

# Analysis of Golden Code in 4G Mobile Communication over Rayleigh Fading Channel

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# Abstract

In Multiple Input Multiple Output (MIMO) system, the symbols per transmit antenna in particular time slot are taken from a coded constellation. Golden code is one of the best coding techniques to form such a kind of constellation. In this paper, Golden code has been analyzed with  $2 \times 1$  and  $2 \times 2$  antenna configurations under AWGN and Rayleigh Fading channel environments. Results show that Golden code with full diversity outperforms the other antenna systems in terms of symbol error rate and diversity gain.

*Keywords:* Golden Code, Orthogonal Space Time Block Code (OSTBC), Symbol Error Rate (SER), Coding Gain, Diversity

# I. INTRODUCTION

The performance of any kind of wireless communication depends on the status of channel environment. As such, the wireless channel is very much dynamic and unpredictable in nature. Moreover, the multipath fading and shadowing are main constrains to degrade the quality of system. So, to achieve high spectral efficiency, it is required to apply multiple antennas at both transmitter and receiver ends. In that case, multiple input and multiple output (MIMO) system is considered as potential candidate in 5G communication system. In MIMO configuration, Golden code plays a vital role in space time (ST) block codes. To form MIMO encoded symbols, generator matrix, channel coefficients and space diversity procedures are the main key activities to measure the performances. In this system, number of transmit antennas are chosen such that those satisfy the rank criterion for getting the advantage of code gain distance (CGD). In order to increase the download throughput along with high data rate various standards like High Speed Downlink Packet Access (HSDPA) have been proposed for Long Term Evolution (LTE) communication systems [1]. This kind of transmission is developed on the concept of adaptive transmission related to channel quality [1]. Due to the channel impairments of radio communication, the application of antenna arrays has received much attention because of the improvement of signal quality, system coverage, capacity and link quality. Although this multiple antenna transmission/reception has

emerged as a key tool to achieve high spectral and power efficiency, the main challenge of this MIMO system is to characterize the different types of frequency selective fading channels. Several novel signal processing along with error correction coding schemes such as orthogonal space time block code (OSTBC), quasi space time block code (QOSTBC) and space time trellis code (STTC) had been formulated that integrate the techniques of space diversity, channel coding including the different fading effect of channel [2]. Several statistical distributions of the channel have been proposed by researchers for channel modeling of the fading envelope under short and long term fading conditions. This paper provides a brief background on MIMO systems followed by an analysis of Golden code for different transmit and receive diversity. We discussed the SER of different coding schemes over various fading channels. In MIMO systems, the Golden Code is a kind of space-time code with two transmit and two receive antenna that achieves both the full rate and the full-diversity [3]. In this paper we have compared its SER performance with other  $2 \times 2$  space-time codes. The main advantage of the Golden code is the independence of the size of signal constellation so that it achieves the optimal diversitymultiplexing performance presented in [4].

## II. SYSTEM MODEL

In channel modeling, researchers have proposed several statistical distributions for channel modeling of fading envelope under short and long term fading conditions [2].





Figure 1. Block diagram of MIMO system

In this paper, a wireless communication system is considered with *N* transmits and *M* receives antennas. A single path is created for each pair of the transmit and receive antenna. There are three elements in a MIMO system which is described as the transmitter (*N*), the channel ( $\alpha$ ) and the receiver (*M*). Figure 1 represents the space time coded MIMO system with *N* transmit and *M* receive antennas. In the space-time coded MIMO systems, bit stream is mapped into symbol stream and a symbol stream of size *N* in space time encoded into  $\{C_i^{(t)}\}_{i=1}^N, t = 1, 2, 3 \dots T$  where *i* is the antenna index and *t* is the symbol time index. At the receiver side, the space time encoded symbol stream is estimated by using the received signals  $\{r_j^{(t)}\}_{j=1}^M$ , t = 1, 2, 3, ..., T. Let us assume that,  $\alpha_{ij}^{(t)}$  denotes the Rayleigh-distributed channel gain from the  $i^{th}$  transmit antenna to the  $j^{th}$  receive antenna over the  $t^{th}$  symbol period where i = 1, 2, ..., N, j = 1, 2,...,M and t = 1, 2,...,T. If  $\{c_i^{(t)}\}$  is the transmitted signal from the  $i^{th}$  symbol period, the received signal at the  $j^{th}$ receive antenna during  $t^{th}$  symbol period is

$$\mathbf{r}_{j}^{(t)} = \sqrt{\frac{E_{x}}{N_{0}N}} [\alpha_{j1}^{(t)}\alpha_{j2}^{(t)}....\alpha_{jN}^{(t)}] \underbrace{\begin{bmatrix} c_{1}^{(t)} \\ c_{2}^{(t)} \\ \vdots \\ \vdots \\ c_{N}^{(t)} \end{bmatrix}}_{\mathbf{C}_{0}^{(t)}} + \eta_{j}^{(t)}$$
(1)

where  $\eta_j^{(t)}$  is the noise process at the *j*<sup>th</sup> receive antenna during *t*<sup>th</sup> symbol period and **E**<sub>x</sub> is the average energy of each transmitted signal [5].

#### **III. THE GOLDEN CODE**

Belifore *et al.* [8] were the first to introduce the space time Golden code by using the generator matrix based on the golden number in a  $2 \times 2$  MIMO system. They showed that this code is a full rate, full rank code satisfying the nonvanishing determinant condition. It is a  $2 \times 2$  space time code whose performance depends on the Golden number. We first illustrate the construction of the Golden code because of the key role played by the Golden number  $\frac{1+\sqrt{5}}{2}$  in the construction. Let us consider the information symbols  $s_1, s_2, s_3, s_4 \in S \subset \mathbb{Z}[i]$  to obtain the finite code *C*. Therefore the codeword can be formed as matrices as shown below [8].

$$C = \begin{cases} X = \frac{1}{\sqrt{5}} \begin{bmatrix} \alpha(s_1 + s_2\theta) \\ i\overline{\alpha}(s_3 + s_4\overline{\theta}) \end{bmatrix} \end{cases}$$

where

$$\theta = \frac{1+\sqrt{5}}{2}, \overline{\theta} = \frac{1-\sqrt{5}}{2}, \alpha = 1+i-i\theta, \overline{\alpha} = 1+i-i\overline{\theta}$$

and  $s_i$  are QAM information symbols, with  $i = 1 \cdots 4$ . Therefore its minimum determinant is defined as [8]:  $\delta_{\min}(C) = \min_{X \in C, X \neq 0} |\det(X)|^2$ . Golden code with non-

$$\frac{\alpha(s_3 + s_4\theta)}{\overline{\alpha}(s_3 + s_4\overline{\theta})} \right\}$$
(2)

vanishing  $\delta_{\min}$  outperforms the other codes generated using previous constructions. Also  $\delta_{\min}$  does not depend on the size of the signal constellation [8]. Therefore, the vectorized generator matrix for  $s_i$  can be written as equation (3)[9].



$$G = \frac{1}{\sqrt{5}} \begin{bmatrix} 1 & -\overline{\theta} & \theta & 1 & 0 & 0 & 0 & 0 \\ \overline{\theta} & 1 & -1 & \theta & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\theta & -1 & 1 & -\overline{\theta} \\ 0 & 0 & 0 & 0 & 1 & -\theta & \overline{\theta} & 1 \\ 0 & 0 & 0 & 0 & 1 & -\overline{\theta} & \theta & 1 \\ 0 & 0 & 0 & 0 & \overline{\theta} & 1 & -1 & \theta \\ 1 & -\theta & \overline{\theta} & 1 & 0 & 0 & 0 & 0 \\ \theta & 1 & -1 & \overline{\theta} & 0 & 0 & 0 & 0 \end{bmatrix}$$
(3)

The generated codewords are finally transmitted on a multiple antenna channel  $\mathbf{H} = [h_{i,j}]$  with *N* transmit and *M* receive antennas. Here,  $h_{ij}$  denotes the Rayleigh fading coefficient between the *j*<sup>th</sup> transmit and *i*<sup>th</sup> receive antenna. At the receiver end the vectorized received signal can be written as

 $Y = H_e GZ + \eta$ , where X = GZ and Z is the matrix containing real and imaginary parts of modulated symbols [10]. However for golden code  $H_e$  is defined as below

$$H_{e} = \begin{bmatrix} H & 0\\ 0 & H \end{bmatrix}$$
(4)

At the receiving end, the message *S* is estimated from the given data in *Y* and *H*as stated previously. There are many techniques available for this estimation but most of the decoders are concerned with the maximum likelihood (ML) detector and the approximation thereof. For each coded bit  $C_{k,i}$ , the ML soft decoder generates two matrices:  $\lambda_{c_k=0}^i$  and  $\lambda_{c_k=1}^i$ . These matrices are computed by the log-MAP theorem and are given by  $[10] \lambda^i(c_k) \approx \min_{X \in X_{c_k}^i} ||Y - H_e GZ||^2$ , where

 $X_b^i = GZ$  denotes the constellation subset[10]. In order to decode the information, it is required to find the shortest path according to equation (5)[10].

$$\widehat{C} = \arg\min_{c \in C} \sum_{k'} \lambda(c_k^i)$$
(5)

#### IV PERFORMANCE EVALUATION

The performance of the different antenna configurations of Golden code has been evaluated over an AWGN and Rayleigh channel in terms of SER versus SNR values. All simulations are done in MATLAB using 10<sup>4</sup> symbols upto a 20dB SNR value. Figure 2 shows the comparison of AWGN and Rayleigh fading with  $2 \times 1$  antenna configurations in terms of symbol error rate (SER) performance. For a particular modulation scheme, under  $2 \times 1$  MIMO systems AWGN channel outperforms the Rayleigh fading channel. This is because in the Rayleigh fading channel the signals are transmitted in multipath ways. Fig. 3 shows the performance comparison between  $2 \times 1$  and  $2 \times 2$  antenna configurations under AWGN and Rayleigh fading scheme. If we analyze the response curves,  $2 \times 2$  MIMO schemes exhibit a good performance at high SNR value but at low SNR values Rayleigh Fading channel performs better than the other. On the other hand, increasing the diversity gain,  $2 \times 2$  system outperforms the  $2 \times 1$  systems under both AWGN and Rayleigh fading channel. At the reference point of  $SER = 10^{-10}$  $^{3}$ , 2 × 2 antenna configuration under AWGN channel requires an SNR of ~4 dB less than that of the configuration under Rayleigh fading channel. Similarly, at the value of  $SER = 10^{-10}$ <sup>1</sup>,  $2 \times 1$  scheme under Rayleigh fading requires ~4 dB SNR values more than that of AWGN channel to achieve the same performance. Here the Golden code is full rate and full diversity ST code. But if we analyze the encoding and decoding complexities, Golden code is more complex compared to other ST codes. Full-rate full-diversity orthogonal designs with complex elements in the transmission matrix are difficult to analyze for more than two transmit antennas. For this reason  $2 \times 1$  and  $2 \times 2$  antenna schemes are taken for Golden code.



Figure 2. SER performance of Golden Code in AWGN and Rayleigh Channel with  $2 \times 1$  antennas





Figure 3. Performance comparison of the Golden code for 2 transmit and 2 receive antennas

## V CONCLUSION

In this paper, a new approach for comparing the Golden code with different diversity schemes has been proposed. Simulation results show that the Golden code under AWGN channel with  $2 \times 2$  system has the best performance as compared to  $2 \times 1$  configuration under Rayleigh channel.

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