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Weighted Intutionistic Fuzzy Additive Multiobjective Linear Model for Vendor Selection Problem

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Abstract

Vendor selection is a key success factor for many organizations. The organization aims to find a vendor that has the most compatible specifications with the buyer's requirements. In this paper, we propose a weighted intuitionistic fuzzy additive multiobjective linear model for vendor selection problem. We assign weights to the three objectives namely cost, quality, and on-time delivery. The formulation of the weighted IFO model is done by constructing intutionistic membership and non membership function for the three objective functions and converted to a crisp optimization problem for the solution. A numerical example illustrates the methodology. Comparing the results of fuzzy and intuitionistic fuzzy model we observe that the IFO model gives better results in terms of selection and order allocations to the vendors.

Keywords: *Vendor Selection, Intutionistic Fuzzy Sets, Multiobjective.*

I. INTRODUCTION

In today's world, we require vendors (**Dickson (1960)**) who can stand and deliver under enormous pressure, change over quickly to new product programs or master new technology to make even high design robust components. Managing supplier is no longer a task of purchasing managers. Purchasing managers have advocated the award of two or more vendors for the supply of materials etc. More than one supplier leads to competition between them and drives the price down and a big company is not dependent on a single supplier. Carefully selected and managed suppliers offer the greatest guarantee of consistently high quality and commitment to the product. The vendor selection problem is multiobjective (**Weber and Current (1993)**) where various tangible and intangible factors cause uncertainty, vagueness, and ambiguity in the decision-making process. Fuzzy sets or Intuitionistic fuzzy sets (IFS) are efficient tools to handle uncertainty effectively. Fuzzy sets (**Zadeh (1965)**) are defined by membership functions which represent the degree of acceptance but no means to represent non-membership functions.IFS (**Atanassov (1986)**) represent both the values and hesitancy along with it. Application of IFS on optimization problem proves to be a rich apparatus for the formulation of optimization problems (**Angelov (1995)**).

IFO has been applied in several optimization problems of linear type like solving multiobjective linear programming problems using IFO by **Parvathi and Malathi (2012)**, **Bharatiand and Singh (2014),** IFO techniques for Pareto optimal solution of manufacturing inventory models with shortages was applied by **Chakraborty, Pal, and Nayak (2013). Bhaya, Pal, and Nayak (2014)** used the IFO technique in the EOQ model with two types of imperfect quality items. **Bharati and Singh (2014)** applied the IFO technique in agriculture production planning. **Mahapatra and Roy (2014)** in the reliability optimization of complex systems used the IFO technique. **Nishad and Singh (2015)** solved the multiobjective decision-making problem under an intuitionistic fuzzy environment. **Kaur and Rachna (2016)** used the IFO technique in the vendor selection problem. **Roy et al.(2018)** solved the Intuitionistic fuzzy multiobjective transportation problem .The advantage of IFO is that the decision-maker can minimize the worst scenario and maximize the better scenario as well. If weights are assigned to objective functions and the model becomes a weighted additive IFO model. To improve the performance of supply chain, manage the flow of supply materials, components, and finished product to improve quality, service and reduced cost, different weights are assigned to several criteria (**Amid, Ghodsypour and Brien (2009)).**

The organization of the paper is as follows: Section1 introduces the problem. Section 2 basic concepts of multiobjective optimization and application of intuitionistic fuzzy sets in it. Section 3 methodology and algorithm of steps. Section 4 discusses the algorithm of steps to be applied in the numerical example. Section 5 is illustration of a numerical example and Section 5 results and discussions of the solutions obtained in the model. Section 6 is conclusions of the work.

II. PROPOSED ALGORITHM

2.1 Multiobjective linear programming (MOLP)-

A multiobjective optimization problem containing p objectives, q constraints and n decision variables is defined as follows:

$$\left.\begin{array}{l} \text{Max } Z_1(X), Z_2(X),\ldots,Z_p(X) \\ \text{subject to:} \\ g_j(X)\leq 0, j=1,2,\ldots,q. \\ X=x_1,x_2,\ldots,x_n, \\ x_i\geq 0, i=1,2,\ldots,n \end{array}\right\} \quad (1)$$

2.2 Complete Solution of MOLP -

x_0 is said to be a completely optimal solution for the above problem if there exist $x_0 \epsilon$ X such that $f_k(x_0) \leq f_k(x)$ for all x ϵ X. Complete solutions that maximize all of the multiobjective function do not exist when objective functions are conflicting in nature. A solution called Pareto optimality was introduced in MOLP.

2.3 Pareto Optimality -

$x^0 \epsilon$ X is said to be Pareto optimal solution for the above problem (1) if there does not exist another x ϵ X such that $f_k(x^0) \leq f_k(x)$ for all k=1,2,...,p and $f_j(x^0) \leq f_j(x)$ for at least one j=1,2,..,p.

2.4 Intutionistic Fuzzy Optimization [4]-

An IFO problem comprises of an objective function and subject to constraints. Here either the objective functions or constraints or both are intutionistic fuzzy sets. An IFO problem is formulated as follows:

We maximize the degree of acceptance of IF objectives and constraints and minimize the degree of rejection of IF objectives and constraints.

$$\begin{array}{l} \text{Max } \{\mu_i(x)\}\ x\epsilon R^n,\ i=1,\ldots,p+q, \\ \text{Min } \{v_i(x)\}\ x\epsilon R^n,\ i=1,\ldots,p+q, \\ \text{subject to} \\ v_i(x)\geq 0, i=1,\ldots,p+q, \\ \mu_i(x)\geq v_i(x), i=1,\ldots,p+q, \\ \mu_i(x)+v_i(x)\leq 1, i=1,\ldots,p+q \end{array}$$

Converting an IFO to deterministic form is as:

$$\begin{array}{l} \text{Max } (\alpha-\beta) \\ \text{subject to:} \end{array}$$

$$(2) \quad \left.\begin{array}{l} \alpha\leq \mu_i(x), i=1,\ldots,p+q \\ \beta\geq v_i(x), i=1,\ldots,p+q \\ \alpha\geq\beta, \beta\geq 0 \\ g_i\geq 0 \\ \alpha+\beta\leq 1 \end{array}\right\}$$

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2.5 Intutionistic Fuzzy Pareto Optimal Solution-

$x^* \in R^n$ is said to be an intutionistic fuzzy pareto optimal solution to the above problem

if and only if there does not exist another $x \in R^n$ such that

$\mu_i(f_i(x)) \geq \mu_i(f_i(x^*))$, $v_i(f_i(x)) \geq v_i(f_i(x^*))$ for all i and

$\mu_j(f_j(x)) \neq \mu_j(f_j(x^*))$, $v_j(f_j(x)) \neq v_j(f_j(x^*))$, for at least one j, $j \in \{1,2,...,p+q\}$.

2.6 Formulation of the weighted intuitionistic fuzzy additive multiobjective linear model for VSP-.

The weighted additive model is widely used in multiobjective optimization techniques to reflect the importance of the objectives (**Tiwari, Dharmar and Rao (1987)**).The objective the function is formulated by multiplying each membership and non-membership functions with a suitable weight and adding them together. This leads to the formulation below as the model of (**Mahapatra (2012)**) in IFO model for VSP and converting to an equivalent LPP is given as follows:

$$\left.\begin{array}{l} \text{Max } \sum_k w_k(\mu_i(x) - v_i(x)) \\ \text{subject to:} \\ \sum_k w_k = 1 \\ w_k \geq 0, k=1,2,..., p+q,\ x \in R^n \\ \mu_k(x) \geq v_k(x) \geq 0, k=1,2,..., p+q,\ x \in R^n \\ x_i \geq 0,\ i=1,2,...,n. \end{array}\right\} \quad (3)$$

Assigning weights to the objective functions is a difficult task. We have taken weights as assigned by **Amid, Ghodspour and Brien (2009)**.

2.6.1 Construction of membership and non-membership functions for objective functions-

In IFO, the fuzzy objective functions are defined by their membership and non-membership functions which are linear. The advantage of linear membership function is that its simplicity and fixing the upper and lower levels of acceptability .For construction we first determine the lower and upper limits of the membership function.

$$\left.\begin{array}{l} \text{For membership functions:} \\ U_k^\mu = \max(Z_r(x)) \\ L_k^\mu = \min(Z_r(x)) \\ \text{For non-membership functions:} \\ U_k^\mu = U_k^\mu - \lambda(U_k^\mu - L_k^\mu) \\ L_k^v = L_k^\mu \text{ where } 0<\lambda<1 \end{array}\right\} \quad (4)$$

The construction of membership and non-membership functions for the minimization of the three objective functions is as follows:

$$\mu_k(Z_k(x)) = \begin{cases} 0, & if\ z_k \geq U_k^\mu \\ \dfrac{U_k^\mu - z_k(x)}{U_k^\mu - L_k^\mu}, & if\ L_k^\mu < z_k(x) < U_k^\mu \\ 1, & if\ z_k(x) \leq L_k^\mu \end{cases} \quad (5)$$

$$v_k(z_k(x)) = \begin{cases} 1, & if\ z_k(x) \geq U_k^v \\ \dfrac{z_k(x) - L_k^v}{U_k^v - L_k^v}, & if\ L_k^v < z_k(x) < U_k^v \\ 0, & if\ z_k(x) \leq L_k^v \end{cases}$$

2.6.2 *Algorithm of Steps-*

Step1: Solve each of the objective functions subject to constraints of the model. We obtain maximum and minimum values of each objective function and its decision variables.

Step 2: Using equations (3) and (4) we determine upper and lower limits for membership and non-membership functions.

Step 3: We determine membership and non-membership functions for each objective function by using equation (5).

Step 4: The IFO-MOLP model is formulated and fitted in equation (4).The weights considered for each objective functions namely net price, net rejections and net late deliveries are w_1=0.5,w_2=0.15 and w_3=.25.The above model is solved by Tora 2.1.

III NUMERICAL EXAMPLE AND RESULT

The numerical example has been taken from (**Kumar et al. (2006)**). The four vendor profiles are shown in Table 1 below.The management of the firm wants to be efficient in purchasing, relook at its sourcing strategies,reduce inventory and its vendors .They shortlisted four vendors for selection and allocate orders to vendors. The three objectives in the model are: minimizing the net cost, net rejections and net late deliveries, subject to constraints as demand of the item, vendors' capacity limitations, vendors' budget allocations etc.

Table 1- Vendor Data

Vendor no.	p_i	q_i	l_i	U_i	r_i	f_i	B_i
1	3	0.05	0.04	5000	.88	0.02	25000
2	2	0.03	0.02	15000	.91	0.01	100000
3	7	0	0.08	6000	.97	0.06	35000
4	1	0.02	0.01	3000	.85	0.04	5500

The mathematical formulation (**Kumar et al. (2006)**) of the problem is as follows:

$$\left.\begin{array}{l}
\text{Minimize } Z_1=3x_1+2x_2+7x_3+x_4 \\
\text{Minimize } Z_2=0.05x1+0.03x2+0.02x4 \\
\text{Minimize } Z_3=0.04x_1+0.02x_2+0.08x_3+0.01x_4 \\
\text{Subject to constraints:} \\
x_1+x_2+x_3+x_4=20000 \\
x_1\leq 5000 \\
x_2\leq 15000 \\
x_3\leq 6000 \\
x_4\leq 3000 \\
.88x1+.91x2+.97x3+.85x4 \geq 18{,}400 \\
0.02x1+0.01x2+0.06x3+0.04x4\leq 600 \\
3x_1\leq 25000 \\
2x_2\leq 10000 \\
7x_3\leq 35000 \\
x_4\leq 5500 \\
x_1 , x_2, x_3, x_4\geq 0
\end{array}\right\} \quad (6)$$

The solution procedure of the above formulation is as follows:

Step 1: From the formulation of (6) we obtain the maximum and minimum values of the objective functions as follows:

Min Z_1= 60000 Min Z_2= 433.33 Min Z_3=641.67

Max Z_1=65120.48 Max Z_2= 466.67 Max Z_3=702.41

Step 2: Finding the lower and upper bounds for Z_1, Z_2 and Z_3 and using equation (4), we obtain

For membership functions:

U_1^μ = 65120.48 , L_1^μ = 60000, U_2^μ = 466.67, L_2^μ =433.33, U_3^μ=702.41 L_3^μ =641.67

For non-membership functions:

U_1^ν = 62610.9648

$L_1^\nu = L_1^\mu$ = 60000 , $L_2^\nu = L_2^\mu$ =433.33, $L_3^\nu = L_3^\mu$ =641.67

U_2^ν =450.3 U_3^ν = 672.65

λ=.49

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Step3: We construct membership and non-membership functions for Z_1 only is as follows:

$$\mu_1(Z_1(x))=\begin{cases} 0, & if\ z_1 \geq 65120.48 \\ \frac{65120.48-3x1-2x2-7x3-x4}{5120.48}, & if\ 6000 < z_1(x) < 65120.48 \\ 1, & if\ z_1(x) \leq 6000 \end{cases}$$

$$v_1(z_1(x))=\begin{cases} 1, & if\ z_1(x) \geq 62610.97 \\ \frac{3x1+2x2+7x3+x4-60000}{2610.97}, & if\ 60000 < z_1(x) < 62610.97 \\ 0, & if\ z_1(x) \leq 60000 \end{cases}$$

Similarly the membership and non-membership function for the other two objective function is constructed.
The formulated IFO model as follows

$$\text{Max}[\ 0.5(\frac{65120.48 - 3x_1 - 2x_2 - 7x_3 - x_4}{5120.48}) + 0.5(\frac{3x_1+2x_2+7x_3+x_4 - 60{,}000}{2610.965}]+$$
$$[0.15(\frac{-0.05x_1-0.03x_2-0.02x_4+466.67}{33.34})+0.15(\frac{-4333.33+0.05x_1+0.03x_2+0.02x_4}{26.97})]+$$
$$[0.25(\frac{-0.04x_1-0.02x_2-0.08x_3-0.01x_4+702.41}{60.74} + 0.25(\frac{-641.67+0.04x_1+0.02x_2+0.08x_3+0.01x_4}{30.98})]$$

$$\left.\begin{array}{l} \text{Subject to:} \\ x_1+x_2+x_3+x_4=20000 \\ x_1 \leq 5000 \\ x_2 \leq 15000 \\ x_3 \leq 6000 \\ x_4 \leq 3000 \\ 0.08x_1+0.91x_2+0.97x_3+0.85x_4 \geq 18400 \\ 0.02x_1+0.01x_2+0.06x_3+0.04x_4 \leq 600 \\ 3x_1 \leq 25000 \\ 2x_2 \leq 10000 \\ 7x_3 \leq 35000 \\ x_4 \leq 5500 \\ \sum_k w_k = 1, w_k \geq 0, k=1,2,3, x \in R^n \\ \mu_k(x) \geq v_k(x) \geq 0, k=1,2,3, x \in R^n \\ x_i \geq 0,\ i=1,2,3,4. \end{array}\right\} \quad (7)$$

On solving the formulation by Lingo 9.0, we obtain the optimal solution for the three objective functions and decision variables as follows: Z_1=60000.0023, Z_2=466.6666, Z_3=641.6666 and x_1=0, x_2=15000, x_3=4166.667, x_4=833.33.The weighted intutionistic fuzzy additive multiobjective linear model for vendor selection problem gives better results than a fuzzy multiobjective linear programming model. The degree of achievement of the objective function is high. Also comparing by **Kumar et al. (2006)** paper our results are still better. The table below gives a comparative study chart for the two different approaches. By our approach gives reduced values of objective functions z_1 and z_3. Also, the maximum order allocation went to vendor 2 then vendor 3 and vendor4 for all the three cases.

Table 2- Comparison of two methods in VSP

Value of objectives and order allocation	Linear Intuitionistic Fuzzy Weighted Additive model	Kumar et al.(2006) Linear Fuzzy model
Z_1(cost)	60000.0023	61818
Z_2(rejected items)	466.6666	448
Z_3(Late deliveries)	641.6666	665
Order to vendor 1	0	0
Order to Vendor 2	15000	14092
Order to vendor 3	4166.667	4621
Order to vendor 4	833.33	1287

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IV. CONCLUSIONS

We formulated an intutionistic fuzzy weighted additive model for the vendor selection problem. We have defined a linear membership and non-membership for IFO problem. Linear memberships are the most simple and popular in various approaches. They fix the lower and upper bounds of both acceptance and rejection region. We compared our formulation with a fuzzy and intutionistic fuzzy model. The intutionistic fuzzy weighted additive model gave better results compared to others. Future scope is working with various membership functions like non-linear ones where results could be better.

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