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Infinite Solutions on Ramanujan's House Problem through C-Program

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Abstract:
Srinivasa Ramanujan's house problem was well known to many Mathematicians. This problem has infinite solutions. Some people tried to calculate few solutions manually , though number of calculations involved in it. Some got few more solutions by using continued fraction and Pell's equation. In this paper we write a Pseudo code to get infinite solutions for Srinivasa Ramanujan's house problem up to the desired number.

Keywords: *Continued fraction , Pell's equation.*

I. Introduction

Srinivasa Ramanujan's house problem was about a street in the town of Louvain in Belgium. One of the house numbers had the peculiar property that the total of the house numbers lower than it was exactly equal to the total of the numbers above it.Furthermore , the peculiar number was greater than 50 but less than 500. Let n be the number of houses in the street and x be the number of peculiar house.The pair that has $50 \le x \le 500$ *is* $(x,n) = (204,288)$. This is particular pair. This pair was extended to general values for (x,n).

The location of the house is 1 , 2 ,3,..........., x-1, $[x]$, $x+1$, $x+2$,........,n . Then the problem can be written as Sum of 1 *to* $x-1$= Sum of $x+1$ *to* n.

Using the sum of first natural numbers formula, which is equivalent to

$$\frac{x^2-x}{2}=\frac{n^2+n}{2}-\frac{x^2+x}{2} \qquad ....(1)$$

This implies

$$8x^2=(2n+1)^2-1 .... \qquad (2)$$

To find x,n Butcher[1] used the theorems (a) and (b) on irrational numbers and continued fraction , where the continued fraction is defined as

$$a_0+\cfrac{b_0}{a_1+\cfrac{b_1}{a_2+\cfrac{b_2}{a_3+\cfrac{b_3}{a_4+.....}}}}$$

This can be written as $(a_{0:}\ b_0,a_1,b_1,a_2,b_2....)$

Theorem (a) : If $r>0$ is irrational and

$$\left|\frac{p}{q}-r\right|\le\frac{1}{2q^2}\ r>0 \text{where } (p,q)=1 .$$

Then $\frac{p}{q}$ is a continued fraction convergent for r

Theorem (b) : If $\left\{\frac{p_k}{q_k}\right\}$ is the sequence of continued fraction convergent for an irrational $r>0$, then r is between $\frac{p_k}{q_k}$ and $\frac{p_{k+1}}{q_{k+1}}$ and $\frac{p_{k+1}}{q_{k+1}}$ - $\frac{p_k}{q_k}=\frac{(-1)^k}{q_{k+1}+q_k}$

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Butcher estimated a continued fraction approximation for $\sqrt{8}$ which is (2: 1,4,1,4,1...). And with the help four recurrence relations (which are equations 3, 5a, 5b and 6 in

[1] with large number of mathematical calculations Butcher had the following combinations

k	1	3	5	7	9	--------
$\frac{p_k}{q_k}$	$\frac{3}{1}$	$\frac{17}{6}$	$\frac{99}{35}$	$\frac{577}{204}$	$\frac{3363}{1189}$	--------
$\frac{n_k}{x_k}$	$\frac{1}{1}$	$\frac{8}{6}$	$\frac{49}{35}$	$\frac{288}{204}$	$\frac{1681}{1189}$	--------

Without using Theorems (a) , (b) and recurrence relations we can find infinite solutions for x only when we use the continued fraction for approximation of $\sqrt{8}$ is (3: -1,6,-1,6,-1,6,-1...).

In [2] to get the values of x and n Continued fraction of $\sqrt{2}$ and Pell's equation were used. Pell's equation is any Diophantine equation of the form

$$z^2 - my^2 = 1,$$

m is a non square positive integer and integer solutions are sought for z and y.

The algorithm for $\sqrt{2}$ is as follows:

$\sqrt{2} = 1 + x$, squaring and simplification leads to a recurrence relation $x = \frac{1}{2+x}$.

Continued fraction of $\sqrt{2}$ which is simple and infinite Continued fraction

$$1 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2 + x}}}}$$

which is simple and infinite continued fraction. The convergent , formed by truncating the continued fractions at successive terms are

$$1,\ 1 + \frac{1}{2} = \frac{3}{2},\ 1 + \cfrac{1}{2 + \cfrac{1}{2}} = \frac{7}{5},$$

$$1 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2}}} = \frac{17}{12},$$

$$1 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2.}}}} = \frac{41}{29}, ----$$

used in [2] .This $\frac{z}{y}$ also form solutions to the Pell's equation $z^2 - 2y^2 = \pm 1$. The peculiar house number $x$is product of of$z$ *and* y and n is provided by z^2 and $2y^2$ alternatively.

In both the articles [1], [2]mathematical calculations are extremely high. If n is more than 7 digit number then it can't be imagined how much human power is required. Now , here we are giving a pseudo code for this problem , from this infinite solutions can be obtained at once when it will be converted to *c-program*

II. PSEDUO CODE

1. Begin
2. Initialize i , j, k and Left sum and Right sum.
3. Input, number of houses in the street . Let it be m.$(m > n)$
4. Start *for loop* with $i = 2$ and $i \leq m$, $i{+}{+}$
5. Assign $k = i - 1$.
6. Calculate Left sum = $k.(k+1)/2$.
7. Right Sum = 0.
8. Perform *forloop* with $j = i + 1$ to m by incrementing j that is $j{+}{+}$.
9. Calculate Right Sum = Right Sum + j.
10. Check the condition whether Right Sum ≥ Left sum , if it is true then go to 13.
11. Continue j *for loop*
12. Now check whether Left sum equals to Right sum

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if it is true , then print the peculiar house number that is *x* and *j - 1*that gives *n*

13. Continue *ifor loop*
14. Stop.

OUT PUT :

x	*n*
6	8
35	49
204	288
1189	1681
6930	9800
40391	57121
235416	332928
1372105	1940449

III. Reference

[1] http//math.auckland.ac.nz/~butcher/miniature/miniature2
[2] House number puzzle –pi mu Epsilon Journal Vol.14 , F6 pp 389-97, 2017
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