TEST Engineering & Management

January - February 2020
ISSN: 0193 - 4120 Page No. 11451 - 11457

Application of Dempster-Shaffer Theory to Study the Student Performance in Post Graduate Examinations

[1] Ananth.Y.N [2] , Dr.Narahari.N.S,
] Senior Research Fellow and Ph.D student, Jain University, Bangalore
,[2] Professor, Department of Industrial Engineering and Management, R.V.College of Engineering
Autonomous institution affiliated to VTU Belagavi. [1]aninotes@gmail.com,[2] nsnarahari@gmail.com

Article Info
Volume 82
Page Number: 11451 - 11457
Publication Issue:
January-February 2020

Article History
***Article Received**: 18 May 2019*
***Revised**: 14 July 2019*
***Accepted**: 22 December 2019*
***Publication**: 21 February 2020*

Abstract

Dempster- Shafer theory is a radically different kind of theory when it comes to probabilistic systems. It proposes a system where in the outcome of an experiment can be studied or determined depending on the extent of belief in the system. Instead of assigning probabilities to outcomes, their occurrence is measured with the extent of belief one has got regarding it. The belief extents from different sources can be combined together to assign a probabilistic occurrence, indicated by the belief in that outcome .

In the current study, the post graduate entrance examination scenario is studied. Here, the students entering into the post graduate courses are expected to possess certain high end skills like analytical skills, mathematical skills and so on. The students who take the test may not possess these skills to the required extent. In the current study a correlation has been found between the qualifying degree marks and entrance test marks and also the pattern in which the students answer the question paper. Dempster Shafer theory can be used as a framework to understand the patterns that exist in the answering of the students. This is done in two phases consisting of two different data sets of the student marks of different examinations. This work is further used as an input to build a decision support system, where in , based upon the classification and their results, the questions in the question paper get revised.

***Keywords:** Dempster Shafer theory, performance, Post-graduate courses, Student aptitude.*

I. INTRODUCTION

Dempster- Shafer theory is a radically different kind of theory when it comes to probabilistic systems. It proposes a a system where in the outcome of an experiment can be studied or determined depending on the extent of belief in the system. Instead of assigning probabilities to outcomes, their occurrence is measured with the extent of belief one has got regarding it. The belief extents from different sources can be combined together to assign a probabilistic occurrence, indicated by the belief in that outcome .

In the current study, the post graduate entrance examination scenario is studied. Here, the students entering into the post graduate courses are expected to possess certain high end skills like analytical skills, mathematical skills and so on. The students who take the test may not possess these skills to the required extent. In the current study a correlation has been found between the qualifying degree marks and entrance test marks and also the pattern in which the students answer the question paper. Dempster Shafer theory can be used as a framework to understand the patterns that exist in the answering of the students. This is done in two phases consisting of two different data sets of the student marks of different examinations. This work is further used as an input to build a decision support system, where in , based upon the classification and their results, the questions in the question paper get revised.

Related work-literature review:

January - February 2020
ISSN: 0193 - 4120 Page No. 11451 - 11457

The authors themselves have been studying this problem for quite a long time and they have applied different techniques to study and find patterns in the marks distribution. Probabilistic and predictive methods have been applied in order to predict the performance in the entrance test based upon the degree marks. Regression techniques have been applied to study the dependency of the scores in the entrance test examinations on the qualifying degree marks. Strong theoretical foundation of Dempster Shafer theory has been discussed in [1] and [2].DST has been used in classification problems in [3].Dempster's rule of combination has been used to combine different results from several different classifiers in [4][5]DST has been used in statistical pattern recognition and machine learning.DST has been used for evidential calibration , which is the process of converting the decisions of statistical classifiers(such as support vector machines)into belief functions.[6][7]DST has been used in designing evidential classifiers in [8].As one good example of classification problem, Xu and Krzyzak applied the DST theorem to handwriting recognition problem[9].Application of DST in educational data mining is quite new and the authors have found the method a novel one and hence attempted to use it in the context of their problem, with the main focus of classification problem .

Problem definition:

The entrance examinations that have been mentioned are mainly containing multiple choice questions and they do not measure the aptitude of a post graduate student sufficiently. Till the qualifying degree, it is mostly containing descriptive and writing answer questions rather than multiple choice questions. The student would have developed his knowledge or a skill based upon such an examination system and continues to do so when he will be pursuing the actual post graduate courses. But the entrance test examination is mostly containing multiple choice questions where the hard skills required in the post graduate courses may not be tested to the required extent. Hence the research question that can be framed out is "Are post graduate entrance examinations testing the aptitude of the students- if so to what extent and if not to what extent".

Towards studying this, in the first phase, the correlation between the two sets of marks will be studied and in the second phase the patterns of scoring in only the entrance examination is concerned. Dempster- Shafer theory has been used as a framework to study these distributions.

Proposed work: Methodology-Dempster-Shafer theory

Dempster- Shafer theory is a theory based upon belief systems. It contains concepts of probability treated in a little different manner. Basically, this theory is based upon assigning degrees of beliefs to the answers of a certain question. The actual belief that gets assigned to a question will be based upon the degree of beliefs assigned to related questions.

There are essentially two ideas in this framework. One is to obtain the degree of belief for a question based upon the subjective probabilities for related questions. The second is to combine all these probabilities for a particular question and generate a belief function for it.

Belief and Plausibility:

In DST, to start with, a domain is defined as a set of all the propositions present in that domain are identified. This domain is called the frame of discernment. Then all the subjective probabilities for the propositions are assigned. For a particular proposition, the mass function or the belief function is the sum total of all the subjective probabilities assigned to it. This is called as the belief function or a mass function and is denoted as the bel function. It is the amount of belief that directly supports a hypothesis and can take values from 0 to 1.It is the lower bound on the cases favoring the hypothesis. Plausilbilty is 1 minus the mass functions of all sets whose intersection with the hypothesis is empty Or, it could be derived as the sum of all mass functions of sets whose interactions with the hypothesis is not empty.

Belief <=plausibility.

Plausibility – denoted by Pl is given by

$Pl(p) = 1- Bel(\sim p)$

TEST Engineering & Management
January - February 2020
ISSN: 0193 - 4120 Page No. 11451 - 11457

Plausibility also ranges from 0 to 1 and denotes the extent to which evidence in favor of ~p leaves room for p.

Supposing that there is a hypothesis that a business man is travelling from Bangalore to New York by flight and there is a flight from Bangalore to London and there is a connecting flight from London to NewYork.If there is a hypothesis that he will be successfully able to catch the connecting flight from London to Newyork ,there can be a belief with probability of 0.4 associated with this hypothesis.The opposite of this hypothesis is that he will not be able to catch the connecting flight.Let us say this hypothesis is having a belief function whose value is 0.5, we can associate a belief function and plausibility to each of these hypothesis as shown in the flowing table.

Table1:

Hypothesis	Mass	Belief	Plausibility
Null(Neither catches the flight nor doesn't	0	0	0
Catches flight	0.2	0.4	0.5
Doesn't catch flight	0.3	0.5	0.5
Either(catch flight or doesn't catch flight)	0.4	1.0	1.0

The above table shows that the Null hypothesis is having no values for any probability. The hypothesis "catches flight " is having probabilities 0.4 and 0.5.The hypothesis "doesn't catch flight ' has got probabilities 0.5 and 0.5.The hypothesis "either" has got probabilities 1.0 and 1.0 which means that there is a connecting flight but whether the person catches it or not doesn't matter.

DST formalisms:

Let X is the set of all propositions under consideration. Then

2^X denotes the power set of X which consists of all the subsets of X.For example if X= {p,q} then, the power set

$2^{X} = \{\Phi, \{p\},\{q\},\{X\}\}$

The elements of the power set can be taken as the set of propositions concerning the actual state of the system.

The theory of evidence assigns a belief mass to each of the above propositions as in the following assignment m: $2^X \rightarrow [0,1]$.

This is called as the Basic belief assignment and has got two basic properties which are :

1. The mass of the empty set is zero – m(Φ) = 0
2. The masses of the remaining members of the power set add upto 1- $\sum_{A \in 2X} m(A) = 1$.

This gives the fact that the particular class of proposition A has got a mass m (A) and no other class has got it in the domain concerned.

From the mass assignment, the upper and lower probabilities can be determined. This includes the precise probability of a set of interest and is bounded by two measures called belief and plausibility.

Bel(A) <= P(A) <= Pl(A)

The belief for a set A is defined as the sum of all the masses for the subsets of A.

$Bel (A) = \Sigma\, m (B)$

B|B is the subset of A.

The plausibility Pl (A) is the sum of all the masses of B that intersect the set of interest A.

$Pl(A) = \Sigma\, m(B)$

B|Bis the intersection with A.

The two measures are related as follows.

Pl(A)= 1-bel(A~)

Dempster's rule of combination:

In the event of different sources of beliefs express their beliefs over the frame in terms of belief constraints such as in giving hints or preferences then, Dempster's rule of combination can be applied there. Specifically, the combination or the joint mass is calculated from two masses in the following manner.

$m_{1,2}(\Phi)=0$

$m_{1,2}(A) = (m1+ m2) (A) = 1/1-k\ \Sigma\ m1(B)\ m2(C)$

TEST Engineering & Management

January - February 2020
ISSN: 0193 - 4120 Page No. 11451 - 11457

B intersection C = A <> Φ

Where K= Σ m1(B) m2(C)

Application of DST to the current problem:

In the current problem DST is applied over in two different phases. One is the learning phase and the other is the predictive phase. In the learning phase, the data collected is the marks distribution of the students in the past years. The question paper consists of 2 marks and 1 mark questions. In the first step, Regression analysis is performed over the marks data to locate the clusters of students who have scored marks in particular range by answering a certain number of one mark and two marks questions. For example, if 30 students have scored marks from 40 to 50 uniformly, by scoring in 15 two marks and 15 one mark questions correctly, then they fall into a cluster. They would have a certain belief system, in which each of the students belonging to the cluster would have a certain belief value and plausibility value for the questions that they have answered. At a deeper of analysis, this kind of a matrix shows what is the aptitude of the students in answering a certain type of questions. For example, a student might have a belief of 0.8 for questions on analytical geometry and plausibility of 0.4 for the same and a belief of 0.7 for questions on trigonometry and 0.5 value of plausibility for the same. The following table shows such a sample distribution of belief and plausibility values which can be formed , if we go by the method of DST.

Table2:

Student score	Question range	Belief	Plausibility	Question topic
10-20	1-10	0.5	0.6	Analytical geometry
10-20	11-20	0.4	0.7	Trigonometry
10-20	21-30	0.7	0.8	English grammar
21-30	1-10	0.5	0.6	Basic Computer science
21-30	11-20	0.4	0.7	Programming
21-30	21-30	0.7	0.8	General awareness

The above table says that a particular student set getting marks in the range 10-20 by answering questions from 10-20 has got a belief of 0.5 and plausibility of 0.6 and so on. The intervals are illustrative and not limited to only those shown.

Such information can also be got by comparing the scores of the students in the qualifying degree examination with the corresponding score in the entrance test. That becomes another source of belief for the system. The degree examination has got a different syllabus and set of questions, which are suggestive of the aptitude of the student in computer science, but are not directly related to the questions and syllabus of the entrance examination. So, the marks of the student in the degree examination is taken as a default indicator of the aptitude of the student and no further study of the performance in a topic –by-topic manner is done. Although, belief functions can be separately used for studying the performance of the students in the degree examinations, later, combination of the belief function values from these two domains becomes difficult and impossible as these are quite disjoint domains.DST combination rule does not allow for combining belief function values of disjoint domains.

In the predictive phase, the same kind of mechanism is used, but the source of belief is different. In this phase, the capability of the students is being predicted with respect to the various questions. Here, the examiner will have a belief that the set of students will and can answer the questions with a certain value for belief and plausibility, again going by the topic of the questions. For example , he might believe that the students would answer the questions on English grammar with a belief of 0.7 and plausibility of 0.3.In this part, essentially, we speak of the belief systems of the examiner and not the students. So, it is not

January - February 2020
ISSN: 0193 - 4120 Page No. 11451 - 11457

guaranteed always that the same values of belief an plausibility hold for , when looked at, from the stand point of the students. Every examiner can have his own values for the believes and plausibility and the expertise of the examiner in teaching and setting papers will speak of the accuracy of the belief systems .A sample of how a belief systems are framed is shown below

Table3: Examiner 1 : Belief table

Student score	Questions range	Belief	Plausibility	Question topic
20-30	1-10	0.5	0.6	Analytical geometry
20-30	11-20	0.4	0.7	Trigonometry
31-40	21-30	0.7	0.4	English grammar
15-20	1-10	0.4	0.6	Limits and Calculus
25-35	11-20	0.4	0.7	Programming
20-30	21-30	0.7	0.8	Basic computer concepts

Here, each of the rows show the belief and plausibilty of a set of students for set of questions mentioned –"as according to the belief of a particular examiner".

In the first phase such a set of beliefs and plausibility's can be drawn for as many examiners and as many students. In the second phase, for the validation, the actual marks which have been scored by the students can be compared to what had been predicted in the first phase. Also, the belief matrices can be combined to get the combination of all the beliefs and to get a composite value of the beliefs and plausibilities.This value gives us the overall value of the belief and plausibility for the entire question paper which is subjected to this type of analysis. A calculation of such a combining process is illustrated below.

From the Dempster's rule of combination, we have

$m_{1,2}(\Phi)=0$

$m_{1,2}(A) = (m1+ m2) (A) = 1/1\text{-}k\ \Sigma\ m1(B)\ m2(C)$

B intersection C = A <> Φ

Where K= Σ m1(B) m2(C)

For the students' belief table given above we form the sets

A<-Analytical Geometry

B<-Programming

Then m1=0.5 and m2=0.4

If we combine m1 and m2 as $m_{1,2}(A)$,then

$m_{1,2}(A)=0.5*0.4/1-0.5*0.4$

=0.20/1-0.20

=0.20/0.8=0.25

Combining the question types in the same way ,pairwise topicwise which are related and finally integrating all the types of questions , we get the bel function value for the entire paper as follows.

$m_{3,4=}0.7*0.6/1-(0.7*0.6)$

=4.2/1+4.2

=4.2/5.2

=0.80

$m_{5,6=}0.8*0.8/1-(0.8*0.8)$

0.64/1-0.64=0.64/0.36=0.18

Mtotal=(0.25*0.80*0.18)/1-(0.25*0.80*0.18)

0.036/0.964=0.037

In a similar way , For the examiner belief table the bel function would be calculated and is found to be having the value.

M1,2,3=(0.6*0.6*0.7)/1-(0.6*0.6*0.7)=0.337

M4,5=(0.7*0.8)/1-(0.7*-0.8)=1.273

Mtotal = (0.337*1.273*0.4)/1-(0.337*1.273*0.4)

0.215

Once we get the final bel function value for a certain paper from both the students and the examiner's belief systems , we can try to predict, with what belief or confidence level that the students of similar nature will score in the future year question paper. The main basis for this deduction is the analysis of the degree question paper in DST way and assignment of aptitude levels for the students in individual topics. The idea is to create a balanced question paper wherein the final m ass function value is at an average of 0.5 to 0.6 thus generating a question paper that is not too difficult or too easy. But, the advantage of

January - February 2020
ISSN: 0193 - 4120 Page No. 11451 - 11457

DST is that the belief values can be adjusted to any feasible and the question paper can be constructed.

In the actual experiment that was conducted one sample table is constructed, for both the examiners and the students given below for the question paper of the year 2014. This can be used to generate the questions which are finally meant for the examination. The tables and the calculation of the values are shown below.

Examiner's belief table:

Student score	Question range	Belief	Plausibility	Question topic
60-70	1-10	0.6	0.7	Analytical geometry
70-80	11-20	0.3	0.7	Trigonometry
80-90	21-30	0.7	0.4	English grammar
70-80	1-10	0.3	0.6	Limits and Calculus
90-100	21-30	0.7	0.3	Basic computer concepts

Mtotal for this table =0.056

Students' belief table:

Student score	Questions range	Belief	Plausibility	Question topic
60-70	1-10	0.7	0.4	Analytical geometry
70-80	11-20	0.5	0.4	Trigonometry
80-90	21-30	0.9	0.3	English grammar
70-80	1-10	0.5	0.4	Limits and Calculus
90-100	21-30	0.7	0.3	Basic computer concepts

Mtotal for this table =0.029

As we can see, the students' belief function value is quite less than the examiners' value as the students cannot guage their own performance before the examination. In the actual experiment, the examiner's table and the students' table were constructed paralleley- the difference in the bel values shows the actual way that the belief systems and expectations work.

Discussions:

In the literature, both supportive and contradictive arguments for DST are present. It is argued that because DST is based upon subjective probabilities, it can be biased. There are also arguments in favor of DST in certain situations. The authors argue that in the context of setting up of examination papers and answering them by the students, belief works to quite an extent. It is the belief of the student in answering a question that makes up whether he can answer it successfully or not. Similarly an expert evaluator can substantially determine through his belief in students' capabilities , gained over a period of several years , whether a student can answer a certain set of questions or not. Of course,. Other factors like knowledge, skills come into play while answering the questions, but belief also plays a major role in answering the questions.

Conclusions:

Essentially Dempster Shafer theory is usable when belief systems are in play, although with a large computational overhead. It is difficult to maintain the beliefs and plausibilities of several sources and use them to the situation concerned. Also, beliefs change over time and it is difficult to maintain them. Nevertheless, DST works well in the current problem and hence usable.

II. References

[1] RONALD .R. YAGER- ON THE DEMPSTER-SHAFER FRAMEWORK AND NEW COMBINATION RULES- AVAILABLE ATHTTP://WWW.SCIENCEDIRECT.COM/SCIENCE/ARTICLE/PII/0020025587900077

[2] A mathematical theory of evidence- Glenn Shafer available at http://pup.princeton.edu/titles/2439.html

[3] Data classification using the Dempster-Shafer method. Qi Chen, Amanda Whitbrook, Uwe Aickelin and Chris Roadknight Available from: https://www.researchgate.net/publication/26529501

2_Data_classification_using_the_Dempster-Shafer_method
[4] G. Rogova, "Combining the results of several neural network classifiers", Neural Networks 7 (5) (1994) 777-781.
[5]L. Xu, A. Krzyzak, C. Y. Suen, "Methods of combining multiple classifiers and their applications to handwriting recognition", IEEE Transactions on Systems Man and Cybernetics 22 (3) (1992) 418-435.
[6] P. Xu, F. Davoine, H. Zha, and T. Denœux, "Evidential calibration of binary SVM classifiers," International Journal of Approximate Reasoning, vol. 72, pp. 55–70, 2016.
[7] P. Minary, F. Pichon, D. Mercier, E. Lef`evre, and B. Droit, "Face pixel detection using evidential calibration and fusion," International Journal of Approximate Reasoning, vol. 91, pp. 202–215, 2017.
[8] Thierry Denoeux. Logistic Regression, Neural Networks and Dempster-Shafer Theory: a New Perspective. 2018. ffhal-01830389f
[9] L. Xu, A. Krzyzak, and C.Y. Suen, Methods of Combining Multiple Classifiers and Their Applications to Handwriting Recognition, IEEE Transactions on Systems, Man, and Cybernetics, Vol. 22, 1992, pp. 418-435.