ratio as 0.3 is presented.

Calibration of model material has been done by determining the material stress fringe value using circular disk under diametral compression, Fig. 8. Material stress fringe value, f_σ($f_\sigma = 8P/\Pi DN$, at the center of the disk, N being the fringe order) is determined as 11.94 kg/cm.

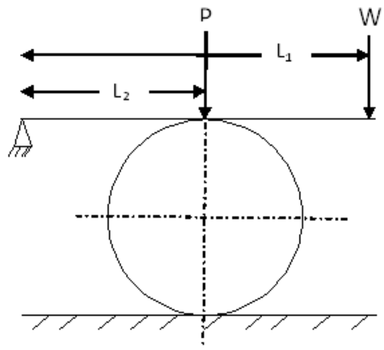

Fig.8.a. Circular disc under compression

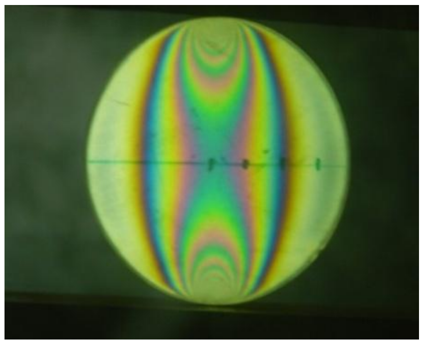
Fig.8.b. Iso-chromatic fringes in Circular disc (L_1 = 52.5mm and L_2 = 18.5 mm)

Plate of size 42 mm*100 mm with hole of 12.5 mm diameter is considered for plate model with D/A as 0.3. The model has been modified by providing one

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set of auxiliary holes of diameter 8 mm at peripheral distance of 16.5 mm from center of main hole. The plates have been analyzed in dark field and light field arrangements in photo elastic bench. Plates have been analyzed by varying the loads and the fringe orders have been recorded, SCF has been determined using stress optic law.

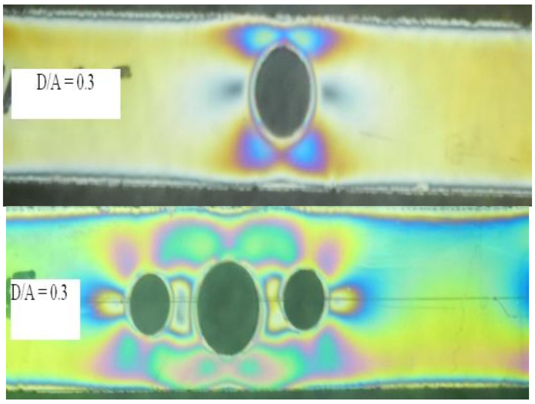

Fig.9. Fringes in axially loaded Epoxy Plate with main hole and auxiliary holes

By determining fringe order at the point of interest, maximum stress and SCF has been calculated for plate with single hole and plate with one set of additional auxiliary holes and the results are compared with results of finite element analysis, Table 2 and 3.

Table.2. Experimental Results of SCF for Plate with circular hole

S.NO.	W	P	N	SCF EXP	SCF MEAN
1	6.980	14.18	1.0	2.48	
2	14.08	28.59	2.0	2.46	
3	20.78	42.18	3.0	2.50	
4	14.22	28.87	2.0	2.44	
5	7.170	14.56	1.0	2.42	
6	3.940	8.010	0.5	2.20	**2.36**
7	11.05	22.44	1.5	2.35	
8	17.62	35.78	2.5	2.46	
9	11.37	23.09	1.5	2.29	
10	4.360	8.850	0.5	1.99	

Table.3. Experimental Results of SCF for Plate with additional one set of auxiliary holes

S.NO.	W	P	N	SCF EXP	SCF MEAN
1	5.72	16.18	1	2.18	2.15
2	11.46	32.43	2	2.17	
3	17.16	48.55	3	2.18	
4	11.70	33.10	2	2.13	
5	5.95	16.83	1	2.09	
6	2.72	7.70	0.5	2.29	
7	9.07	25.68	1.5	2.06	
8	14.33	40.57	2.5	2.17	
9	9.15	25.89	1.5	2.04	
10	2.82	7.98	0.5	2.21	

These results have been compared with the results obtained from Mitigation Polynomials and finite element analysis result, Table 4. Experimental results establish good agreement with computational results.

Table.4. Comparison of SCF with experimental and FEM results

D/A	Order of polynomial	SCF	SCF EXP	%Variation	SCF FEM	%Variation
0.3	1 ST	2.259	2.15	0.6	2.12	1.98
0.3	2ND	2.175				
0.3	3RD	2.199				
0.3	4TH	2.199				
0.3	5TH	2.182				
0.3	6TH	2.173				
0.3	7TH	2.163				
0.5	1 ST	2.109	1.99	0.8	1.96	2.31
0.5	2ND	2.009				
0.5	3RD	2.128				
0.5	4TH	2.210				
0.5	5TH	2.183				
0.5	6TH	2.188				
0.5	7TH	2.006				

6. CONCLUSIONS

Mitigation polynomials are formulated for optimum SCF around circular hole in rectangular plate in

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plane loading with one set of auxiliary holes around main hole. These polynomials can be directly applied for determining the optimum size and position of the auxiliary holes for maximum mitigation of SCF.

The polynomial expressions enable the designer to determine the other parameters i.e. R_1 and δ for maximum mitigation in SCF for different D/A ratios. The 3D surface plots gives relation between radius of auxiliary hole, peripheral distance and SCF. These plots enable the designer to read any of the unknown parameter for given D/A ratio.The experimental results establish good agreement with computational results and hence establishes the proposed findings.

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