January – February 2020
ISSN: 0193-4120 Page No. 13220 - 13222

Spinor Interaction with Gravitational Field in Sigma Model

AL-Baidhani Ahmed Kamil [1]

[1]Institute for Physical Research and Technology, Peoples' Friendship University of Russia, RUDN, Moscow
[1]ahmedkamilkh@yandex.ru

Article Info
Volume 82
Page Number: 13220 - 13222
Publication Issue:
January-February 2020

Article History
***Article Received**: 18 May 2019*
***Revised**: 14 July 2019*
***Accepted**: 22 December 2019*
***Publication**: 24 February 2020*

Abstract

A unify of all interaction in Faddeev-Skyrme model could be based on a higher - dimensional theory including only spinor fields.16-spinor realization in the chiral Sigma model to describe leptons and baryons as topological charge. At large distances Schwarzschild solutions are applied in our case. At small distances the close-string appear in strong gravity. The model is based on the special 8-spinor identity introduced by Brioschi.

Keywords; *16-spinor, chiral model, topological solitons.*

I. INTRODUCTION

In this work we use the Einstein idea [1] to describe particles or in other words, the soliton concept of Einstein-de Broglie according to this hypothesis both light and matter have dual character as wave and as particle has worked well in physics. When electron is taken as wave, it must be associated with wavelength, frequency and energy then according to plank's equation. If electron is taken as particle then its energy is given by Einstein equation. The extension of these ideas for the three-dimensional case was expressed by the British physicist Skyrme, who owns a great idea of considering baryons as topological solitons, in 1954. Extended topological structure such as heavy (baryons) and light (leptons) particles.

In the first model (Skyrme, 1954) [2,3] topological charge takes structure form: $Q = \deg(\mathrm{S}^3 \to \mathrm{S}^3)$, has been interpreted as the baryon number B and it played the role of the generator of the Homotopy group, $(\pi_3(S^3) = \mathbb{Z})$ Skyrme model.

In the second model (Faddeev, 1972) [4,5] Hopf invariant Q_H interpreted as lepton number L and it played the role of the generator of the Homotopy group,$\pi_3(S^2) = \mathbb{Z}$ Faddeev model.

The topological charge increases geometric characteristic of the field manifold model. It is the characteristic of chiral model in which the field takes values in a group or field of a homogeneous space. There are basically two different types of topological solitons: the chiral solitons, for physical fields which tend to infinity, and the Higgs solitons, basically for fields with nontrivial and different asymptotic behavior at infinity and a degenerated classical vacuum [6,7].We introduce 16-spinor filed: $(\Psi = \Psi_1 \oplus \Psi_2, \lambda_\mathrm{i} = \mathrm{I}_4 \otimes \sigma_\mathrm{i} \otimes \mathrm{I}_2)\Psi_i$, $i = 1{,}2$ being 8-spinor and σ_i stands for Pauli matrices for each 8-spinor Ψ the following Brioschi identity form:

$$j_\mu j^\mu - \tilde{j}_\mu \tilde{j}^\mu = s^2 + p^2 + \vec{v}^2 + \vec{a}^2 \qquad (1)$$

where the following spinor takes form:

$$s = \overline{\Psi}\Psi, \quad p = i\overline{\Psi}\gamma_\mu\Psi, \quad \vec{v}^2 = \overline{\Psi}\lambda\Psi, \quad \vec{a} = \overline{\Psi}\gamma_\mu\lambda\Psi$$

$$j_\mu = \overline{\Psi}\gamma_\mu\Psi, \quad , \quad \tilde{j}_\mu = \overline{\Psi}\gamma_\mu\gamma_5\Psi,$$

Published by: The Mattingley Publishing Co., Inc.

January – February 2020
ISSN: 0193-4120 Page No. 13220 - 13222

In this work we construct at ourLagrangian density of our model, that is corresponding with Skyrme and Faddeev models can reads:($\mathcal{L} = \mathcal{L}_{spin} + \mathcal{L}_g$),therefor interaction of spinor with the gravitational spinor field can reads:

$$\nabla_v \Psi = (\mathrm{D}_v - \Gamma_v)\Psi \qquad (2)$$

Our aim in this work to study structure of metric in geometric Toroidal Coordinates and find gravitation connectionΓ_v.

Interaction with Gravitational Field

Now let us use special geometric toroidal coordinates ($x^0 = t, x^1 = r, x^2 = \vartheta, x^3 = \Phi$) with boundary conditions of coordinates ($0 \leq r \leq \infty, \vartheta \in [\pi, -\pi], \Phi \in [0,2\pi]$), where the scale parameter a inserted in cylindrical coordinates by using substitution $z = rcos\vartheta,\ \rho = a + rsin\vartheta$, therefore metric will take the form:

$$ds^2 = \mathrm{e}^{2\eta} dt^2 - e^{-2\alpha} dr^2 - e^{-2\beta} d\vartheta^2 - e^{-2\gamma} d\Phi^2 \qquad (3)$$

these function $(\eta, \alpha, \beta, \gamma)$ depend only on two parameters (r and ϑ). The spinor affine connection matricesΓ_u are defined as:

$$\Gamma_u = \frac{1}{8}\left[\partial_u \gamma_v - \Gamma^{\rho}_{uv}\gamma_\rho, \gamma^v\right] \qquad (4)$$

where the tetrad ($e^0_0 = \mathrm{e}^{\eta}, e^1_1 = e^{\alpha}, e^2_2 = e^{\beta}, e^3_3 = e^{\gamma}$) corresponed to metric in Eq (1) and (Γ^{ρ}_{uv})Christopher symbolwhich means of comparing vectors in two nearby tangent spaces while the actual objects used. All basis vectors and their expanded in terms of a number. Affine connection is related to the metric for torsion free manifold also can be used in expressions involved differential operators. Let us write Dirac matrix in high dimension for metric:

$$\gamma_{(0)} = \mathrm{e}^{\eta}(I_2 \otimes \sigma_1 \otimes I_4)$$

$$\gamma_{(1)} = -e^{\alpha}(\tau_1 \otimes i\sigma_2 \otimes I_4)$$

$$\gamma_{(2)} = -e^{\beta}(\tau_2 \otimes i\sigma_2 \otimes I_4)$$

$$\gamma_{(3)} = -e^{\gamma}(\tau_3 \otimes i\sigma_2 \otimes I_4)$$

contravariant components of Dirac matrix:

$$\gamma^{(0)} = \mathrm{e}^{-\eta}(I_2 \otimes \sigma_1 \otimes I_4)$$

$$\gamma^{(1)} = -\mathrm{e}^{-\alpha}(\tau_1 \otimes i\sigma_2 \otimes I_4)$$

$$\gamma^{(2)} = -\mathrm{e}^{-\beta}(\tau_2 \otimes i\sigma_2 \otimes I_4)$$

$$\gamma^{(3)} = -\mathrm{e}^{-\gamma}(\tau_3 \otimes i\sigma_2 \otimes I_4)$$

Where τ_i are Pauli matrices for specical geometric Toroidal Coordinates:

$$\tau_1 = \begin{bmatrix} \cos\vartheta & \sin\vartheta\, e^{-i\Phi} \\ \sin\vartheta\, e^{i\Phi} & -\cos\vartheta \end{bmatrix}$$

$$\tau_2 = \begin{bmatrix} -\sin\vartheta & \cos\vartheta\, e^{-i\Phi} \\ \cos\vartheta\, e^{i\Phi} & \sin\vartheta \end{bmatrix}$$

$$\tau_3 = \begin{bmatrix} 0 & -ie^{-i\Phi} \\ ie^{i\Phi} & 0 \end{bmatrix}$$

At large distance our metric has form:

$$ds^2 = dt^2 - (dr^2 + r^2 d\theta^2 + (a + rsin\vartheta)^2 d\Phi^2) \qquad (5)$$

Let us find solution of Spinor affine matrices for Eq (5):

$$\Gamma_0 = \frac{1}{2}(\mathrm{e}^{\eta-\alpha}\partial_1\eta N_1 + \mathrm{e}^{\eta-\beta}\partial_2\eta N_2)$$

$$\Gamma_1 = -\frac{1}{2}\mathrm{e}^{-2\beta}\partial_2\alpha M_3$$

$$\Gamma_2 = -\frac{1}{2}M_3(\partial_1\beta\mathrm{e}^{\beta-\alpha} - 1)$$

$$\Gamma_3 = \frac{1}{2}M_2(sin\vartheta\mathrm{e}^{-\gamma-\alpha} - \mathrm{e}^{-2\alpha}\partial_1\gamma) - \frac{1}{2}M_1\left(cos\vartheta\mathrm{e}^{-\gamma-\beta} - \mathrm{e}^{-2\beta}\partial_2\gamma\right)$$

At small distance strong gravity[8].is an non mainstream theoretical approach to particle confinement have both a particle scale gravity and cosmology. The young Quantum Chromodynamics (QCD) theory by several theorists including Weinberg and Glashow [9,10], showed that the particle level gravity approach can probably produce

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confinement and asymptotic freedom while not required a force, our metric has form:

$$ds^2 = (a + rsin\vartheta)^2 dt^2 - dr^2 - r^2 d\vartheta^2 - (a + rsin\vartheta)^2 d\Phi^2 \qquad (6)$$

Let us find solution of Spinor affine matrices for Eq (6):

$$\Gamma_0 = \frac{1}{2}\begin{bmatrix} 0 & e^{-i\Phi} \\ e^{i\Phi} & 0 \end{bmatrix} \otimes \sigma_3 \otimes I_4$$

$$\Gamma_i = 0$$

CONCLUSION

Using special geometric Toroidal coordinates and introduce metric which has two forms and studying behavior of fields at small and large distance. found solutions of spinor affine connection matricesΓ_u at large and small distance respectively. In our case Lagrangian ($\mathcal{L} = \mathcal{L}_{sp} + \mathcal{L}_g$) field has two parts spinor and gravity. We consider leptons sector within the scope of the united model for an axially-symmetric field. For this purpose, the vacuum condition at a large distance is analyzed.

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