TEST Engineering & Management

January - February 2020
ISSN: 0193 - 4120 Page No. 14877 - 14882

Unique isolated perfect domination in graphs

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Article Info
Volume 82
Page Number: 14877 – 14882
Publication Issue:
January-February 2020

Article History
Article Received*: 18 May 2019*
Revised*: 14 July 2019*
Accepted*: 22 December 2019*
Publication*: 28 February 2020*

Abstract

A vertex subset S of a graph G is said to be an isolate dominating set(IDS) of G if S is a dominating set and there is at least one isolated vertex in the induced subgraph < S > [8]. An isolated dominating set S of a graph G is called as unique isolate perfect dominating set(UIPDS) of G if there exists exactly one isolated vertex in < S > and the set S is a perfect dominating set. If no proper subset of S is an UIPDS, then S is said to be minima UIPDS. The UIPD number, denoted by $\gamma_{0,p}^{u}$ (G) ,is the minimum cardinality of a minimal UIPDS of G. This paper includes some fundamental properties of UIPDS and contains the UIPD number of paths, complete k-partite graphs and disconnected graphs. At the end, the role of UIPDS in the domination chain has been discussed in detail.

AMS Subject Classification: 05C69

Keywords: *isolated domination, unique isolated perfect domination, unique isolated perfect domination number*

I. INTRODUCTION

By a graph $G = (V, E)$, we mean a finite, non-trivial, undirected graph with neither loops nor multiple edges. For graph theoretic terminology, we refer to the book by Chartrand and Lesniak[3].

For $v \in V$, the open neighborhood of v is $N_G(v) = \{u \in V \,|uv \in E\}$ and the closed neighborhood of v is $N_G(v) = \{v\} \cup N(v)$. The degree of v is $deg_G(v) = |N_G(v)|$

The minimum and maximum degree of the graph G is given by $\delta(G) = \min_{v\in V}\{\deg(v)\}$ and $\Delta(G) = \max_{v\in V}\{\deg(v)\}$ respectively. A vertex of degree one is called a pendent vertex. The diameter of a graph G, denoted by $diam(G)$, is the maximum distance between pairs of vertices of G.

A vertex subset S of G is a dominating set of G if for every vertex in V(G)-S there is a neighbor in S. The minimum cardinality of a dominating set of G *is* called the domination number, denoted by $\gamma(G)$ and the maximum cardinality of a dominating set of G *is* called the upper domination number, denoted by $\Gamma(G)$. A dominating set of cardinality $\gamma(G)$ is said to be $\gamma(G)$-set. A perfect dominating set S is a dominating such that every vertex in $V(G)$-S has exactly one neighbor in S. A dominating set S is called a total dominating set if $<S>$ has no isolated vertex. The total domination number $\gamma_t(G)$ is the minimum cardinality of a total dominating set of G. We refer a total dominating set of cardinality $\gamma_t(G)$ as a $\gamma_t(G)$-set.

The notion of isolate domination was introduced and studied by I. Sahul Hamid et al. [8, 7]. An isolate dominating set S is dominating set in which $<S>$ has at least one isolated vertex. The isolate domination number $\gamma_0(G)$ and the upper isolate domination number $\Gamma_0(G)$ are respectively the minimum and maximum cardinality of a isolate dominating set. An isolate domination set S with $|S| = \gamma_0(G)$ is called a γ_0-set of G and an isolate domination set S with $|S| = \Gamma_0(G)$ is called a Γ_0-set of G. In 2017, Nader Jafari Rad studied the complexity of the isolate domination in graphs and answered some open problems [6]. By using the definition of isolate domination, a few domination parameters have been defined and studied [10, 2].

In this paper, we define a new domination parameter, namely "Unique Isolate Perfect Domination" (UIPD). A dominating set S of a graph G is called as an "Unique Isolate Perfect Dominating set" (UIPDS) of G if the induced subgraph $<S>$ contains exactly one isolated vertex and S is a perfect dominating set of G. Also S is minimal if any proper subset of S is not an UIPDS. The minimum cardinality of a minimal UIPDS of G is called the UIPD number $\gamma^{u}_{0,p}(G)$ and the maximum cardinality of a minimal UIPDS of G is called the upper UIPD number $\Gamma^{U}_{0,p}(G)$. An UIPDS of cardinality $\Gamma^{U}_{0,p}$ is called a $\Gamma^{U}_{0,p}$-set.

Observe that the path graph P_4 will not admit UIPDS but it will admit IDS. Lot of difference between these two domination parameters and this we have discussed later. This paper includes some fundamental properties of UIPDS and contains the UIPD number of paths, complete k-partite graphs and disconnected graphs. At the end, the role of UIPDS in the domination chain has been discussed in detail.

II. EXTENDED DOMINATION CHAIN

A dominating set $S \subseteq V$ is said to be an independent dominating set of G if every two vertices $u, v \in S$, $uv \notin E$. The minimum cardinality of the independent dominating set is called an independent domination number, denoted by $i(G)$. The independence number $\beta_0(G)$ of a graph G is the maximum cardinality of an independent set of G. For a set S of vertices, v is called private neighbor of $u \in S$ with respect to S if $N[v] \cap S = \{u\}$. Furthermore, we define the private neighbor set of u, with respect to S, to be $pn[u, S] = \{v : N[v] \cap S = \{u\}\}$. Notice that $u \in pn[u, S]$ if u is an isolate in $<S>$, in which case we say that u is its own private neighbor.

When every vertex in S has at least one private neighbor with respect to S, then S is said to be irredundant. The minimum cardinality of a maximal irredundant set is called the irredundance number $ir(G)$ and maximum cardinality of a maximal irredundant set is called as the upper irredundance number $IR(G)$. This is an inequality chain connecting these parameters [4].

$ir(G) \leq \gamma(G) \leq i(G) \leq \beta_0(G) \leq \Gamma(G) \leq IR(G)$ (1)

For more detailed information about the domination chain, one can refer to [5]. By extending the dominating chain by introducing new parameters in between ir(G) and IR(G) is a new direction. In this manner, Sahul Hamid [8] obtained the following chain.

$ir(G) \leq \gamma(G) \leq \gamma_0(G) \leq i(G) \leq \beta_0(G) \leq \Gamma_0(G) \leq \Gamma(G) \leq IR(G)$ (2)

In this section, we study the position of UIPDS in the domination chain.

THEOREM 1. *For any graph G, we have* $\gamma_0(G) \leq \gamma^{u}_{0,p}(G)$

Proof. The proof follows immediately from the definitions of UIPDS and IDS.

REMARK 2. Consider the two graphs H_1 and H_2.

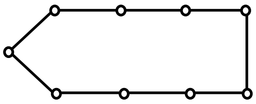

Fig .1. H_1

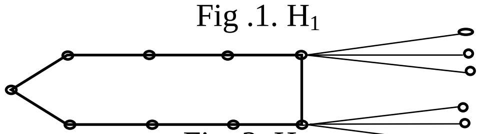

Fig .2. H_2

From Table-1, it is easy to observe that the parameter

$\gamma^{u}_{0,p}(G)$ is not a comparable parameter with other parameters such as $\beta_0(G)$, $\Gamma(G)$, $i(G)$, $\Gamma_0(G)$ and $IR(G)$.

	Graphs	
Parameter	H_1	H_2
$\gamma_{0,p}^{U}$	5	3
i	4	8
β_0	4	8
Γ	4	13
Γ_0	4	13
IR	4	9

Table- 1: Comparison of domination parameters

From Theorem 1 and Remark 2, we obtained a different domination chain as follows.

LEMMA 3. *For any simple graph G, the following is true*

$ir(G) \leq \gamma(G) \leq \gamma_0(G) \leq \gamma^{u}_{0,p}(G) \leq \Gamma^{U}_{0,p}(G)$.

III. UNIQUE ISOLATE PERFECT DOMINATING SET

This section obtains the UIPD number of graphs such as complete k-partite graphs and paths. Further it contain some fundamental properties of UIPDS.

REMARK 4. *(a). Let G be a graph and S be an UIPDS of G. Then <S> has only one isolated vertex and any other vertex of S is adjacent to another vertex in S.*

(b). If u is isolated in <S> and $d(u, y) \leq 2$, *then* $y \notin S$

THEOREM 5. *Let G be a graph and x be not a full vertex in G such that* $d(x, y) \leq 2$ $\forall y \in G$. *Then x will not be in any UIPDS of G.*

Proof. Suppose x is an isolated vertex in <S>. Since x is not a full vertex, there exist $v \in G$ such that $v \notin N[x]$. To dominate v, S must have one more vertex in S, which is not possible by Remark 4(b). Suppose x is not isolated in <S>. Since $d(x, y) \leq 2$, by Remark 4(b), there exists no isolated vertex in <S>, a contradiction.

THEOREM 6. *Let G be a graph and n be the order of G with* $diam(G) \leq 2$ *and* $\Delta(G) < n$-1. *Then G does not admit UIPDS.*

Proof. On the contrary, suppose G admits UIPDS, say S. Let w be the only isolate vertex in <S>. Since $\deg_G(w) < n$-1, there are some undominated vertices in the graph <G-$N[w]$>. Thus S must have one more element $v \neq w$. Since $diam(G) \leq 2$ and w is isolated in <S>, S could not be perfect, a contradiction.

THEOREM 7. *Any tree T of* $diam(T)$=3 *does not admit UIPDS.*

Proof. On the contrary, Suppose S is an UIPDS of G and u is the only isolate in <S>. Since $diam(T)$ = 3, $deg_T(u)$<n-1. Thus there are some undominated vertices in the graph <G-$N[u]$>. Thus S must have one more element $v(\neq u)$. Then by Remark 4(b), $d(u,v)$ = 3 and so v is also an isolated vertex in <S>, a contradiction.

A Mob $M_n(n \geq 1)$ is a tree which is obtained from P_4, a path on 4 vertices by adding n pendant edges with one end of P_4.

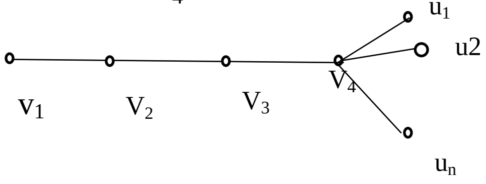

Fig. 3 M_n

THEOREM 8. *A tree with* $diam(T) = 4$ *admits an UIPDS if and only if* $T \simeq M_n$ *for some integer* $n(\geq 1)$.

Proof. Suppose the tree T admits an UIPDS, let it be S and let x be isolated vertex in <S>. Let P: $v_1, v_2, \ldots, v_5$ be a largest path in T. Then v_1 and v_5 must be pendant vertices. Since $d(v_3, y) \leq 2$ for all $y \in T$, by Theorem 5, $v_3 \notin S$.

Case:(i) Suppose v_3 is adjacent with some other vertices, say $u_1, u_2, \ldots, u_k$, $k \geq 1$. Suppose $x \in N(v_3)$, then by Remark 4(b), either v_1 or v_5 will be another isolated vertex in$^{\leq}$<S>. Thus $d(v_3, x)$ must be equal to 2, without loss of generality, let us take $x = v_1$. Since S is uniquely isolated, to dominate v_4, S must include both v_4 and v_5. Then $u_1 \notin S$. Thus to dominate u_1, there must exists another vertex w_1 in T such that $w_1 \neq v_3$, w_1 and u_1 are adjacent. In this case w_1 will be another isolated vertex in <S>.

Case:(ii) Suppose v_2 is adjacent with at least one vertex, say w_2 and v_4 is adjacent with at least one vertex, say w_4. Since $diam(T) = 4$, both w_2 and w_4 are vertices. Since $v_3 \notin S$, either v_2 or v_4 must be in S, without loss of generality, let $v_2 \in S$. Since S is perfect, $v_4 \notin S$. Thus to dominate the pendant vertices v_5 and w_4, S must include both v_5 and w_4 in it. In this case, both v_5 and w_4 are isolated vertices in <S>, a contradiction. Hence $T \simeq M_n$.

Conversely, suppose $T \simeq M_n$. Then $\{v_1, v_4, u_1\}$ is $\gamma^{u}_{0,p}$ set(see Figure 3).

LEMMA 9. *Let G be a graph and S be a UIPDS of G. Suppose all the non-isolated vertex in <S> has*

□

a private neighbor with respect to S. Then S is minimal.

Proof. Let $w \in S$.

Case 1: If w is isolate in $<S>$. In this case, the vertex w is not dominated by $S - \{w\}$.

Case 2: If $w \in <S>$ is not an isolated vertex. In this case, there exist a vertex $x \in S$ with the property that x is the private neighbor of w in S. Here, $S - \{w\}$ will have two isolated vertices in $<S - \{w\}>$. Thus S is minimal.

The corona of two graphs G and H, denoted by $G \circ H$, is the graph obtained by taking one copy of G of order n and n copies of H, and then joining the i-th vertex of G to all the vertices of i-th copy of H [1]. For any vertex $v \in V$, we mean by H^v, the copy of H in which all the vertices of H are joined to the vertex v.

REMARK 10. *The converse of the above Lemma 9 is not true. Consider the cycle* C_8 *with* $V(C_8) = \{v_i: 1 \le i \le 8\}$. *Here* $S = \{v_1, v_4, v_5, v_6\}$ *is UIPDS and minimal, where as the vertex* v_4 *do not have a private neighbor in S.*

REMARK 11. *(a). Let G be a graph G and x is a full vertex in G. Here, the singleton set {x} is an UIPDS of G and* $\gamma^u_{0,p}(G) = 1$. *As a corollary, it follows that the complete graphs, star graphs and the wheel graphs admit UIPDS.*

(b). Since each UIPDS of G has exactly one isolated vertex, $\gamma^u_{0,p}(G)$ cannot be equal to 2.

LEMMA 12. *Let* P_n *be a path graph of order* $n \ge 1$ *and* $n \ne 4$*Then*

(a). $\gamma^u_{0,p}(Pn) = 2t + 1$ when $n = 4t + 3$ *or* $4t + 2$ *or* $4t + 1$ *for some integer* $t \ge 0$.

(b). $\gamma^u_{0,p}(Pn) = 2t$ when $n = 4t$ *for some integer* $t \ge 2$.

Proof. (a). Let $V(P_n) = \{v_1, v_2, \ldots, v_n\}$

Suppose $n = 4t + 3$ or $4t + 2$ or $4t + 1$ for $t \ge 0$. When $t = 0$, each of the graphs P_1, P_2 and P_3 has a full vertex and hence by Remark 11(a),

$\gamma^u_{0,p}(P_1) = \gamma^u_{0,p}(P_2) = \gamma^u_{0,p}(P_3) = 1 = 2t + 1$

Suppose $n = 4t + 1$ for $t \ge 1$. Let S be an UIPDS of the path P_n and x be the only isolated vertex of $<S>$. Note that x will dominate at most 3 vertices. Thus $S - \{x\}$ will dominate all the remaining $n - 3$ vertices. Note that any vertex of $S - \{x\}$ has a neighbor in $S - \{x\}$. Further, two adjacent vertices in $S - \{x\}$ will dominate at most 4 vertices.

Since $n = 3 + 4(t - 1) + 2$, to dominate the $3 + 4(t - 1)$ vertices, S must include $1 + 2(t - 1) = 2t - 1$ vertices. Let v_a and v_b be two vertices of P_n which are not dominated by these $2t - 1$ vertices.

Case 1: If v_a and v_b are adjacent, then there exists a path P with at least 3 vertices such that $v_a, v_b \in P$ and $P \cap S = \varphi$. If we choose a vertex of P to dominate all the vertices of P, then this vertex should be isolate of $<S>$, a contradiction. This means, to dominate the vertices of P, we need to include two more vertices in S.

Case 2: Suppose v_a and v_b are not adjacent. Suppose there exists a vertex v_c such that v_c is adjacent with both v_a and v_b. Then v_c is not in S and we can et a path P which contain more than two vertices such that v_a, v_b are in P and $P \cap S = \varphi$. Thus, as discussed above, S need at least two more vertices to dominate all the vertices of P_n.

In the other case, S must include at least two more vertices to dominate the vertices v_a and v_b. In all cases, $|S| = 1 + 2(t - 1) + 2 = 2t + 1$and so $\gamma^u_{0,p}(P_n) \ge 2t + 1$. Also the set $\{v_1\} \cup \{v_{4i}, v_{4i+1}: i = 1, 2, 3, \ldots, t\}$ is an UIPDS with cardinality 2t + 1. Thus $\gamma^u_{0,p}(P_n) \le 2t + 1$.

By using $4t + 2 = 3 + 4(t-1) + 3$ and $4t + 3 = 3 + 4t$, we can have $\gamma^u_{0,p}(P_n) = 2t + 1$ when $n = 4t + 2$ and $n = 4t + 3$ respectively.

(b). Let $n = 4t$ for $t \ge 2$. Note that $n = 4t = 3 + 4(t-1) + 1$. As discussed in the proof of (a), $|S| \ge 1 + 2(t - 1) + 1 = 2t$. Therefore $\gamma^u_{0,p}(P_n) \ge 2t$. Further, $\{v_2, v_{4t-1}\} \cup \{v_{4i+1}, v_{4i+2} / i = 1, 2, 3, \ldots, t-1\}$ is an UIPDS with cardinality $2t$. Hence $\gamma^u_{0,p}(P_n) \le 2t$.

LEMMA 13. *Let $G = K_{m_1, m_2,\ldots,m_k} = (M_1, M_2, \ldots, M_k)$ be a complete k-partite graph and $k \geq 2$ be an integer. The graph G admits an UIPDS if, and only if, $m_i = 1$ for some $1 \leq i \leq k$.*

Proof. Let S be an UIPDS of G admits an UIPDS. On the contrary, suppose $m_i \geq 2$ for all i with $1 \leq i \leq k$.

Let x be the unique isolated vertex in $<S>$. Without loss of generality, let us assume $x \in M_1$. Since $|M_1| \geq 2$, there exists a vertex $y \in M_1$ such that y is not equal to x. Note that any vertex of $M_2, M_3, \ldots, M_k$ is not an vertex of S(otherwise x is not isolated in $<S>$). Now, to dominate y, the set S must include y and so $<S>$ has two isolated vertices namely x and y, a contradiction.

Remark 11(a) gives the proof for the converse part.

THEOREM 14. Suppose $n \geq 2$ is *a positive integer and G is a disconnected graph with n components $G_1, G_2, \ldots, G_n$ such that the first r components $G_1, G_2, \ldots, G_r$ admit UIPDS. Then* $\gamma^u_{0,p}(G) = \min\limits_{1\leq i\leq r} \{t_i\}$ where $t_i = \gamma^u_{0,p}(G_i) + \sum\limits_{i=1, i\neq j}^{n} \gamma_t(G_j)$ for $1 \leq i \leq r$

Proof. Without loss of generality, let $t_1 = \min\limits_{1\leq i\leq r} \{t_i\}$. Let S be a $\gamma^u_{0,p}$ - set of G_1 and D_i be a γ_t - set of G_i for all i with $2 \leq i \leq n$. Thus the set $S \cup (\bigcup\limits_{i=2}^{n} D_i)$ is an UIPDS of G with number of vertices $\gamma^u_{0,p}(G_1) + \sum\limits_{i=2}^{n} \gamma_t(G_i)$ and so $\gamma^u_{0,p}(G) \leq \gamma^u_{0,p}(G_1) + \sum\limits_{i=2}^{n} \gamma_t(G_i) = t_1$. Let S be an UIPDS of G which is minimal. Then $S \cap V(G_i)$ is not empty for each i with $1 \leq i \leq n$. Also, there exists j with the property that $1 \leq j \leq r$ and $S \cap V(G_j)$ is a minimal UIPDS of G_j. Further, for any I with $1 \leq i \leq n$, $i \neq j$, the set $S \cap V(G_i)$ is a minimal and is a total dominating set in G_i. Thus $|S| \geq \gamma^u_{0,p}(G_j) + \sum\limits_{i=1, i\neq j}^{n} \gamma_t(G_i) \geq t_1$ and so $\gamma^u_{0,p}(G) = \min\limits_{1\leq i\leq r} \{t_i\}$.

THEOREM 15. *For an integer $k(\neq 2) \geq 1$, there is a graph G in which $\gamma(G) = \gamma^u_{0,p}(G) = k$.*

Proof. Let P_{k+2} be a path $V(P_{k+2}) = \{u_1, u_2, \ldots, u_{k+2}\}$. Add one pendant edge at each vertex of $V(P_{k+2})$- $\{u_1, u_2, u_3\}$, and name the new graph by G.

Let S be any dominating set of G. To dominate all the pendant vertices in G, S must have include at least $k - 1$ vertices from $V(G) - \{u_1, u_2, u_3\}$. Further to dominate the vertex u_1, S must include at least one more vertex. Therfore $|S| \geq (k-1) + 1 = k$. Thus $\gamma(G) \geq k$. Now, $S = \{u_1, u_4, u_5, \ldots, u_{k+2}\}$ is an UIPDS with vertices and u_1 is unique isolated vertex in $<S>$. Thus $\gamma^u_{0,p}(G) \leq k$. Since $\gamma(G) \leq \gamma^u_{0,p}(G)$ it follows that $\gamma(G) = \gamma^u_{0,p}(G) = k$.

THEOREM 16. *For two integers $b>a\geq 2$, there is a graph G with $\gamma(G) = a$ and $\gamma^u_{0,p}(G) = b$.*

Proof. Let C_{a+2} be a cycle with $V(C_{a+2}) = \{u_1, u_2, \ldots, u_{a+2}\}$. Let H be a graph such that $\gamma^u_{0,p}$ (H) $= \gamma(H) = b - a + 1$(by Theorem 15). Let $G = (C_{a+2} \circ H) - (H^{u2} \cup H^{ua+2})$.

Let S be any dominating set in G.

Then S must have u_i or at least one vertex from H^{ui} for each i with $1 \leq i \leq a+1$ and i is not equal to 2.-----(1)

Thus $\gamma(G) \geq a$. Since $V(C_{a+2}) - \{u_2, u_{a+2}\}$ is a dominating set of G, it follows that $\gamma(G) \leq a$ and so $\gamma(G) = a$

Let S be an UIPDS of G and u be the unique isolated in $<S>$.

Suppose that $u \in V(C_{a+2})$ and $u = u_1$. Then $u_2, u_3 \notin S$(by Remark 4(b)). To dominate all the vertices of $V(H^{u3})$, S have to include at least $b - a + 1$ vertices from $V(H^{u3})$(since $\gamma(H^{u3}) = b - a + 1$). Further, to dominate all the vertices of $V(H^{ui})$ for each I with $4 \leq i \leq a+1$, S must include at least one vertex of $V(H^{ui})$ or u_i. Thus $|S| \geq 1 + b - a + 1 + a - 2 \geq b$. Suppose $u \in V(C_{a+2})$ and $u = u_i$ for some $2 \leq i \leq a$. By Remark 4(b), no vertex of $\{u_{i+1}\} \cup V(H^{ui+1})$ will be in S, a contradiction to (1).

Suppose $u \in V(C_{a+2})$ and $u = u_i$ for $i = a + 1$ or $a + 2$. Then by Remark 4(b), no vertex of $\{u_{i-1}\} \cup V(H^{ui-1})$ will be in S, a contradiction to (1).

Suppose that $u \in V(H^{ui})$ for some i with $1 \leq i \leq a + 1$ and $i \neq 2$. Then $u_i \notin S$. To dominate all the vertices of V (H^{ui}), S have to include minimum b-a+1 vertices from V (H^{ui})(since $\gamma(H^{ui}) = \gamma_{0,p}^{u}$ (H^{ui}) = b- a + 1). To dominate all the vertices of V (H^{uj}) for each j with $1 \leq j \leq a + 1$ and $j \neq i$, S must include minimum one vertex of V (H^{uj}) or u_j. Therefore, $|S| \geq b - a + 1 + a - 1 \geq b$.

In all the cases, $|S| \geq b$ and hence $\gamma_{0,p}^{u}(G) \geq b$.

Let S be a $\gamma_{0,p}^{u}$ set of H^{u1}. Then $|S| = b - a + 1$ and $D = \{u_3, u_4, . . ., u_{a+1}\} \cup S$ is an UIPDS of G with $|D| = a - 1 + b - a + 1 = b$. Hence $\gamma_{0,p}^{u}(G) \leq b$.

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