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Study of Nasty Numbers and Some Properties

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Abstract

Linear programming is a technique for defining an optimum schedule of interdependent events in observation of existing resources. A short summary of nasties, the interesting properties of nasty number and Ramanujan numbers are summarized in this paper and a new technique of correlating Ramanujan theory with nasty numbers using various technique to get a nasty number is portrayed in a clear way.

Keywords; *Nasty number, Ramanujan number*

I. INTRODUCTION

If the nasty numbers is applied in linear programming, can get the optimal solution as nasty numbers. Double nasties and twin nasties are also applied in linear programming problem.

Some positive integers are double nasties. That is, there are some positive integers which can be the areas of two primitive Pythagorean triangles. For example 210 is the area of two primitive. Pythagorean triangles (20, 21, 29) and (35, 12, 37).

Some consecutive multiples of 6 are nasties like 24, 30; 54, 60; 210, 216; 330, 336; 480, 486; 540, 546; 720 and 726. They are called twin nasties.

A nasty numbers cannot be obtained from alternative nasty number by multiplying $f_q{}^2$ where f_q is any positive integer is said to be an independent nasty number. For example 6 is an independent nasty number while 24 is not. From the list of nasty numbers 6, 30, 60, 84, 180, 210, 330, 504, 546, 630, 924 and 990 are independent nasty numbers.

An outer layer view about Nasty numbers, Ramanujan numbers and its interesting properties are being explored along with few examples. A different procedure for getting nasty number using Ramanujan number theory.

Several researches [1-10] have suggested various procedures to obtain optimal solution.

II. NASTY NUMBERS

to the difference between the factors in next set of nasty numbers and the nasty number is equal to the product of each pair.

Thus a non- negative integer F^1 is a nasty number.

If $F^1 = f_1 * f_2 = f_3 * f_4$ and

$$f_1 + f_2 = f_3 - f_4.$$

Where f_1, f_2, f_3, f_4 are non-negative integers.

EXAMPLE:

Consider, 210

210 =1, 2, 3, 5, 6, 7, 10, 14, 15, 21, 30, 35, 42, 70, 105, 210.

$$210 = 6 \times 35 = 14 \times 15 \quad \text{and}$$

$$35 - 6 = 14 + 15 = 29.$$

Therefore 210 is a nasty number.

January – February 2020
ISSN: 0193-4120 Page No. 13299 - 13303

III. RAMANUJAN NUMBER

A general rule for generating Ramanujan numbers is given below. Let g, h be integers. Define a, b, c, d by using the following formulas:

$a = 1 - (g - 3h)(g^2 + 3h^2)$

$b = -1 + (g + 3h)(g^2 + 3h^2)$

$c = (g + 3h) - (g^2 + 3h^2)^2$

$d = -(g - 3h) + (g^2 + 3h^2)^2$

Then the following is identically true: $a^3 + b^3 = c^3 + d^3$.

IV. PROPERTIES OF NASTY NUMBERS AND THEOREMS

1.If F1 is a nasty number, then obviously 2 F1 is said to be a nasty number 0.

2.If x, y, z are in arithmetic progression (AP), then F1   is a nasty number.

3. is a nasty number. Where tp is positive integer.

4. is a nasty number.   Where tp is positive integer.

5.Multiplication of every three successive integers is a nasty number.

6.A number F1 is nasty if and only if it is of the form (gh) (g2 – h2), where (g,h) = 1, g > h and g and h are natural numbers of different parity and   is some positive integer.

7.Every nasty number is divisible by 6.

8.If 6p is a nasty number and p is a prime then p = 5.

9.A nasty number is never a square.

10.No nasty number except 6 is perfect.

11.No nasty number can have 2 or 8 in its unit's place.

V. PREPARE YOUR PAPER BEFORE STYLING

Theorem: 4.1

$3\,(1.2 + 2.3 + 3.4 + \ldots [t_p\,(t_p + 1)]$ is a nasty number, for every integer.

PROOF:

Let us assume that F^1 is of the given form

That is, $F^1 = 3\,(1.2 + 2.3 + 3.4 + \ldots + t_p\,(t_p + 1))$

$$F^1 = \frac{3(t_p\,(t_p + 1)\,(t_p + 2)}{3}$$

$F^1 = t_p\,(t_p + 1)\,(t_p + 2)$

$F^1 = a_0\, b_0\, c_0\, d_0$

Let $a_0 = t_p$

$b_0 = t_p + 1$

$c_0 = t_p + 2$

$d_0 = 1$

Now a_0, b_0, and c_0 are in AP with common difference d_0.

Let $f_1 = (t_p + 1)\, t_p$

$f_2 = (t_p + 2)$

$f_3 = (t_p + 1)\,(t_p + 2)$

$f_4 = t_p$

$F^1 = f_1 f_2 = f_3 f_4$

$f_1 f_2 = [(t_p + 1)\, t_p]\,[(t_p + 2)]$

$= t_p(t_p + 1)(t_p + 2)$ (1)

$f_3 f_4 = [(t_p + 1)\,(t_p + 2)]\,[t_p]$

$= t_p\,(t_p + 1)\,(t_p + 2)$ (2)

(2)

From (1) and (2) we have $f_1 f_2 = f_3 f_4$

$f_1 + f_2 = (t_p + 1)\, t_p + (t_p + 2)$

$= (t_p)^2 + t_p + t_p + 2$

January – February 2020
ISSN: 0193-4120 Page No. 13299 - 13303

$= (t_p)^2 + 2t_p + 2$ (3)

$f_3 - f_4 = (t_p + 1)(t_p + 2) - t_p$

$= (t_p)^2 + 2t_p + t_p + 2 - t_p$

$= (t_p)^2 + 2t_p + 2$ (4)

From (3) and (4) we have $f_1 + f_2 = f_3 - f_4$

Theorem: 4.2

If $F^1 = nP_3$ for all integer $n \geq 3$, then F^1 is a nasty number.

PROOF:

Given that $F^1 = nP_3$

That is, $F^1 = \dfrac{n!}{(n-3)!}$

$$F^1 = \frac{n(n-1)(n-2)(n-3)\ldots(3)(2)(1)}{(n-3)(n-4)\ldots(3)(2)(1)}$$

$F^1 = n(n-1)(n-2)$

$F^1 = a_0\, b_0\, c_0\, d_0$

Let $a_0 = n$

$b_0 = n - 1$

$c_0 = n - 2$

$d_0 = 1$

Now a_0, b_0, and c_0 are in AP with common difference d_0.

Let $f_1 = (n-1)(n-2)$

$f_2 = n$

$f_3 = (n-1)n$

$f_4 = (n-2)$

$F^1 = f_1 f_2 = f_3 f_4$

$f_1 f_2 = [(n-1)(n-2)][n]$

$= (n)(n-1)(n-2)$ (1)

$f_3 f_4 = [(n-1)n][(n-2)]$

$= n(n-1)(n-2)$ (2)

From (1) and (2) we have $f_1 f_2 = f_3 f_4$

$f_1 + f_2 = (n+1)n + (n+2)$

$= (n)^2 - 2n - n + 2 + n$

$= (n)^2 - 2t_p + 2$ (3)

$f_3 - f_4 = (n-1)n - (n-2)$

$= (n)^2 - n - n + 2$

$= (n)^2 - 2n + 2$ (4)

From (3) and (4) we have $f_1 + f_2 = f_3 - f_4$

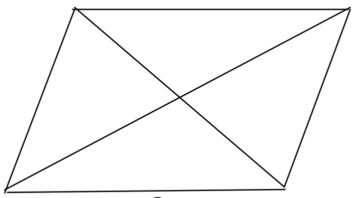
Theorem: 4.3

The area of a rhombus with integer sides is a nasty number.

PROOF:

a

bb



a

The sides of a rhombus with integer sides are given by

$a = 2 f_q g h$,

$b = f_q (g^2 - h^2)$

where g and h are natural numbers of different parity, $g > h$ with $(g, h) = 1$ and f_q is some positive integer.

Area of the rhombus

$= (1/2)[(2 f_q g h)][f_q (g^2 - h^2)]$

$= f_q^2\, g\, h\, (g^2 - h^2)$

Now g - h, g and g + h are in AP with common difference h.

Now $t_p = (g - h)(g(g + h)h)$

January – February 2020
ISSN: 0193-4120 Page No. 13299 - 13303

$t_p = g\,h\,(g^2 - h^2)$

Therefore t_p is a nasty number.

But if t_p is a nasty number then $f_q^{\,2} t_p$ is also a nasty number.

Therefore $f_q^{\,2} t_p = f_q^{\,2}\, g\, h\, (g^2 - h^2)$ is a nasty number.

Therefore Area of a rhombus is a nasty number.

Theorem: 4.4

(a) F^1, $F^1 + 8$, $F^1 + 16$ and $F^1 + 24$ cannot be simultaneously nasty.

(b) F^1, $F^1 + 16$ and $F^1 + 32$ cannot be simultaneously nasty.

PROOF:

(a) F^1 being nasty number, it must have 0, 4 or 6 in its unit's place.

(i) If F^1 has 0 in its unit's place, then $F^1 + 8$ has 8 in its unit's place.

So $(F^1 + 8)$ cannot be nasty number.

(ii) If F^1 has 4 in its unit's place, then $F^1 + 24$ has 8 in its unit's place.

So $F^1 + 24$ cannot be nasty number.

(iii) If F^1 has 6 in its unit's place then $F^1 + 16$ has 2 in its unit's place.

So $F^1 + 16$ cannot be nasty number.

Hence F^1, $F^1 + 8$, $F^1 + 16$ and $F^1 + 24$ cannot be simultaneously nasty.

(b) F1 being nasty it must have 0, 4, or 6 in its unit's place.

(i) If F^1 has 0 in its unit's place, then $F^1 + 32$ has 2 in its unit's place.

So $F^1 + 32$ cannot be nasty number.

(ii) If F^1 has 6 in its unit's place, then $F^1 + 16$ has 2 in its unit's place.

So $F^1 + 24$ cannot be nasty number.

Hence F^1, $F^1 + 16$ and $F^1 + 32$ cannot be simultaneously nasty.

Property: 4.5

Nasty numbers in Terms of Twice of Ramanujan number 1729

Below nasty numbers are in terms of digits of 1729 used twice, only with the operations addition, subtraction and multiplication.

$6 = 17 \times 2 - 91 + 72 - 9$

$24 = 17 \times 2 - 91 + 72 + 9$

$30 = (1 - 7 + 2 + 9) \times (17 - 2 - 9)$

$54 = 17 + 2 - 9 - 1 + (7 - 2) \times 9$

$60 = 1 + 7 + 29 + 1 - 7 + 29$

$84 = 1 \times 72 + 9 - 1 - 7 + 2 + 9$

$96 = 1 \times 7 - 2 + 91 + 7 + 2 - 9$

Property: 4.6

Third spoke of a hexagonal spiral is defined as

$M(t_p) = 3\,t_p^{\,2} + 1;\ t_p = 0, 1, 2, \ldots$

This gives

$M(24) = 1729.$

$t_p = 24$ is nasty number.

Property: 4.7

Square numbers product of 6 is called nasty number.

That is, $S_n = t_p^{\,2}$

$6\,t_p^{\,2}$ = nasty numbers.

Example:

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$F^1 = (6t_p + 2)(2t_p + 1)(8t_p + 3)(4t_p + 1)$ is not an independent nasty number, if $3t_p + 1$ is a perfect square.

PROOF:

$$F^1 = (6t_p + 2)(2t_p + 1)(8t_p + 3)(4t_p + 1)$$

$$= 2(3t_p + 1)(2t_p + 1)(8t_p + 3)(4t_p + 1)$$

Let $3t_p + 1$ be a perfect square say $(3t_p + 1) = k^2$

$$N = 2k^2 (2t_p + 1)(8t_p + 3)(4t_p + 1)$$

$$= k^2 (4t_p + 2)(8t_p + 3)(4t_p + 1)$$

$$= k^2 (F^1)^1 ((F^1)^1 + 1)(2 (F^1)^1 + 1),$$

where $(F^1)^1 = 4t_p + 1$

$$= k^2 (F^1)^{11} \text{ where } (F^1)^{11} \text{ is a nasty number.}$$

F^1 is not an independent.

VI. CONCLUSION

In this paper properties of nasty numbers and some theorems were discussed. Some Ramanujan properties also satisfies the nasty number properties.

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