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Certain Operation Approaches on $\alpha_{(\gamma,\gamma')(\beta,\beta')}$ -Continuous Mappings in Topological Spaces

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Abstract

In this article we introduce the concepts of $\alpha_{(\gamma,\gamma')(\beta,\beta')}$ -continuous mappings, $\alpha_{(\gamma,\gamma')(\beta,\beta')}$ -homeomorphism and study some of their basic properties

Keywords; $\alpha_{(\gamma,\gamma')}$ *-open set,* $\alpha_{(\beta,\beta')}$ *-open set* $\alpha_{(\gamma,\gamma')(\beta,\beta')}$ *-continuous mappings,* $\alpha_{(\gamma,\gamma')(\beta,\beta')}$ *- homeomorphism*

I. INTRODUCTION

Njastad [7], Kasahara [4,5], Ogata [8,9] and Kalaivani, SaiSundara Krishnan[1,2,3] introduced respectively the α-open sets, operation on topological spaces, and α-(γ,γ')-open sets. Bioperations-this concept was introduced by Maki &Noiri [6], Umehara, Maki&Noiri[10] and Umehara[11].

In this paper, the concept of $\alpha_{(\gamma,\gamma')(\beta,\beta')}$-continuous mappings have been introduced and its properties are studied. Further the $\alpha_{(\gamma,\gamma')(\beta,\beta')}$ -homeomorphism has been introduced and properties are discussed.

The following notations are used: $\alpha_{(\gamma,\gamma')}$-open set denotes the α **-(**γ**,**γ'**)**-open set , $\tau_{\alpha_{(\gamma,\gamma')}}$ denotes $\tau_{\alpha-(\gamma,\gamma')}$ - the collection of all $\alpha_{(\gamma,\gamma')}$-open sets, X_{tosp} - the topological space (X,τ), Y_{tosp} - the topological space (Y,τ), ope se - open set ,clo se - closed set, ss-subset, C_o -continuous, M_a -mapping, C_oM_a -continuous mapping.

II. PRELIMINARIES

Definition 2.1. A set $G \subseteq X_{tosp}$ is said to be an $\alpha_{(\gamma,\gamma')}$-opese if and only if $G \subseteq \tau_{\alpha_{(\gamma,\gamma')}}$ -int($\tau_{\alpha_{(\gamma,\gamma')}}$ -cl($\tau_{\alpha_{(\gamma,\gamma')}}$ -int(G))).

Theorem 2.1. Every (γ,γ') -ope se in X_{tosp} is an $\alpha_{(\gamma,\gamma')}$-ope se. However the converse need not be true.

Theorem 2.2. Let $\tau_{\alpha_{(\gamma,\gamma')}}$ be the family of $\alpha_{(\gamma,\gamma')}$-opeses in X_{tosp} .Then $\bigcup_{\alpha\in J} A_\alpha$ is also an $\alpha_{(\gamma,\gamma')}$ -ope se.

Remark 2.1. If U , V are any two $\alpha_{(\gamma,\gamma')}$-opeses in X_{tosp} , then the following example shows that $U \cap V$ need not be an $\alpha_{(\gamma,\gamma')}$-ope se.

Remark 2.2. The concepts of α -ope se and $\alpha_{(\gamma,\gamma')}$ -ope se are independent.

Remark 2.3. If X_{tosp} is a (γ,γ')-reg spa, then the idea of $\alpha_{(\gamma,\gamma')}$-opes e and α -ope se coincide.

Definition 2.2. A subset M of X_{tosp} is said to be an $\alpha_{(\gamma,\gamma')}$-clo se iff $X_{tosp} - M$ is an $\alpha_{(\gamma,\gamma')}$-ope se , which is equivalently M is an $\alpha_{(\gamma,\gamma')}$-clo se if and only if $M \supseteq \tau_{\alpha_{(\gamma,\gamma')}}$-cl ($\tau_{\alpha_{(\gamma,\gamma')}}$-int($\tau_{\alpha_{(\gamma,\gamma')}}$-cl (M))).

Definition 2.3. (i) Let $P \subseteq X_{tosp}$. Then $\tau_{\alpha_{(\gamma,\gamma')}}$-interior of P is the union of all $\alpha_{(\gamma,\gamma')}$-opeses contained in P, denoted by $\tau_{\alpha_{(\gamma,\gamma')}}$-int (P). That is given as $\tau_{\alpha_{(\gamma,\gamma')}}$-int (P) = $\cup$ { S : S is an $\alpha_{(\gamma,\gamma')}$- ope se and $S \subseteq P$ }

Remark 2.4. Let $C \subseteq X_{TS}$. Then the following holds good:

(i) $\tau_{\alpha_{(\gamma,\gamma')}}$-int (C) is the largest $\alpha_{(\gamma,\gamma')}$-ope se contained in C.

(ii) C is an $\alpha_{(\gamma,\gamma')}$-opese if an on if $\tau_{\alpha_{(\gamma,\gamma')}}$-int (C) = C.

(iii) $\tau_{\alpha_{(\gamma,\gamma')}}$ -int ($\tau_{\alpha_{(\gamma,\gamma')}}$-int (C)) = $\tau_{\alpha_{(\gamma,\gamma')}}$-int (C)

Theorem 2.3. Let E , $F \subseteq X_{tosp}$. Then:

(i) If $E \subseteq F$ then $\tau_{\alpha_{(\gamma,\gamma')}}$-int (E) $\subseteq$ $\tau_{\alpha_{(\gamma,\gamma')}}$-int (F)

(ii) $\tau_{\alpha_{(\gamma,\gamma')}}$-int (E) $\cup$ $\tau_{\alpha_{(\gamma,\gamma')}}$-int (F) $\subseteq$ $\tau_{\alpha_{(\gamma,\gamma')}}$-int ($E \cup F$)

Definition 2.4. Let $Q \subseteq X_{tosp}$. Then $\tau_{\alpha_{(\gamma,\gamma')}}$- closure of Q is the intersection of all $\alpha_{(\gamma,\gamma')}$-closescontaining Q, denoted by $\tau_{\alpha_{(\gamma,\gamma')}}$-cl (Q). That is $\tau_{\alpha_{(\gamma,\gamma')}}$-cl (Q) = $\cap$ { H : H is an $\alpha_{(\gamma,\gamma')}$- clo se and $Q \subseteq H$ }

Remark 2.5. Let X_{tosp} be a TS and γ, γ' are operations on τ. Then

(i) The $\tau_{\alpha_{(\gamma,\gamma')}}$-cl (P) is an $\alpha_{(\gamma,\gamma')}$-clo se containing P .

(ii) P is an $\alpha_{(\gamma,\gamma')}$-close if an on if $\tau_{\alpha_{(\gamma,\gamma')}}$-cl (P) = P.

Theorem 2.5. Let $\{A_\alpha : \alpha \in J\}$ be the family of $\alpha_{(\gamma,\gamma')}$-opeses in X_{tosp} .Then $\bigcup_{\alpha \in J} A_\alpha$ is also an $\alpha_{(\gamma,\gamma')}$-ope se in X_{tosp} .

Definition 2.5. A M_a is said to be an $\alpha_{(\gamma,\gamma')(\beta,\beta')}$- $C_o M_a$ if an on if for every $\alpha_{(\beta,\beta')}$-ope se, M of Y_{tosp}, the invima f_{ma}^{-1} (M) is an $\alpha_{(\gamma,\gamma')}$ -ope se in X_{tosp} .

Theorem 2.6. A M_a $f_{ma} : X_{tosp} \to Y_{tosp}$ is an $\alpha_{(\gamma,\gamma')(\beta,\beta')}$ - $C_o M_a$ if an on if for each point e in X_{tosp} and each $\alpha_{(\beta,\beta')}$-neighbourhood D of f_{ma} (e), there is an $\alpha_{(\gamma,\gamma')}$ -neigh E of b such that f_{ma} (E) $\subseteq D$.

Theorem 2.6. Let f_{ma} be a M_a. Then the following statements are equivalent:

(i) $f_{ma} : X_{tosp} \to Y_{tosp}$ is an $\alpha_{(\gamma,\gamma')(\beta,\beta')}$ - $C_o M_a$;

(ii) f_{ma} ($\tau_{\alpha_{(\gamma,\gamma')}}$ -cl(E)) $\subseteq$ $\sigma_{\alpha_{(\beta,\beta')}}$-cl(f_{ma} (E)) holds for every subset E of X_{tosp} ;

(iii) For every $\alpha_{(\beta,\beta')}$-clo se, F of Y_{tosp} , f_{ma}^{-1} (F) is an $\alpha_{(\gamma,\gamma')}$ -clo se in X_{tosp} .

Theorem 2.7. A space X_{tosp} is an $\alpha_{(\gamma,\gamma')}$ -spa if an on if for each $d \in X_{tosp}$, $\{d\}$ is $\alpha_{(\gamma,\gamma')}$-clo se or $\alpha_{(\gamma,\gamma')}$ -ope se in X_{tosp} .

III. $\alpha_{(\gamma,\gamma')(\beta,\beta')}$-CONTINUOUS MAPPINGS

Definition 3.1. A M_a $f_{ma} : X_{tosp} \to Y_{tosp}$ is said to be an $\alpha_{(\gamma,\gamma')(\beta,\beta')}$ - $C_o M_a$ if an on if for any every $\alpha_{(\beta,\beta')}$- ope se U_{os} of Y_{tosp}, the invima f_{ma}^{-1} (U_{os}) is an $\alpha_{(\gamma,\gamma')}$- ope se in X_{tosp}.

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Definition 3.2. A M_a $f_{ma}: X_{tosp} \to Y_{tosp}$ is said to be an $\alpha_{(\gamma,\gamma')(\beta,\beta')}$ - irresolute if f_{ma}^{-1} (U_{os}) is an $\alpha_{(\gamma,\gamma')}$ - ope se in X_{tosp} for every $\alpha_{(\gamma,\gamma')}$ - ope se in Y_{tosp}.

Example 3.1. Let $X_{tosp} = \{a,b,c\}$, $\tau = \{\phi, X, \{a\}, \{c\}, \{a,b\}, \{a,c\}\}$ $Y_{tosp} = \{a,b,c\}$, $\sigma = \{\phi, Y, \{a\}, \{b\}, \{a,b\}, \{a,c\}\}$.

The operation γ, γ' on τ is defined as $A^{\gamma} = A \cup \{a\}$ and $A^{\gamma'} = \begin{cases} A \text{ if } A = \{a\} \\ A \cup \{c\} \text{ if } A \neq \{a\} \end{cases}$ for every $A \in \tau$.

The operation β, β' on σ is defined as $A^{\beta} = \begin{cases} A \text{ if } b \notin A \\ cl(A) \text{ if } b \in A \end{cases}$ and $A^{\beta'} = \begin{cases} A \text{ if } b \in A \\ cl(A) \text{ if } b \notin A \end{cases}$ for every $A \in \sigma$.

We define f_{ma} as f_{ma} (a) = 1, f_{ma} (b) = 2 and f_{ma} (c) = 3. Then the invima of each $\alpha_{(\beta,\beta')}$-ope se is an $\alpha_{(\gamma,\gamma')}$ - ope se under f_{ma}. Hence f_{ma} is an $\alpha_{(\gamma,\gamma')(\beta,\beta')}$ - $C_o M_a$.

The following Remarks 3.1 and 3.2 shows that the concept of $\alpha_{(\gamma,\gamma')(\beta,\beta')}$ - $C_o M_a$ and $((\gamma,\gamma'),(\beta,\beta'))$ - irresolute M_a are independent but when X_{tosp} is a (γ,γ')-reg spa and Y_{tosp} is a (β,β')-reg spa both the concepts coincide.

Remark 3.1. The concepts of $\alpha_{(\gamma,\gamma')(\beta,\beta')}$ - $C_o M_a$ and $((\gamma,\gamma'),(\beta,\beta'))$ -irresolute M_a are independent.

Let $X_{tosp} = \{a,b,c\}$, $Y_{tosp} = \{1,2,3\}$, $\tau = \{\phi, X_{tosp}, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}\}$ and $\sigma = \{\phi, Y_{tosp}, \{1\}, \{3\}, \{1,2\}, \{1,3\}\}$.

The operation γ, γ' on τ is defined as $A^{\gamma} = A \cup \{a\}$ and $A^{\gamma'} = \begin{cases} A \text{ if } A = \{a\} \\ A \cup \{c\} \text{ if } A \neq \{a\} \end{cases}$ for every $A \in \tau$.

The operation β, β' on σ is defined as $A^{\beta} = \begin{cases} A \text{ if } b \notin A \\ cl(A) \text{ if } b \in A \end{cases}$ and $A^{\beta'} = \begin{cases} A \text{ if } b \in A \\ cl(A) \text{ if } b \notin A \end{cases}$ for every $A \in \sigma$.

We define f_{ma} as f_{ma} (a) = 1, f_{ma} (b) = 2 and f_{ma} (c) = 3. Then f_{ma} is a $((\gamma,\gamma'),(\beta,\beta'))$ - $C_o M_a$. But $f_{ma}^{-1}(\{1,2\}) = \{a,b\}$ is not an $\alpha_{(\gamma,\gamma')}$ -ope set under f_{ma}. Hence f_{ma} is not an $\alpha_{(\gamma,\gamma')(\beta,\beta')}$ - $C_o M_a$.

Remark 3.2. If X_{tosp} is a (γ,γ') - reg spa and Y_{tosp} is a (β,β') - reg spa then the concept of $((\gamma,\gamma'),(\beta,\beta'))$ - irresoluteness and $\alpha_{(\gamma,\gamma')(\beta,\beta')}$ -continuity coincide.

Definition 3.3. A ss P_{ss} of X_{tosp} is said to be an $\alpha_{(\gamma,\gamma')}$-neigh of a point c $\in X_{tosp}$ if there exists an $\alpha_{(\gamma,\gamma')}$ - ope set U_{os} such that c $\in U_{os} \subseteq P_{ss}$.

Theorem 3.1. A M_a $f_{ma}: X_{tosp} \to Y_{tosp}$ is an $\alpha_{(\gamma,\gamma')(\beta,\beta')}$ - $C_o M_a$ if an on if for each c in X_{tosp}, the inv of every $\alpha_{(\beta,\beta')}$ -neigh of f_{ma} (c) is an $\alpha_{(\gamma,\gamma')}$ -neigh of c.

Proof. Let c$\in X_{tosp}$ and B_{neigh} be an $\alpha_{(\gamma,\gamma')}$ - neigh of f_{ma} (c). By Definition 3.2, the exi a $V_{ele} \in \sigma_{\alpha_{(\beta,\beta')}}$ suctha f_{ma} (c)$\in V_{ele} \subseteq B_{neigh}$. This imp tha c$\in f_{ma}^{-1}$ (V_{ele}) $\subseteq f_{ma}^{-1}$ (B_{neigh}). Since f_{ma} is an $\alpha_{(\gamma,\gamma')(\beta,\beta')}$ - $C_o M_a$, f_{ma}^{-1} (V)$\in \tau_{\alpha_{(\gamma,\gamma')}}$. Hence f_{ma}^{-1} (B_{neigh}) is an $\alpha_{(\gamma,\gamma')}$ -neigh of c.

Conversely, let $B_{ele} \in \sigma_{\alpha_{(\beta,\beta')}}$, $A_{ele} = f_{ma}^{-1}$ (B_{ele}) and c$\in A_{ele}$. Hence by Definition 3.3 the exian $A_c \in \tau_{\alpha_{(\gamma,\gamma')}}$ suctha c$\in A_c \subseteq A_{ele}$. This imp tha $A_{ele} = \bigcup_{c \in A_{ele}} A_c$. By Theorem 2.2 A_{ele} is an $\alpha_{(\gamma,\gamma')}$-ope set in X_{tosp}. Therefore f_{ma} is an $\alpha_{(\gamma,\gamma')(\beta,\beta')}$ - $C_o M_a$.

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Theorem 3.2. A M_a $f_{ma}: X_{tosp} \to Y_{tosp}$ is an $\alpha_{(\gamma,\gamma')(\beta,\beta')}$ - C_oM_a if an on if for each point c in X_{tosp} and each $\alpha_{(\beta,\beta')}$-neigh B_{neigh} of f_{ma} (c), there is an $\alpha_{(\gamma,\gamma')}$-neigh A_{neigh} of c suctha f_{ma} (A_{neigh})$\subseteq B_{neigh}$.

Proof. Let c $\in X_{tosp}$ and B_{neigh} be an $\alpha_{(\beta,\beta')}$- neigh of f_{ma} (c). Then the exi an $O_{f(c)} \in \sigma_{\alpha_{(\beta,\beta')}}$ suctha f_{ma} (c) $\in O_{f(c)} \subseteq B_{neigh}$. It foltha c$\in f_{ma}^{-1}$ ($O_{f(c)}$)$\subseteq f_{ma}^{-1}$ (B_{neigh}). By hypothesis, f_{ma}^{-1} ($O_{f(c)}$) $\subseteq \tau_{\alpha_{(\gamma,\gamma')}}$. Let A_{ele} = f_{ma}^{-1} (B_{neigh}). Then it foltha A_{ele} is an $\alpha_{(\gamma,\gamma')}$-neigh of c and f_{ma} (A_{ele}) = f_{ma} (f_{ma}^{-1} (B_{neigh})) $\subseteq B_{neigh}$.

Conversely, let $U_{ele} \in \sigma_{\alpha_{(\beta,\beta')}}$. Take $W_{ele} = f^{-1}$ (U_{ele}). Let c$\in W_{ele}$. Then f_{ma} (c) $\in U_{ele}$. Thus U_{ele} is an $\alpha_{(\beta,\beta')}$- neigh of f_{ma} (c) and hence the exi an $\alpha_{(\gamma,\gamma')}$-neigh V_c of c suctha f_{ma} (V_c)$\subseteq U_{ele}$. Thus it foltha c $\in V_c \subseteq f_{ma}^{-1}$ (f (V_c))$\subseteq f_{ma}^{-1}$ (U_{ele}) = W_{ele}. Since V_c is an $\alpha_{(\gamma,\gamma')}$-neigh of c, which imp tha the exi a $W_c \in \tau_{\alpha_{(\gamma,\gamma')}}$ suctha c$\in W_c \subseteq W_{ele}$. This imp tha $W_{ele} = \bigcup_{c \in W_{ele}} W_c$. By Theorem 2.2, W_{ele} is an $\alpha_{(\gamma,\gamma')}$-ope set in X_{tosp}. Thus f_{ma} is an $\alpha_{(\gamma,\gamma')(\beta,\beta')}$- C_oM_a.

Theorem 3.3. Let $f_{ma}: X_{tosp} \to Y_{tosp}$ be a M_a. Then the statements described below are comparable:

(i) $f_{ma}: X_{tosp} \to Y_{tosp}$ is an $\alpha_{(\gamma,\gamma')(\beta,\beta')}$- C_oM_a.

(ii) f_{ma} ($\tau_{\alpha_{(\gamma,\gamma')}}$-cl ($A_{ss}$))$\subseteq \sigma_{\alpha_{(\beta,\beta')}}$-cl ($f_{ma}$ (A_{ss})) holds for every ss A_{ss} of X_{tosp}.

(iii) For every $\alpha_{(\beta,\beta')}$- clo set V_{clse} of Y_{tosp}, f_{ma}^{-1} (V_{clse}) is an $\alpha_{(\gamma,\gamma')}$-clo se in X_{tosp}.

Proof. (i)$\Rightarrow$ (ii) Let d$\in f_{ma}$($\tau_{\alpha_{(\gamma,\gamma')}}$-cl ($A_{ele}$)) and V_{opse} be any $\alpha_{(\beta,\beta')}$-ope se containing d. Using Theorem 3.2, the exi a point c$\in X_{tosp}$ and an $\alpha_{(\gamma,\gamma')}$-ope set U_{opse} suctha c$\in U_{opse}$ with f_{ma} (c) = d and f_{ma} (U_{opse})$\subseteq V_{opse}$. Since c$\in \tau_{\alpha_{(\gamma,\gamma')}}$-cl ($A_{ele}$), we have $U_{ele} \cap A_{ele} \neq \phi$ and hence $\phi \neq f_{ma}$ ($U_{ele} \cap A_{ele}$)$\subseteq f_{ma}$ (U_{ele})$\cap f_{ma}$ (A_{ele})$\subseteq V_{ele} \cap f_{ma}$ (A_{ele}). This imp tha d $\in \sigma_{\alpha_{(\beta,\beta')}}$-cl ($f_{ma}$ (A_{ele})).Theref f_{ma} ($\tau_{\alpha_{(\gamma,\gamma')}}$-cl ($A_{ele}$)) $\subseteq \sigma_{\alpha_{(\beta,\beta')}}$-cl ($f_{ma}$ (A_{ele})).

(ii) $\Rightarrow$ (iii) Let V_{ele} be an $\alpha_{(\beta,\beta')}$- clo set in Y_{tosp}. Then $\sigma_{\alpha_{(\beta,\beta')}}$-cl ($V_{ele}$) = V_{ele}. By (ii) f_{ma} ($\tau_{\alpha_{(\gamma,\gamma')}}$- cl ($f_{ma}^{-1}$ (V_{ele}))) $\subseteq \sigma_{\alpha_{(\beta,\beta')}}$-cl ($f_{ma}$ (f_{ma}^{-1} (V_{ele}))) $\subseteq \sigma_{\alpha_{(\beta,\beta')}}$-cl ($V_{ele}$) = V_{ele} holds. Therefore $\tau_{\alpha_{(\gamma,\gamma')}}$-cl ($f_{ma}^{-1}$ (V_{ele})) $\subseteq f_{ma}^{-1}$ (V_{ele}) and f_{ma}^{-1} (V_{ele}) = $\tau_{\alpha_{(\gamma,\gamma')}}$-cl ($f_{ma}^{-1}$($V_{ele}$)). Hence f^{-1} (V_{ele}) is an $\alpha_{(\gamma,\gamma')}$-clo set in X_{tosp}.

(iii)$\Rightarrow$ (i) Let B_{ele} be any $\alpha_{(\beta,\beta')}$-ope set in Y_{tosp}. Consider V_{ele} = $Y_{tosp} - B_{ele}$. Then V_{ele} is $\alpha_{(\beta,\beta')}$- clo set in Y_{tosp}. By (iii) f_{ma}^{-1} (V_{ele}) is an $\alpha_{(\gamma,\gamma')}$-clo set in X_{tosp}. Hence f_{ma}^{-1} (B_{ele}) = $X_{tosp} - f_{ma}^{-1}$ ($Y_{tosp} - B_{ele}$) = $X_{tosp} - f_{ma}^{-1}$ (V_{ele}) is an $\alpha_{(\gamma,\gamma')}$-ope set in X_{tosp}. Hence f_{ma} is an $\alpha_{(\gamma,\gamma')(\beta,\beta')}$ - C_oM_a.

Theorem 3.4. Let $f_{ma}: X_{tosp} \to Y_{tosp}$ be an $\alpha_{(\gamma,\gamma')(\beta,\beta')}$ - C_o and injective M_a. If Y_{tosp} is an $\alpha_{(\beta,\beta')}$- T_2 spa (resp. $\alpha_{(\beta,\beta')}$ - T_1 spa), then X_{tosp} is $\alpha_{(\gamma,\gamma')}$- T_2 spa (resp. $\alpha_{(\gamma,\gamma')}$ - T_1 spa).

Proof. Suppose Y_{tosp} is an $\alpha_{(\beta,\beta')}$- T_2 spa. Let g and h be two distinct points of X_{tosp}. Then, the exi two $\alpha_{(\beta,\beta')}$-ope sets E_{opse} and F_{opse} suctha f_{ma} (g) $\in E_{opse}$, f_{ma} (h) $\in F_{opse}$ and $E_{opse} \cap F_{opse} = \phi$. Since f_{ma} is an $\alpha_{(\gamma,\gamma')(\beta,\beta')}$- C_oM_a, for E_{opse} and F_{opse}, the exi two $\alpha_{(\gamma,\gamma')}$- ope sets W_{opse} and S_{opse} suctha g$\in W_{opse}$ and h $\in S_{opse}$, f_{ma}(W_{opse}) $\subseteq E_{opse}$ and f_{ma} (S_{opse}) $\subseteq F_{opse}$, imp tha $W_{opse} \cap S_{opse} = \phi$. Hence X_{tosp} is an $\alpha_{(\gamma,\gamma')}$-$T_2$

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spa. In a similar way we can prove that X_{tosp} is an $\alpha_{(\gamma,\gamma')}$ - T_1 spa whenever Y_{tosp} is an $\alpha_{(\beta,\beta')}$ - T_1 spa.

Theorem 3.5. Let $f_{ma}: X_{tosp} \to Y_{tosp}$ and $g_{ma}: Y_{tosp} \to Z_{tosp}$ be two M_a s. If f_{ma} is an $\alpha_{(\gamma,\gamma')(\beta,\beta')}$ - $C_o M_a$ and g_{ma} is an $\alpha_{(\beta,\beta')(\delta,\delta')}$ - $C_o M_a$, then $g_{ma} \circ f_{ma}: X_{tosp} \to Z_{tosp}$ is an $\alpha_{(\gamma,\gamma')(\delta,\delta')}$ - $C_o M_a$.

Proof. Proof follows from the Definition 3.1.

Definition 3.4. Let A_{ss} be a subset of X_{tosp} and p be any point in X_{tosp}. Then p is called an $\alpha_{(\gamma,\gamma')}$-limit point of A_{ss} if $U_{opse} \cap (A_{ss} - \{p\}) \neq \phi$, for any $\alpha_{(\gamma,\gamma')}$-ope set U_{opse} containing p. The set of all $\alpha_{(\gamma,\gamma')}$-limit points of A_{ss} is called an $\alpha_{(\gamma,\gamma')}$-derived set of A_{ss} and is denoted by $d_{\alpha_{(\gamma,\gamma')}}(A_{ss})$.

Remark 3.3. Let A_{ss}, B_{ss} are any two subsets of X_{tosp}. Then,

(i) if $A_{ss} \subseteq B_{ss}$, then $d_{\alpha_{(\gamma,\gamma')}}(A_{ss}) \subseteq d_{\alpha_{(\gamma,\gamma')}}(B_{ss})$

(ii) p$\in d_{\alpha_{(\gamma,\gamma')}}(A_{ss})$ if and only if p$\in \tau_{\alpha_{(\gamma,\gamma')}}$-cl $(A_{ss} - \{p\})$.

Proof: Follows from the Definition 3.4.

Theorem 3.6. Let A_{ss} and B_{ss} are any two subsets of X_{tosp}. Then the following statements hold good.

(i) $A_{ss} \cup d_{\alpha_{(\gamma,\gamma')}}(A_{ss}) \subseteq \tau_{\alpha_{(\gamma,\gamma')}}$-cl (A_{ss})

(ii) $d_{\alpha_{(\gamma,\gamma')}}(A_{ss} \cup B_{ss}) = d_{\alpha_{(\gamma,\gamma')}}(A_{ss}) \cup d_{\alpha_{(\gamma,\gamma')}}(B_{ss})$

(iii) $\bigcup_i d_{\alpha_{(\gamma,\gamma')}}(A_{(ss)i}) = d_{\alpha_{(\gamma,\gamma')}}(\bigcup_i (A_{(ss)i}))$

(iv) $d_{\alpha_{(\gamma,\gamma')}}(d_{\alpha_{(\gamma,\gamma')}}(A_{ss})) \subseteq d_{\alpha_{(\gamma,\gamma')}}(A_{ss})$

(v) $\tau_{\alpha_{(\gamma,\gamma')}}$-cl $(d_{\alpha_{(\gamma,\gamma')}}(A_{ss})) = d_{\alpha_{(\gamma,\gamma')}}(A_{ss})$

Proof.(i)Suppose that e$\in A_{ss} \cup d_{\alpha_{(\gamma,\gamma')}}(A_{ss})$, then to prove that e$\in \tau_{\alpha_{(\gamma,\gamma')}}$-cl (A_{ss}). If e$\in A_{ss}$ then e $\in \tau_{\alpha_{(\gamma,\gamma')}}$-cl (A_{ss}). If e $\notin A_{ss}$, then to prove that e$\in \tau_{\alpha_{(\gamma,\gamma')}}$-cl (A_{ss}).Suppose not , if there is an $\alpha_{(\gamma,\gamma')}$- clo se C_{clse} containing A_{ss} but not containing e. Then e$\in X_{tosp} - C_{clse}$, which is an $\alpha_{(\gamma,\gamma')}$-ope se and $U_{opse} \cap A_{ss} = \phi$. This implies that e$\notin d_{\alpha_{(\gamma,\gamma')}}(A_{ss})$. This contradiction proves that e$\in \tau_{\alpha_{(\gamma,\gamma')}}$-cl$(A_{ss})$. Hence $A_{ss} \cup d_{\alpha_{(\gamma,\gamma')}}(A_{ss}) \subseteq \tau_{\alpha_{(\gamma,\gamma')}}$ - cl (A_{ss}).

(ii) Let e$\in d_{\alpha_{(\gamma,\gamma')}}(A_{ss} \cup B_{ss})$. By the Definition 3.4, we obtain that $\phi \neq U_{opse} \cap ((A_{ss} \cup B_{ss}) - \{e\}) = U_{opse} \cap [(A_{ss} - \{e\}) \cup (B_{ss} - \{e\})] = [U_{opse} \cap (A_{ss} - \{e\})] \cup [U_{opse} \cap (B_{ss} - \{e\})]$ and hence either e$\in d_{\alpha_{(\gamma,\gamma')}}(A_{ss})$ or $d_{\alpha_{(\gamma,\gamma')}}(B_{ss})$.Therefore $d_{\alpha_{(\gamma,\gamma')}}(A_{ss} \cup B_{ss}) \subseteq d_{\alpha_{(\gamma,\gamma')}}(A_{ss}) \cup d_{\alpha_{(\gamma,\gamma')}}(B_{ss})$. The result $d_{\alpha_{(\gamma,\gamma')}}(A_{ss}) \cup d_{\alpha_{(\gamma,\gamma')}}(B_{ss}) \subseteq d_{\alpha_{(\gamma,\gamma')}}(A_{ss} \cup B_{ss})$, follows by the Remark 3.3(i).

(iii) Proof follows from the Remark 3.3 and (ii)

(iv) Suppose that, e$\notin d_{\alpha_{(\gamma,\gamma')}}(A_{ss})$. Then e $\notin \tau_{\alpha_{(\gamma,\gamma')}}$-cl $(A_{ss} - \{e\})$. This imp tha the exi an $\alpha_{(\gamma,\gamma')}$- ope se U_{opse} suctha e$\in U_{opse}$ and $U_{opse} \cap (A_{ss} - \{e\}) = \phi$. We have to prove that e$\notin d_{\alpha_{(\gamma,\gamma')}}(d_{\alpha_{(\gamma,\gamma')}}(A_{ss}))$. Suppose on the contrary that e$\in d_{\alpha_{(\gamma,\gamma')}}(d_{\alpha_{(\gamma,\gamma')}}(A_{ss}))$. Then e$\in \tau_{\alpha_{(\gamma,\gamma')}}$-cl $(d_{\alpha_{(\gamma,\gamma')}}(A_{ss}) - \{e\})$. Since e$\in U_{opse}$, we have $U_{opse} \cap (d_{\alpha_{(\gamma,\gamma')}}(A) - \{e\}) \neq \phi$. Therefore there is a q $\neq$e suctha q $\in U_{opse} \cap (d_{\alpha_{(\gamma,\gamma')}}(A_{ss}))$. This imp tha q$\in (U_{opse} - \{e\}) \cap (d_{\alpha_{(\gamma,\gamma')}}(A_{ss}) - \{e\})$. Hence $((U_{opse} - \{e\}) \cap (d_{\alpha_{(\gamma,\gamma')}}$

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$(A_{ss}) - \{e\}$) $) \neq \phi$, a contradiction to the fact that $U_{opse} \cap$ ($d_{\alpha(\gamma,\gamma')}$ $(A_{ss}) - \{e\}$) $= \phi$. This imptha e $\notin$ $d_{\alpha(\gamma,\gamma')}$ ($d_{\alpha(\gamma,\gamma')}$ (A_{ss})) and hence $d_{\alpha(\gamma,\gamma')}$ ($d_{\alpha(\gamma,\gamma')}$ $(A_{ss}$ $)) \subseteq$ $d_{\alpha(\gamma,\gamma')}$ (A_{ss}).

(v) This follows from the Definition 3.4.

Theorem 3.7. A M_a $f_{ma}: X_{tosp} \to Y_{tosp}$ is an $\alpha_{(\gamma,\gamma')(\beta,\beta')}$ - $C_o M_a$ if an on if f_{ma} ($d_{\alpha(\gamma,\gamma')}$ $(A_{ss})) \subseteq \sigma_{\alpha(\beta,\beta')}$ -cl ((A_{ss})) , for all $A_{ss} \subseteq X_{tosp}$.

Proof. Let $f_{ma}: X_{tosp} \to Y_{tosp}$ be an $\alpha_{(\gamma,\gamma')(\beta,\beta')}$ - $C_o M_a$. Let $A_{ss} \subseteq X_{tosp}$, and e $\in$ $d_{\alpha(\gamma,\gamma')}$ (A_{ss}). Assume that f_{ma} (e) $\notin$ f_{ma} (A_{ss}) and let V_{neigh} denote an $\alpha_{(\beta,\beta')}$ - neigh of f_{ma} (e). Since f_{ma} is an $\alpha_{(\gamma,\gamma')(\beta,\beta')}$ - $C_o M_a$, by Theorem 3.2 , the exi an $\alpha_{(\gamma,\gamma')}$ - neigh U_{neigh} of e such that f_{ma} $(U_{neigh}) \subseteq V_{neigh}$. From e $\in$ $d_{\alpha(\gamma,\gamma')}$ (A_{ss}), it foltha $U_{neigh} \cap A_{ss} \neq \phi$. The exi at least an element a $\in U_{neigh}$ $U \cap A_{ss}$, imp tha f_{ma} (a) $\in$ f_{ma} (A_{ss}) and f_{ma} (a) $\in V_{neigh}$. Since f_{ma} (e) $\notin$ f_{ma} (A_{ss}), we have f_{ma} (a) $\neq$ f_{ma} (e). Thus every $\alpha_{(\beta,\beta')}$ - neigh of f_{ma} (e) contains an element f_{ma} (a) of f_{ma} (A_{ss}) different from f_{ma} (e). Hence , f_{ma} (e) $\in$ $d_{\alpha(\beta,\beta')}$ (f_{ma} (A_{ss})). Using Theorem 3.6 (i) f_{ma} ($d_{\alpha(\gamma,\gamma')}$ $(A_{ss}$)) $\subseteq \sigma_{\alpha(\beta,\beta')}$ - cl (f_{ma} (A_{ss})).

Conversely, suppose that f_{ma} is not an $\alpha_{(\gamma,\gamma')(\beta,\beta')}$ - $C_o M_a$. Then by Theorem 3.2 , the exi e $\in X_{tosp}$ and an $\alpha_{(\beta,\beta')}$ - neigh V_{neigh} of f_{ma} (e) suctha every $\alpha_{(\gamma,\gamma')}$ - neigh U_{neigh} of e contains at least one element a $\in$ U_{neigh} , for which f_{ma} (a) $\notin V_{neigh}$. Let A_n = {a $\in X_{tosp}$: f_{ma} (a) $\notin V_{neigh}$ }. Since f_{ma} (e) $\in V_{neigh}$, therefore e $\notin$ A_n and hence f_{ma} (e) $\notin$ f_{ma} (A_n). Since f_{ma} $(A_n) \cap$ (V_{neigh} – f_{ma} (e)) $= \phi$, therefore f_{ma} (e) $\notin$ $d_{\alpha(\beta,\beta')}$ (f_{ma} (A_n)). It foltha f_{ma} (e) $\in$ f_{ma} ($d_{\alpha(\beta,\beta')}$ (A_n)) – (f_{ma} $(A_n) \cup$ $d_{\alpha(\beta,\beta')}$ (f_{ma} (A_n))) $\neq \phi$, which is a contradiction to the given condition. Hence f_{ma} is an $\alpha_{(\gamma,\gamma')(\beta,\beta')}$ - $C_o M_a$.

Theorem 3.8. Let $f_{ma}: X_{tosp} \to Y_{tosp}$ be an one-to-one M_a . Then f_{ma} is an $\alpha_{(\gamma,\gamma')(\beta,\beta')}$ - $C_o M_a$ if and only if f_{ma} ($d_{\alpha(\gamma,\gamma')}$ $(A_{ele})) \subseteq$ $d_{\alpha(\beta,\beta')}$ (f_{ma} (A_{ele})), for all A_{ele} $\subseteq X_{tosp}$.

Proof. Let $A_{ele} \subseteq X_{tosp}$, e $\in$ $d_{\alpha(\gamma,\gamma')}$ (A_{ele}) and V_{neigh} be an $\alpha_{(\beta,\beta')}$ -neig of f_{ma} (e). Since f_{ma} is an $\alpha_{(\gamma,\gamma')(\beta,\beta')}$ - $C_o M_a$ by Theorem 3.2, the exi an $\alpha_{(\gamma,\gamma')}$ - neigh U_{neigh} of e suctha f_{ma} $(U_{neigh}) \subseteq V_{neigh}$. But e $\in$ $d_{\alpha(\gamma,\gamma')}$ (A_{ele}) gives the exi an element a $\in U_{neigh} \cap A_{ele}$ sucthaa $\neq$ e, f_{ma} (a) $\in$ f_{ma} (A_{ele}) and since f_{ma} is one-to-one, f_{ma} (a) $\neq$ f_{ma} (e). Thus every $\alpha_{(\beta,\beta')}$ -neigh V_{neigh} of f_{ma} (e) contains an element f_{ma} (a) of f_{ma} (A_{ele}) different from f_{ma} (e). Consequently f_{ma} (e) $\in$ $d_{\alpha(\beta,\beta')}$ (f_{ma} (A_{ele})). Therefore f_{ma} ($d_{\alpha(\gamma,\gamma')}$ $(A_{ele})) \subseteq$ $d_{\alpha(\beta,\beta')}$ (f_{ma} (A_{ele})), for all $A_{ele} \subseteq X_{tosp}$.

Converse part follows from the Theorem 3.7.

Definition 3.5. A M_a $f_{ma}: X_{tosp} \to Y_{tosp}$ is an $\alpha_{(\gamma,\gamma')(\beta,\beta')}$ - homeomorphism, if is a bijective , $\alpha_{(\gamma,\gamma')(\beta,\beta')}$ - C_o M_a and f_{ma}^{-1} is an $\alpha_{(\gamma,\gamma')(\beta,\beta')}$ - $C_o M_a$.

Remark 3.4 Every bijective, $\alpha_{(\gamma,\gamma')(\beta,\beta')}$ - C_o and $\alpha_{(\gamma,\gamma')(\beta,\beta')}$ -clo M_a is an $\alpha_{(\gamma,\gamma')(\beta,\beta')}$ -homeomorphism.

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Theorem 3.9. Let $f_{ma}: X_{tosp} \to Y_{tosp}$ be an $\alpha_{(\gamma,\gamma')(\beta,\beta')}$ - homeomorphism. If X_{tosp} is an $\alpha_{(\gamma,\gamma')}$ - $T_{\frac{1}{2}}$ -spa then Y_{tosp} is an $\alpha_{(\beta,\beta')}$ - $T_{\frac{1}{2}}$ -spa.

Proof. Let $\{b\}$ be a singleton se of Y_{tosp}. Then the exi a point a of X_{tosp} such that $b = f_{ma}(a)$. By 2.7 Theorem, it folltha the singleton se $\{b\}$ is either an $\alpha_{(\beta,\beta')}$ -ope se or an $\alpha_{(\beta,\beta')}$ -clo se. Therefore Y_{tosp} is an $\alpha_{(\beta,\beta')}$ - $T_{\frac{1}{2}}$ -spa.

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