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Hydromagnetic Flow and Mass Transfer of a Visco-elastic Fluid over an Exponentially Stretching Surface through Porous Media: An analytical Approach

N.N.V. Sakuntala[1], W. Sridhar[2], D.Sateesh Kumar[2],G.Balaji Prakash[3]
[1]*Department of Mathematics, Swarnandhra College of Engineering &Technology*,Narsapur,
W.G. Dist., A.P., India.
[2,3,]*Department of Mathematics,KoneruLakshmiah Education Foundation,*
Vaddeswaram, Guntur Dist.,A.P, India.
drsateeshdeevi@gmail.com

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Abstract:
The present study is discussed on the mass transfer of viscoelastic MHD fluid flow through porous medium over an exponentially stretching surface. Equations of governing boundary layer are translated into ode using correct similarity transformations. The subsequent equations are iron out throughHomotopy Analysis Method (HAM) analytical method. The concurrence of solution is explicitly discussed. The results are represented through the graphs for various physical criterion like magnetic parameter, Schmidt number and modified Grashof number for velocity and concentration.

Keywords:*MHD, Porous Medium, Visco-Elastic Fluid, HAM, Exponentially Stretching Sheet.*

I. INTRODUCTION

The flow over a stretching sheet with boundary layer has wide range of practical utilizations in industry and engineering processes. The mass transfer effect is recognized as very important in the field of chemical processes. The studied of boundary layer on a continuous flat surface[1] has been extended by L.J. Crane [2] to a stretching sheet when the speed and distance are proportional from the slit in drawing of plastic film. With this motivation, P.S.Gupta [3] studied the impact of heat & mass transfer on viscous flow that are subject to suction or blowing. H.I. Anderson [4] observed the effects of viscoelasticity and magnetic fields through analytical results. E. Magyari et al. [5] described the characteristics of mass with heat transfer over an exponential stretching sheet through numerically and analytically. All the studies listed above are only on non-porous medium heat transfer. Considering the porous medium, K. Vajravelu [6] studied the heat transfer in the flow of viscous fluid in a saturated porous medium. Most recently, Subhas. A and P. Veena [7] studied heat transfer through porous medium. Afterwards, Subhas Abel [8] et al. proposed a numerical approach in this process. Later, a study is performed by P.S. Datti et al. [9] to MHD flow over a non-isothermal stretching surface. In the present work, we adopt Homotopy Analysis Method to visco-elastic MHD fluid flow and heat transfer through porous medium over an exponentially stretching sheet. The governing equations are transformed into ODE by using suitable transformations. Further through Homotopy Analysis Method (HAM) [11] and evaluated numerical values. Finally Porosity parameter, visco-elastic parameter, Magnetic parameter, modified Grashof number etc., are discussed with the assistance of graphs.

MATHEMATICAL FORMULATION

Consider 2-dimensional flow of mass transfer over an exponentially stretching sheet of an electrically run visco-elastic fluid in porous medium. The flow is limited to y>0. The x-axis is taken in the direction

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of motion along the stretching surface, and to it y-axis is perpendicular. The stretching sheet undergoes by the application of magnetic field strength. In addition to this we also considered the chemically reactive breed in the field of flow. The equations that govern the flow are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

(1)

$$u*\frac{\partial u}{\partial x}+v*\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}-k_0\left\{u*\frac{\partial^3 u}{\partial x\partial y^2}+v*\frac{\partial^3 u}{\partial y^3}+\frac{\partial u}{\partial x}*\frac{\partial^2 u}{\partial y^2}-\frac{\partial u}{\partial y}*\frac{\partial^2 u}{\partial x\partial y}\right\}-\frac{\upsilon}{k^*}u-\frac{\sigma B_0^2 u}{\rho}+g*\beta_C*(C-C_\infty)$$

(2)

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}$$

(3)

andthe boundary conditions:

$$u = U_w(x) = U_0 e^{x/l}$$
$$v = 0,$$
$$C = C_W = C_\infty + C_0 e^{x/2l} \quad when\ y = 0$$
$$u = 0,\ u_y = 0,\ C = C_\infty \quad when\ y \to \infty$$

(4)

Stream function $\psi(\boldsymbol{x},\boldsymbol{y})$satisfies (1)

$$u = \frac{\partial\psi}{\partial y},\quad v = -\frac{\partial\psi}{\partial x}$$

(5)

by introducing the similarity transformations:

$$\eta = y\sqrt{\frac{U_0}{2\nu l}}\, e^{\frac{x}{2l}}$$

$$\psi(x,y)=\sqrt{2\nu lU_0}\,f(x.\eta)e^{x/2l}, \qquad \varphi(\eta) = \frac{C - C_\infty}{C_W - C_\infty}$$

(7)

Here η is referred as similarity variable $\boldsymbol{f}$ is referred as stream function (dimensionless), $\theta(\eta)$ is referred as temperature (dimensionless).

The momentum & energy equations are converted into

$$f'''-2f'^2+ff''-k_1\left\{3ff'''-\frac{1}{2}ff^{iv}-\frac{3}{2}f''^2\right\}-(M+k_2)f'+G_c\phi=0$$

(8) $$\phi'' = Sc(\phi f' - f\phi')$$

(9)

boundary conditions are

$$at\ \eta=0 \quad f=0,\quad f'=1,\quad \varphi=1$$
$$as\ \eta\to\infty \quad f'=0,\quad f''=0,\quad \varphi=0 \qquad (10)$$

Where $k_1 = \frac{k_0 U_w}{\upsilon l}$, $k_2 = \frac{2\upsilon l}{k^* U_w}$ $M = \frac{2\sigma B_0^2 l}{\rho U_0 e^{x/l}}$

$S_c = \frac{\nu}{D}$ and $G_c = \frac{2g\beta_c(C-C_\infty)}{U_w^2}$.

are dimensionlessPorosity parameter,Visco-elastic-parameter, Magnetic-parameter,Schmidt number and modified Grashof Number.

HOMOTOPY ANALYSIS SOLUTION:

We apply HAM to interpret solutions of the equations (9) - (10) under the conditions given in (11). We must take initial guess for $\boldsymbol{f}_0$, ϕ_0 as follows

$$f_0(\eta)=1-\exp(-\eta)$$

(11)

$$\phi_0(\eta)=\exp(-\eta)$$

(12)

The linear operator is taken

$$L(f(\eta))=f'''(\eta)-f'(\eta)$$

(13)

$$L(\varphi(\eta))=\varphi''(\eta)-\varphi(\eta)$$

(14)

Which have the following property

$$L_f\ [C_1 + C_2\exp(\eta)+C_3\exp(-\eta)]=0$$

(15)

$$L_\varphi[C_4\exp(\eta)+C_5\exp(-\eta)]=0 \quad (16)$$

Where C_i $(i=1,2,3,4,5)$ be arbitrary constants.

Zeroth-order deformation equations are constructed by using q $\in[0,1]$, $\hbar_1$ **and** $\hbar_2$ & $\boldsymbol{H}_1(\eta)$ **and** $\boldsymbol{H}_2(\eta)$ are the embedding-parameter, auxiliary-parameter and the auxiliary functions respectively.

$$(1-p)\,L(\bar{f}(\eta,p)-f_0(\eta))=p\,h_1 H_1(\eta)\,N_1(\bar{f}(\eta,p),\bar{\varphi}(\eta,p))$$

(17)

$$(1-p)\,L(\bar{\varphi}(\eta,p)-\varphi_0(\eta))=p\,h_2 H_2(\eta)\,N_2(\bar{f}(\eta,p),\bar{\varphi}(\eta,p))$$

(18)

Subject to the boundary conditions

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$$\bar{f}(0;p)=0,\ \bar{f}'(0;p)=1\ \ \bar{f}'(\infty;p)=0$$
$$\varphi(0,p)=1,\quad \varphi'(\infty;p)=0$$
(19)

We define non linear operator as

(20)

$$N_2(f(\eta,p),\ \varphi(\eta,p))=\frac{\partial^2\varphi}{\partial\eta^2}+Sc\left(f\frac{\partial\varphi}{\partial\eta}-\frac{\partial f}{\partial\eta}\varphi\right)$$
(21)

For $p=0$ *and* $p=1$, we have

$$f(\eta,0)=f_0(\eta),\qquad f(\eta,1)=f(\eta)$$

$$\phi(\eta,p)=\phi_0(\eta),\qquad \phi(\eta,1)=\phi(\eta)$$
(22)

Thus as 'p' increases from 0 to 1, $\bar{f}(\eta\ p)$ varies from $\bar{f}(\eta\ 0)$ to $f(\eta)$ and $\phi(\eta;p)$ varies from $\varphi(\eta\ 0)$ to $\varphi_0(\eta)$

Now expanding $\bar{f}(\eta,p),\ \bar{\varphi}(\eta,p)$ using Taylor's Theorem with respect to p we have

$$\bar{f}(\eta,p)=f_0(\eta)+\sum_{m=1}^{\infty}f_m(\eta)p^m$$
(23)

$$\bar{\varphi}(\eta,p)=\varphi_0(\eta)+\sum_{m=1}^{\infty}\varphi_m(\eta)p^m$$
(24)

Where $f_m(\eta)=\dfrac{\partial^m\bar{f}(\eta,\ p)}{m!\ \partial p^m}$ at $p=0$
(25)

$$\varphi_m(\eta)=\frac{\partial^m\bar{\varphi}(\eta,\ p)}{m!\ \partial p^m}\quad at\quad p=0$$
(26)

For p=1, the series (17) - (18) converges as

$$f(\eta)=f_0(\eta)+f_1(\eta)+f_2(\eta)+-----$$
(27)

$$\varphi(\eta)=\varphi_0(\eta)+\varphi_1(\eta)+\varphi_2(\eta)+-----$$
(28)

Deformation equations of mth order as follows:

$$L_f[f_m(\eta)-\chi_m*f_{m-1}(\eta)]=h_1*H_1(\eta)*R_m^f(\eta)$$
(29)

$$L_\varphi[\varphi_m(\eta)-\chi_m*\varphi_{m-1}(\eta)]=h_2*H_2(\eta)*R_m^\varphi(\eta),$$
(30)

where

$$f_m(0)=0$$
$$f_m'(0)=0$$
$$f_m'(\infty)=0$$

$$\varphi_m(0)=0$$
$$\varphi_m(\infty)=0$$
(31)

where

(32)

$$R_m^\varphi(\eta)=\varphi''_{m-1}+Sc\sum_{i=0}^{m-1}(f_i*\varphi'_{m-1-i}-\varphi_i*f'_{m-1-i})$$
(33)

$$\chi_m=0\quad if\ m\ is\ less\ than\ 1$$
$$\quad 1\quad other\ wise$$
(34)

We choose the auxiliary function as

$$H_1(\eta)=1\quad H_2(\eta)=1$$
(35)

Consider the special functions $f_m^*(\eta)$, $\phi_m^*(\eta)$ of m-th order, then the solution become

$$f_m(\eta)=f_m^*(\eta)+C_1+C_2e^{-\eta}+C_3e^{\eta}$$
(36)

$$\phi_m(\eta)=\phi_m^*(\eta)+C_4e^{-\eta}+C_5e^{\eta}$$
(37)

By using the boundary conditions (31), we can find the integral constants C_1 to C_5. Using MATHEMATICA software, non-homogeneous equation (32)-(33) can be easily solved.

CONVERGENCE OF HAM:

HAM solution convergence depends mainly on the auxiliary parameters h_i (i=1, 2). For this, the curves of $\hbar_i$ are plotted for order 20^{th} approximations in Fig.1 and Fig.2.The valid ranges of $\hbar_i$ are

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$-1.37 \le \hbar_1 \le 0.15, \quad -2.16 \le \hbar_2 \le 0.16$ respectively.

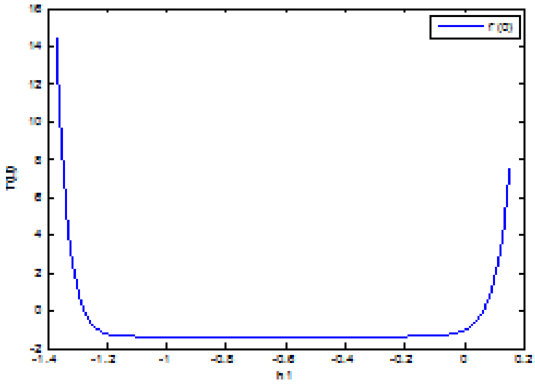

Fig.1 ħ1 curve for f ''(η)

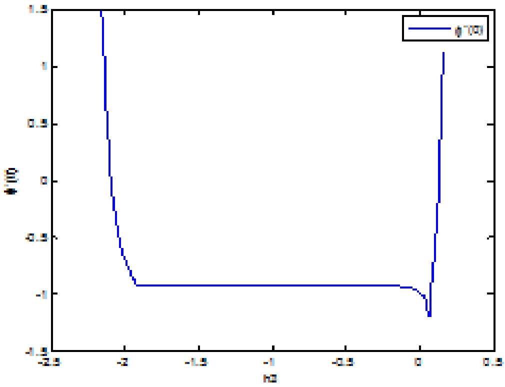

Fig.2 ħ2 curve for φ´(η)

Table 1. Convergence of HAM solution for various $k_1 = 0.1, k_2 = 0.1, Gc = 0.1,$ n, $Sc = 0.96, M = 0.1.$

Order of Approximation	$-f''(0)$	$-\phi'(0)$
5	1.37478	0.920428
10	1.37486	0.920326
15	1.37493	0.920277
20	1.37498	0.920265
25	1.37498	0.920264
30	1.37498	0.920264
35	1.37498	0.920264
40	1.37498	0.920264

RESULTS AND DISCUSSION:

Analytical solutions are discussed for thetemperature, velocity & concentration profiles under variousparameter values of visco-elastic (k_1), porosity (k_2), Grashof Number (Gr) &Magnetic (M), modified Grashof Number (Gc), Schmidt number (S_c) and Prandtl number (Pr). These are showed through graphs.

Fig.3-4 represents the effect of the parametersof visco-elastic k_1 on velocity and concentration profiles. From the figure increasing the visco-elastic (k_1) declines the velocity and enhances the concentration.

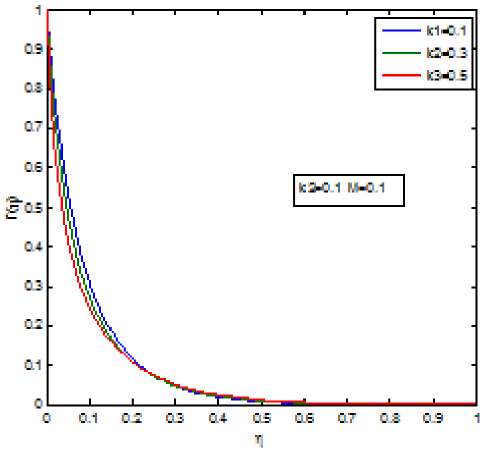

Fig.3 velocity f´ (η) for different values of k_1

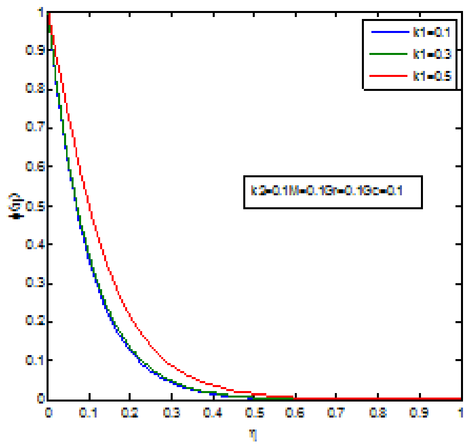

Fig.4 concentration φ (η) for different values of k_1

Fig.5-6 illustrates the relation ofporosity_parameter$k_2$on velocity and concentration profiles. From the figure increasing in the porosityparameter (k_2) declines the velocity and enhances the concentration.

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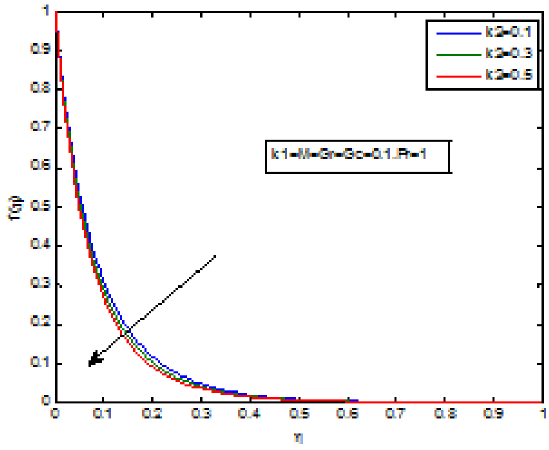

Fig.5 velocity f′ (η) for different values of k_2

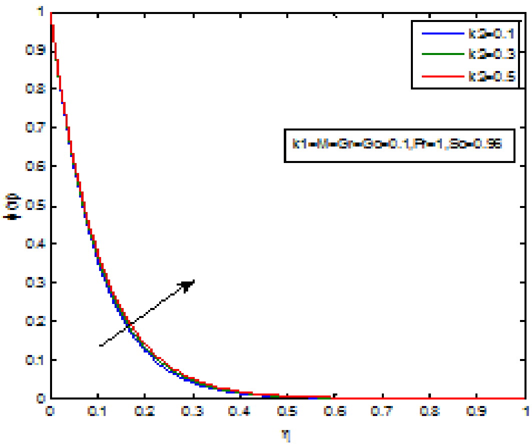

Fig.6 concentration φ (η) for different values of k_2

Fig.7-8 shows the velocity and concentration profiles at various values Gc. It is noticed that as Gc increases, there is increase in velocity and decrease in concentration.

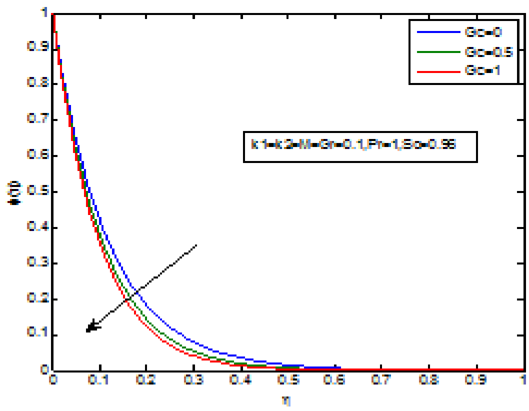

Fig.8 concentration φ(η) for different values of Gc

Fig.9&10 shows the relation between magnetic-parameter M on velocity. It is observed that the velocity desentigrades with increase in magnetic-parameter M, and asScincreases concentration reduces.

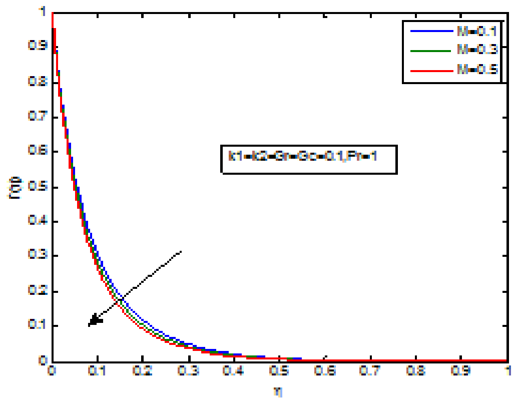

Fig. 9 velocity f′ (η) for different values of M

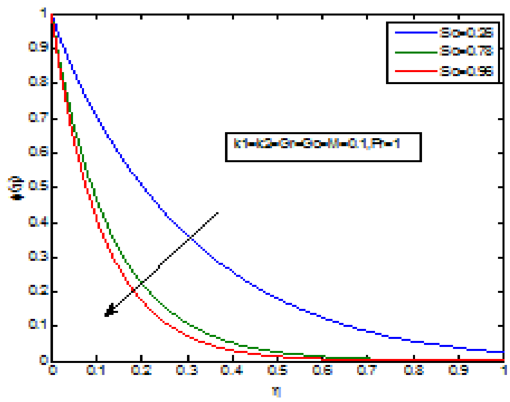

Fig.10 concentration φ (η) for different values of Sc

CONCLUSIONS:

The conclusions of the present study is

* Velocity increases by increasing modified Grashof number whereas concentration decreases.
* Magnetic parameter andVelocity are inversely related
* Concentration of the fluid Reduces as Sc increases
* Mass transfer rate is increased with Parameter of the Porous and visco-elastic parameter.

REFERENCES:

1. B.C. Sakiadis, 1961 Boundary layer behavior on continuous Solid surfaces, A.I.Ch.E. Journal. 7: 221-225.
2. L.J.Crane, 1970, Flow Past a stretching Sheet, Z. Angew. Math. Phys.21: 645-647.
3. P.S.Gupta, A.S. Gupta, Heat and mass transfer on a stretching sheet with suction or Blowing, Canad.J.Chem. Engrg. 55, 744-746, (1977).

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4. H.I.Anderson, MHD flow of a visco-elastic fluid past a stretching surface, Acta Mechanical 95, 227-230, (1992).
5. E.Magyari, B. Keller, Heat and mass transfer in the boundary layers on an exponentially Stretching continuous surface, J. Phys.D., 577-585 (1999).
6. Vajravelu.K, Flow and Heat transfer in a Saturated Porous Medium over a Stretching Surface, ZAMM. 74(12), 605-614 (1994).
7. A.Subhas and P.Veena, Visco-elastic fluid flow and heat transfer in porous medium over a stretching sheet, International Journal of Non- Linear Mechanics, 33(3), 531-540, (1998).
8. M. Subhas Abel, Sujith Kumar Khan, K.V.Prasad, Convective heat and mass transfer in aVisco-elastic fluid flow through a porous medium over a stretching sheet, InternationalJournal of Numerical Methods for Heat & Fluid Flow, 11, 779-792, (2001).
9. P. S. Datti, K.V. Prasad, M. Subhas Abel, Ambuja Joshi, MHD visco-elastic fluid flow over