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Effective Spinor Model of Leptons and Baryons

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Abstract

we study the existing models for describing the particles as topological solitons, the Skyrme model, which allows us to describe baryons as topological solitons and simulate the internal structure of hadrons and light nuclei, as well as the Faddeev model, which allows us to describe the leptons as topological solitons. Combining these two approaches is an interesting problem, in which hypothetically, it is possible to consider hadrons and leptons as two possible phases of some effective spinor model.

We will consider a possible variant of the effective model in which the mechanism of spontaneous disorder of symmetry is eliminated if we use its 16-spinor realization, and we will concretize it on a lepton sector. We use the Higgs potential of the special kind, considering the special properties of 16-spinors mentioned by Cartan.

INTRODUCTION

Structure of Lagrangian chiral models

When designing Lagrangian chiral models, the principles based on spontaneous disturbance of chiral symmetry have been used[1,2]. If you build for the field $g(\vec{r},t): R^{3+1} \to G$ Lagrangian $L(g, \partial_\mu g)$, that need two requirements:

1. Invariance relative to local group H

$$h(\vec{r},t): g(\vec{r},t) \to g(\vec{r},t)h(\vec{r},t)$$

2. Global group invariance G

$$g_0: g(\vec{r},t) \to g_0 g(\vec{r},t)$$

then by virtue of the first condition, the field function for such Lagrangian takes values in the space of the left classes of contiguity G/Hwhich means it's a chiral field. If we limit ourselves to members of the fourth and second order by derivatives, the Lagrangian of the chiral model takes the following form:

$$L = \alpha L_1 + \beta L_1 (1)$$

where, $L_1 = Sp(B_\mu B^\mu)$, $L_1 = Sp(f_{\mu\nu})^2$, $f_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu$

Here are$B_\mu$anda_ν- 4-vectors. To illustrate the application of topological charges, we will describe some models in which the energy of field E is estimated from below through the topological charge or some function from it.

Skyrme model

In 1961, Tony Skyrme[3] proposed a model for a unified description of mesons and baryons, in which the physical field took values on the diversity of the group$SU(2)$. Skyrme's approach was based on original physical hypotheses according to which baryons are topologically non-trivial excitations of the pion field. In this case, the topological charge arising in theory plays the role of a baryon charge.

Group$SU(2)$ three-dimensional isomorphism, and topological charge is interpreted as the degree of display$Q = deg(R^3 \to S^3)$,spatial sphere S^3in the fieldS^3. Accordingly, the field model configurations are classified using the third homotopic group$\pi_3(S^3) = \mathbb{Z}$, where $\mathbb{Z}$ is an Abelian group of integers. In this case, the current density is equal[4]:

$$j^\mu = \frac{1}{12\pi^2} \varepsilon_{ijk} \varepsilon^{abcd} \Phi^a \partial_i \Phi^b \partial_j \Phi^c \partial_k \Phi^d (2)$$

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When calculating the charge, it is convenient to parameterizeS^3angle variables$(\theta,\gamma,\beta)$by putting:

$$\Phi^1 + i\Phi^2 = \sin\theta \sin\beta \, e^{i\gamma}, \;\; \Phi^3 = \sin\theta \cos\beta \, , \; \Phi^4 = \cos\beta \, , (\theta,\beta) \;\; \in (0,2\pi)$$

As a result, we'll find the following:

$$Q = \frac{-1}{2\pi^2} \int d^3\, x \sin^2\theta \sin^2\beta \, (\nabla\theta[\nabla\beta\nabla\gamma]) \qquad (3)$$

To get an energy estimate Eovercharge Q the main chiral field$g(\Phi) \in SU(2)$that is: $g(\Phi) = \Phi^0 + i\tau\Phi^{\mathrm{i}}$ where τ- Paulie's matrices.

Let's make up the left chiral current (right variant). $I_\mu = \partial_\mu g . g^{-1}$, for whom identity is fair$\left[I_\mu \, , I_v\right] = \partial_\mu I_v - \partial_v I_\mu$, and through which the topological current is expressed as follows:

$$j^\mu = \frac{1}{48\pi^2} \varepsilon^{\mu v \sigma\tau} Sp[I_v \, , I_\sigma] I_\tau \qquad (4)$$

Let's write down free Lagrangian first: $\mathcal{L} = \frac{-1}{4\lambda^2} Sp\;(I^i I_i)$As soon as:

$$Q = \frac{-1}{48\pi^2} \int d^3\, x \; \varepsilon^{\mu v \sigma} \, Sp\big(\left[I_\mu \, , I_v\right] I_\sigma\big)$$

Then

$$\frac{\varepsilon}{\lambda}|Q| \le \frac{-1}{2\lambda^2} \int d^3\, x Sp\;(I^i I_i) + \frac{\varepsilon^2}{2} \int d^3\, x Sp\;(\varepsilon^{\sigma\mu v}\left[I_\mu \, , I_v\right])^2 \qquad (5)$$

Therefore, the Skyrme model selects the Lagrangian species density:

$$\mathcal{L}_{\mathrm{Skyrme}} = \frac{-1}{4\lambda^2} Sp\;(I^i I_i) + \frac{\varepsilon^2}{16} Sp\left[I_\mu \, , I_v\right]^2 (6)$$

The energy function is evaluated from below through a linear function from the topological charge[5,6]:

$$H_{Sk}[g(\vec{r}, t)] > 6\sqrt{2}\pi^2 \frac{\varepsilon}{\lambda} |Q[g(\vec{r}, t)]| (7)$$

which ensures the stability of field configurations that implement the lower end of the energy function in each topological class.

The Hamiltonian of the Skyrme model is invariant to the group. $G = diag[SO(2)_s \otimes SO(2)_l]$, where the indices $(S)$and (I)correspond to the spatial needle rotation. The same applies to the Hamiltonian and$SU(2)$ – the Skyrme calibration model.

Faddeev Model

Let's look at the Faddeev model[7]in which the chiral field takes on values in the diversity of the sphereS^2. In this case, the role of the field variable is performed by a vector of unit length on the sphere$S^2$and geometrical sense of topological charge is number of gearings of contours of Hopf invariant. Such vectors may be the direction of the spin or magnetization in magnetic, to get an explicit view of the Hopfinvariant, let's consider a triplet of scalar fields. $n_a(t,\vec{r})$: $R^1 * R^3 \to S^2; a = 1,2,3$, that satisfy the field equation S^2: $\sum_a (n^a)^2 = 1$ and boundary conditions$n_a(t,\vec{r}) \;\to\; \delta_3^a$ at$|\vec{r}| \to \infty$.This last condition ensures the compactization of the coordinate space $R^3 \to S^2$and their subordinate fields are classified in the third homotopic group $\pi_3(S^3) = \mathbb{Z}$.

As a group of invariants, we will select the group$G = diag[SO(2)_s \otimes SO(2)_l]$, matched axle rotation $\mathbb{Z}$ and the third axis in space. Let's write down the condition of the field invariance $n_a(t,\vec{r})$:

$$\vec{\omega}_3\vec{n} - \omega\partial_\mu \vec{n} = 0 \qquad (8)$$

or in components: $\partial_\mu \vec{n}_3 = 0$; $i\partial_\mu \vec{n}_\pm + \vec{n}_\pm = 0$, Where $\vec{n}_\pm = \frac{\vec{n}_1 + \vec{n}_2}{\sqrt{2}}$,

To simplify the recording of the equations, let's enter on the S^2 polar coordinate sphere $n_1 = \sin\beta\cos\gamma, n_2 = \sin\beta\sin\gamma, n_3 = \cos\beta$,the equations will take the following form:

$$\partial_\mu \beta = 0; \qquad \partial_\mu \gamma = 1 \qquad (9)$$

The above substitutions are often used to study the structure of soliton solutions and greatly simplify the analysis of the field equations under consideration. This triplet of scalar fields is comparable to a 4 vector.$a_\mu$by putting it down:

$$f_{\mu v} = \partial_\mu a_v - \partial_v a_\mu = 2\epsilon^{abc} \partial_i n^a \partial_k n^b n^c -$$
Faddeev's tensor (10)

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This model, proposed by L.D.Faddeev for geometrical reasons, has a large degree of certainty, as on theS^2 by means of$\vec{n}$only two independent fields and its first derivatives can be constructed in accordance with (10). $O(3)_I$ - invariant: $(f_{\mu v})^2$and$(\partial_\mu n^a)^2$. So the Lagrangian density can be taken as follows:

$$\mathcal{L}_F = \lambda^2(\partial_\mu n^a)^2 - \frac{\epsilon^2}{4} f_{\mu v} f^{\mu v}, \quad (\lambda, \epsilon) -\text{constant}$$
(11)

To get an estimate of energy in the Hopf model through the Hopf Index, we will only dwell on the end results. The static Hamiltonian of the Faddeev model looks as follows:

$$E = \int d^3x \left\{\lambda^2(\partial_i n^a \partial_k n^b n^c)^2 + \frac{\epsilon^2}{4} f_{\mu v} f^{\mu v}\right\} \quad (12)$$

The Hopf index is the number of meshes of vector lines $\vec{b}$ - fields where $(\vec{b} = rot\vec{a})$. It can be recorded in the form[8]:

$$Q_H = \frac{-1}{8\pi^2}\int d^3x\,(\vec{a} rot \vec{b}\,) \quad (13)$$

Hence, from the Lagrangian structure of the Faddeev model, an estimate for the topological soliton energy follows:

$$H_F[n(\vec{r},t)] > 16\sqrt{2}\pi^2|Q|^{\frac{3}{4}} \quad (14)$$

The Higgs mechanism

To explain the mass of calibration bosons without violating the laws of nature, the concept of spontaneous symmetry disturbance has been used. An additional field - the Higgs field - is entered, which interacts with all other fields and through this interaction informs the mass of calibration bosons.

The problem of using the model of spontaneous symmetry disorder in elementary particle physics is that according to Jeffrey Goldstone's theorem, it predicts a mass-free scalar particle, which is a quantum excitation in the direction of F, the so-called Nambu-Goldstone boson. The energy of such a particle is purely kinetic, which in quantum field theory implies that the particle has no mass. However, no mass-free scalar particles were found.

According to the Noether's theorem, any continuous symmetry of Lagrangian leads to the existence of a remaining current: $\partial_\mu j^\mu = 0$.Under normal circumstances, the law of charge retention follows from this equation. $dQ/dt = 0$ where$Q(t) = \int j_0(\vec{x},t)d^3x$,But with spontaneous symmetry disorder. $\langle 0|\varphi_i|0\rangle \neq 0$, size$Q(t)$, is not well defined, because the field operator in the sub-integral expression does not decrease fast enough on infinity. It follows from the law of current conservation:

$$\int\left[\partial^\mu j_\mu(\vec{x},t), \varphi_0\right]d^3x = 0 \quad (15)$$

For a sufficiently large surface, i.e. for large space-like states, the second summand in the right part turns to zero. Ergo,

$$\frac{d}{dt}[Q(t), \varphi(0)] = 0 \quad (16)$$

If this switch, which is a combination of fields, has a different vacuum average from zero:

$$\langle 0|[Q(t), \varphi(0)]|0\rangle \neq 0 \quad (17)$$

that is said that the symmetry is spontaneously broken. In Lagrangian, there is the definition for Higgs potential. [9]:

$$L_{Higgs} = (D^\mu\varphi)^+(D_\mu\varphi) + m\varphi^+\varphi - \lambda(\varphi^+\varphi)^2 (18)$$

whereφ - the Higgs field, $m$and$\lambda$are positive real numbers,

$D_\mu = \partial_\mu - igT_a A_\mu^a$, where$T_a$is the calibration group generator, and the$A_\mu^a$is for the calibration fields that should create mass through the Higgs mechanism. In this Lagrangian, it is not yet clear how masses appear in the particles. For understanding, it is useful to consider the potential U[10]:

$$U = \lambda(\varphi^+\varphi)^2 - m\varphi^+\varphi \quad (19)$$

This potential describes W -shaped fourth order parabola for the actual field with one component. Sinceφ - complex, it can be represented in 3

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dimensions as the body of rotation of this parabola. Amount F in its basic state is so-called condensation:

$$\langle\varphi\rangle = \sqrt{\frac{m}{2\lambda}}$$

which is obtained by calculating zero states.

Identical relation for 16-spinors and effective spinor field model

First of all, let's consider the existence of a special identity for 16-spinor, obtained by the Italian mathematician Brioschi, according to which in 8 -spinorfield [11]:

$$j_\mu j^\mu - \tilde{j_\mu}\tilde{j^\mu} = s^2 + p^2 + \vec{v}^2 + \vec{a}^2 \qquad (20)$$

where you enter the following designations for spinors squares:

$$s = \bar{\Psi}\Psi\,, \quad p = i\bar{\Psi}\gamma_\mu\Psi\,, \quad \vec{v} = \bar{\Psi}\lambda\Psi, \quad \vec{a} = \bar{\Psi}\gamma_\mu\lambda\Psi\,,$$

$$j_\mu = \bar{\Psi}\gamma_\mu\Psi, \qquad \tilde{j_\mu} = \bar{\Psi}\gamma_\mu\gamma_5\Psi, \qquad \bar{\Psi} = \Psi^+\gamma_0$$

$\lambda -$ of Pauli's matrices in isotopic space: $\lambda_i = I_4 \otimes I_2 \otimes \sigma_i$,The diagonal representation (Weyl representation) is used here:$\gamma_5 = \gamma_5^+$and$\gamma_\mu, \mu = 0,1,2,3.$ are the Dirac unitary matrices that act on spinors:

$$\gamma_0 = I_4 \otimes \sigma_3 \otimes I_2, \quad \gamma_i = \sigma_i \otimes \sigma_2 \otimes I_4\,,$$

If you define 16-spinor as columns:

$$\Psi = \oplus_{i=1}^2 col(\varphi_i \oplus \chi_i \oplus \xi_i \oplus \theta_i),$$

where$(\varphi_i\,, \chi_i, \xi_i, \theta_i)$2-spinor, it's not hard to find what:

$$\left\{\begin{array}{c} j_0 = \sum_i \varphi_i^+\varphi_i + \chi_i^+\chi_i + \xi_i^+\xi_i + \theta_i^+\theta_i, \\ \vec{j} = \sum_i \varphi_i^+\sigma_i\varphi_i - \chi_i^+\sigma_i\chi_i + \xi_i^+\sigma_i\xi_i - \theta_i^+\sigma_i\theta_i, \end{array}\right\} (21)$$

$$\left\{\begin{array}{c} \tilde{j_0} = \sum_i \varphi_i^+\varphi_i - \chi_i^+\chi_i + \xi_i^+\xi_i - \theta_i^+\theta_i, \\ \tilde{\vec{j}} = \sum_i \varphi_i^+\sigma_i\varphi_i + \chi_i^+\sigma_i\chi_i + \xi_i^+\sigma_i\xi_i + \theta_i^+\sigma_i\theta_i, \end{array}\right\} (22)$$

where$\sigma_i -$ are Paulie matrices, from (21) and (22) matrices are easy to get: $j_\mu j^\mu - \tilde{j_\mu}\tilde{j^\mu} = \Omega$, where the designation is entered:

$$\begin{aligned}\Omega = 16[(\varphi_1^+\varphi_1)(\varphi_2^+\varphi_2) - |\varphi_1^+\varphi_2|^2 \\ + (\chi_1^+\chi_1)(\chi_2^+\chi_2) - |\chi_1^+\chi_2|^2 \\ + (\xi_1^+\xi_1)(\xi_2^+\xi_2) - |\xi_1^+\xi_2|^2 \\ + (\theta_1^+\theta_1)(\theta_2^+\theta_2) - |\theta_1^+\theta_2|^2] \geq 0\end{aligned}$$

After folding (20) and (21), (22) we get the resulting identity:

$$2j_\mu j^\mu = s^2 + p^2 + \vec{v}^2 + \vec{a}^2 + \Omega, \qquad (23)$$

which demonstrates the time-like nature of the 4-vectors j_μ, The structure of the ratio (23) leads to the natural conclusion that the Higgs potential in the effective spinor field model can be introduced as a function of the $j_\mu j^\mu$:

$$V = \frac{2D^3}{\lambda^2 G^2 K^2 \varkappa_0{}^2}\left(j^2 - \varkappa_0^2\right)^2 \qquad (24)$$

If we look for a localized soliton-like configuration in the model, we can find a natural boundary condition at infinity:

$$lim_{r\to\infty}\, j_\mu j^\mu = \varkappa_0^2 \qquad (25)$$

As it follows from ratio (23), condition (24) defines a fixed point of space (S^8) (vacuum). A vacuum is a state in which the field has minimal energy.

Using condition (25) and well-known property of homotopic group on the sphere $\pi_3(S^n) = 0$, for $n \geq 4$,we conclude that in the model with the species potential (24) there can be 2 states with non-trivial topological charges. The first state is implemented when $\pi_3(S^3) = \mathbb{Z}$ (Skyrme model), and the second is implemented at $\pi_3(S^3) = \mathbb{Z}$ (Faddeev model).

For example, if the vacuum stateΨ_0 identifies.$s(\Psi_0) \neq 0$, then there may be configurations characterized by a chiral invariant. $(s^2 + \vec{a}^2 = inv)$, sphere defining S^3as a field diversity that matches the Skyrme model. On the contrary, unless$\{v_3(\Psi_0) \neq 0, v_3 = \bar{\Psi}\lambda_3\Psi =$

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$const\}$ then $SO(3)$ invariant $\vec{v}^2$ identifies. S^2[12,13]as field diversity that matches the Faddeev model. So, in view of the above, using the analogy with the Skyrme (Faddeev) model, we will choose the following type of spin density Lagrangian for the effective 16-spinor field model:

$$\mathcal{L}_{spin} = \frac{1}{2\lambda^2}\overline{D_\mu \Psi}\gamma^\nu\, j_\nu D^\mu\, \Psi + \frac{\epsilon^2}{4} f_{\mu\nu} f^{\mu\nu} - \frac{2\mathrm{D}^3}{\lambda^2 \mathrm{G}^2 K^2 \varkappa_0{}^2}\left(j^2 - \varkappa_0^2\right) \quad 2$$

where$(D^\mu = \partial^\mu - ieA^\mu)$involves interaction with the electromagnetic field, and responds to the Faddeev-Skyrme type antisymmetric tensor: $f_{\mu\nu} = \left(\Psi^+\partial_\mu\Psi\right)(\partial_\nu\Psi^+\Psi) - (\Psi^+\partial_\nu\Psi)\left(\partial_\mu\Psi^+\Psi\right)$ where$(\lambda, \epsilon, \varkappa_0)$model constant parameters, G is the gravitational constant, Kis Roman's invariant, and we can imagine D in the form:

$$\mathrm{D} = \overline{D_\mu \Psi}\gamma^\nu\, j_\nu D^\mu\, \Psi$$

CONCLUSION

In this paper, we conducted a study of spinor model of leptons and baryons in sigma model. To obtain results, we used the well-known 8-spinor model, the open Brioschi model, and the Higgs model, whose potential can be introduced into an effective spinor model as a function. we studied the topological charge structure in the lepton and baryon sectors. We combined Skyrme and Faddeev models with the 16-spinor field model..

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