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Fractal Applications in Digital Mammogram Analysis for the Early Detection of Breast Cancer

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Abstract:
Breast cancer is one of the frequent and leading causes of mortality among women in the world. Women with early-stage breast cancers are expected to have greater probability of survival. Digital mammogram is emerged as a most reliable screening technique for the early diagnosis of breast cancer and the presence of masses in mammograms is an important early indication of breast cancer. Fractal geometry is an efficient mathematical approach that deals with self-similar, irregular geometric objects called fractals. As the breast background tissues have high local self-similarity, which is the basic property of fractals, fractal analysis finds its place in the effective analysis of digital mammograms. This chapter emphasizes the recent facts on breast cancer risk and projects the significance of fractal applications in the early diagnosis of breast cancer that includes suppression of pectoral muscles, removal of artifacts, detection and segmentation of masses in digital mammograms. The fractal applications in the analysis of digital mammograms are discussed using suitable illustrative research experiments.

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1. Introduction

The developments in Digital Image Processing (DIP) have opened up a new dimension in medical diagnostics. The image segmentation, an element of DIP deals with subdividing an image into its constituent regions or objects. Moreover, it provides a platform to retain the vital objects/regions of interest and ignore the insignificant details of an image [1, 2], which directly complements the process of detection and diagnosis in medical images. This computer-aided medical image analysis holds greater potential in health diagnosis which has gained popularity over the period.

Radiological imaging such as computed tomography, mammograms, and magnetic resonance imaging, have emerged as an indispensable diagnostic tool for the medical practitioners for the detection/identification of health disorders. These images are formed as a function of illuminating source for X-ray and the transmissivity of the subject to be imaged. As the technological advancements in the computing systems and softwares have enormously enhanced the precision of imaging, the applicability, reliability and accountability of such images are wider [3]. In general, the radiologists arrive at medical conclusions based on the pre-defined specific patterns with reference to the nature and severity of the disorder to be diagnosed [4].

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In both developed and developing countries, the highest incidences of mortality among the women in the age group of 35-55 are due to breast cancer [5]. The viability of breast cancer can be detected at an early stage by the diagnosis of the masses and thereafter classifying them as benign or malignant. Mammography is an invasive medical imaging technique that combines, low-dose radiation and high-contrast, high-resolution film for screening breast. Even though, the purpose of this process is to detect the masses that cannot be physically detected [6], accuracy of the diagnosis is inhibited by several factors of the image such as presence of artifacts and pectoral muscles, poor quality, fatigues, etc. Hence, any computer aided breast cancer diagnosis system requires an effective mechanism to cull out the maximum relevant information from the mammogram by suppressing these factors are a welcome step.

Fractal geometry is an efficient mathematical approach that deals with self-similar, irregular geometric objects called fractals. In the recent years, Fractal-based techniques are being widely applied on several areas of DIP such as image enhancement, image compression, image encoding, image segmentation and texture analysis [7, 8]. Masses/Microcalcifications appear in an inhomogeneous background, describing the structure of the breast tissue is found to possess self-similarity, which qualifies itself to be a fractal. As these images exhibits self-similar structures [9, 10], which is the basic property of a fractal object, fractal approach can be used as an effective feature descriptor in the segmentation, detection and classification of masses/microcalcifications in mammograms. This chapter describes the developments and uses/implementation of fractal techniques as a key factor in the feature description based mammogram analysis for the early diagnosis of breast cancer.

In this Paper, section 2 describes the basics of fractals and section 3 presents the facts about the risks of breast cancer and the statistics, whereas the principle and applications of digital mammography is given in section 4. An overview on the BIRADS classification system and MIAS database is shown in sections 5 and 6 respectively. The fractal applications in the context of mammogram analysis are produced in section 7 while the conclusions are drawn in section 8.

2. Fractals

Fractal geometry was introduced to the world of research in 1982 by Mandelbrot and has gained a significant momentum over the years as its applications are getting progressed widely in the area of Medical Image Processing. The Fractals are self-similar and irregular rough geometric shapes which can be subdivided in parts, each of which is reduced to similar of the whole and those Fractal objects are characterized by their Fractal Dimension which has got information about the geometric structure of the objects [9, 10]. The Fractal-based techniques are being applied broadly in several areas of image processing such as image segmentation, image enhancement, texture analysis, etc [11]. Fractal dimension determines how fractal objects differs from Euclidean objects and it measures the degree of fractal boundary fragmentation or irregularity over multiple scales too.

2.1 Box Counting Method for the calculation of Fractal Dimension

In Euclidean n-space, the bounded set X is said to be self-similar when X is the union of N_r distinct non-overlapping copies of itself, each of which is similar to X scaled down by a ratio r. Fractal Dimension FD of X can be derived from the relation [9,10], as

$$FD = \frac{\log(N_r)}{\log(\frac{1}{r})} \quad (1)$$

The algorithmic description of the proposed method to calculate the Fractal dimension using the Box Counting method is explained as follows.

Algorithm: Computation of Fractal Dimension using Box Counting.

Aim: To Calculate Fractal dimension

Input: A 2-Dimensional image, I
Output: Fractal Dimension, FD of I

1. Read a 2-D input mammogram image I
2. Cover the image with boxes of size r.
3. [L, M] ← IMSIZE [I]
4. If L > M then $r \leftarrow$ L; Else $r \leftarrow$ M
5. Let min. and max. gray levels of the image fall in k and l box respectively.
6. Calculate $n_r(i, j) = l - k + 1$ where $n_r(i, j)$ is the contribution at the $(i,j)^{th}$ grid.
7. Find $N_r = \sum n_r(i, j)$, where N_r is the summation of n_r with respect to r.
8. Compute fractal dimension FD using Eqn.(1)
9. Stop.

2.2 Hurst Coefficient

The assumption of statistical self-affinity implies a linear relationship between fractal dimension, a measure of roughness and Hurst coefficient, a measure of long-memory dependence. Hurst Coefficient is defined as the difference between the topological dimension and fractal dimension. The Hurst coefficient, FH is the only one parameter of interest in the fractal Brownian motion, which can be described as texture features, when we apply it to classify the breast tumors if any. Considering the topological dimension T_d and fractal dimension FD, the Hurst coefficient H can be calculated [9, 10] as

$$H = T_d - FD \quad (2)$$

The application of Fractals has opened up new avenues in medical image processing due to abundant occurrence of fractals in medical images. Also fractal objects can also be used to model the objects of interest in an image [12-15]. Several applications of fractal techniques in segmentation of mammograms confirm the fact that the fractal approach is an efficient method for mammogram segmentation that notably reduces the cost of computation. The presence of fractal objects in digital mammogram, offer greater scope for detection of masses/microcalcifications and hence the early detection of breast cancer through fractal objects will improve the probability of right prognosis [16-18].

In fractal image processing, the image details can be contextually modelled using fractal objects that are attractors of sets of 2-D affine transformations. This chapter focuses to enlighten the applications of fractals in developing a more generic automated computer-assisted diagnostic (CAD) method, for the segmentation, detection and classification of masses/microcalcification clusters in digital mammograms. Fractal applications in the context of mammogram analysis are explained in Section 7.

3. Breast Cancer Risk and Statistics

When a single cell or a group of cells gets changed from its usual control which regulates cellular growth and begins to multiply and spread, Cancer is caused. This results in a mass, tumor or neoplasm. When the abnormal growth of the mass is restricted to a single, circumscribed, expanding mass of cells, those masses are termed as benign; in turn, if the abnormal growth invades the surrounding tissues and gets spread to the other organs of the body, it is termed as malignant [19]. A notable factor in breast cancer is that it appears earlier in life than other types of cancer.

As per the recent estimates of American Cancer Society breast cancer in the United States for 2012 are about 226,870 cases of invasive breast cancer in women and about 39,510 women mortality out of breast cancer. About 1 in 8 (12%) women in the US will develop invasive breast cancer during their lifetime. Currently, there are more than 2.9 million breast cancer survivors in the United States. Recent data from the “Atlas of Cancer in India project – a study to assess nationwide patterns of cancer incidence across urban and rural parts of the country” [20] - suggest that breast cancer is the most common cancer in metropolitan cities and is predicted to be the most common type of cancer in the coming decade. A recent report by the Indian Council of Medical Research (ICMR) predicts the number of breast cancer cases in India to rise to 106,124 in 2015 and to 123,634 in 2020. The breast

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cancer cases in urban Indian women is 25-30 and the age adjusted rate is 30-35 new cases per 1,00,000 women per year. The average increase in breast cancer over a 30 year period in Mumbai, India was 11 per cent per decade. Breast cancer is increasing both in young (11per cent per decade) and old women (16 per cent per decade). There are an estimated 1,00,000-1 ,25,000 new breast cancer cases in India every year and thereby, the number of breast cancer cases in India is estimated to double by 2025.

Breast cancer is the most common cancer in women worldwide. Breast cancer is the second leading cause of cancer death in women, exceeded only by lung cancer. The chance that breast cancer will be responsible for a woman's death is about 1 in 36 (about 3%). It is also the principle cause of death from cancer among women globally. 89% of women diagnosed of breast cancer with high incidence rates are still alive 5 years after their diagnosis in western countries, which is because of an early detection and treatment. The UK and USA have one of the highest incidence rates worldwide (together with the rest of North America and Australia/New Zealand), making these countries a priority for breast cancer awareness. To add, one-third of these breast cancer deaths in women could be controlled if detected and treated early. To count, nearly 400,000 lives could be saved every year worldwide. Hence, the early diagnosis becomes an important means to reduce the future threat with regard to the mortality rate. It is reported by ICMR that one in 22 women in India is likely to suffer from breast cancer during her lifetime. As per the Population Based Cancer Registry (PBCR), Breast cancer accounts for 28.3% of all cancers in women in India.

The World Health Organisation [WHO] has suggested that two components of early detection have been shown to improve cancer mortality:

- **Education**—to help people recognize early signs of cancer and seek prompt medical attention for symptoms.
- **Screening programs**—to identify early cancer or pre-cancer before signs are recognizable, including mammography for breast cancer.

In the UK and US, **effective education and screening could save between 12 to 37 lives per day**, respectively.

Thus, the current situation indicates to have proper efforts to organize awareness programmes, develop strategies for early detection and treatment in order to control breast cancer death-toll. Against this back drop, the present chapter aims to explain and discuss the important fractal applications in the early detection of breast cancer that facilitates the proper diagnosis and therapy.

4. Digital Mammography

A normal mammogram projects the converging patterns of fibroglandular tissues and vessels. Any feature that causes a distortion with reference to the normal pattern is analyzed with suspicion and extra attention is given for those tissue patterns. The breast features such as calcifications, masses, and increase in density level, architectural distortion and asymmetry among the left and right breast images are notified by the mammogram which helps the radiologists in the proper diagnosis of breast cancer. There is no direct observation of breast cancer risk from the low-dose radiation exposure of mammography. The recent technological developments/advances in mammography continue to reduce the radiation exposure while screening mammogram and preserving the quality of the images.

To produce an image of the breast X-rays are used in both Digital and conventional mammography. In digital mammography, an electronic image of the breast is stored as a computer file, whereas in conventional mammography, the image is stored directly on the film. This stored digital information can be enhanced, magnified and processed for further evaluation more easily than the information stored on film. Other than this difference in the way the image is recorded and stored, there is no other difference between the two types of mammography.

The digital mammography helps a radiologist/health practitioner to adjust, store, and retrieve digital images electronically. Hence, the digital mammography is more useful in

- Sharing of image files electronically, which makes long-distance consultations between radiologists and breast surgeons possible?
- Projection of subtle differences between normal and abnormal tissues in an easy way.
- Carrying out of follow-up procedures and repetition of images by reducing the exposure to radiation can be done effectively based on the consultation.

5. Breast Imaging Reporting and Database System (BI-RADS®)

To describe the findings of the mammogram analysis, the American College of Radiology (ACR) has established a uniform way for radiologists. The established system is called BI-RADS which include seven standardized categories. With each category of BI-RADS, there associates an assessment and a follow-up plan, which helps radiologists and other physicians for the arrangement of proper patient's care and treatment. The following *table 1* depicts the categories of BI-RADS and its assessment and follow-up plan.

Table 1: BI-RADS Categories

Breast Imaging Reporting and Database System (BI-RADS)		
Category	**Assessment**	**Follow-up**
0	Need additional imaging evaluation	Additional imaging needed before a category can be assigned
1	Negative	Continue annual screening mammograms (for women over age 40)
2	Benign (non-cancerous)	Continue annual screening of mammograms
3	Probably benign	Receive a 6-month follow-up mammogram
4	Suspicious abnormality	May require biopsy
5	Highly suggestive of malignancy (cancer)	Requires biopsy
6	Known biopsy-proven malignancy (cancer)	Biopsy confirms presence of cancer before treatment begins

6. MIAS Database

The Mammography Image Analysis Society (MIAS) [21], which is an organization of UK research groups interested in the understanding of mammograms, has produced a digital mammography database. The X-ray films in the database have been carefully selected from the United Kingdom National Breast Screening Programme and digitized with a Joyce-Lobel scanning microdensitometer to a resolution of 50 µm × 50 µm, a device linear in the optical density range 0-3.2 and representing each pixel with an 8-bit word. The database contains left and right breast images for 161 patients, and is available on a DAT-DDS tape. Its quantity consists of 322 images, which belong to three types such as Normal, benign and malignant. There are 208 normal, 63 benign and 51 malignant (abnormal) images. It also includes radiologist's `truth'-markings on the locations of any abnormalities that may be present.

For each film, experienced radiologists give the type, location, scale, and other useful information of them. According to these experts' descriptions, the database is conclude with four kinds of abnormalities (architectural distortions, stellate lesions, circumscribed mass and calcifications). The database possesses an introduction file, which included the following information:

- Type: to which kinds mentioned above, the abnormalities belong to.
- Sort: whether the abnormalities are cancer or benign ones.

Location and size: the original coordinates and diameters of the abnormalities.

6.1 Detailed Information

The following list gives the films in the MIAS database and provides appropriate details as below:

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First Column:
MIAS database reference number.
Second Column:
Character of background tissue:
F Fatty
G Fatty-glandular
D Dense-glandular
Third Column:
Class of abnormality present:
CALC Calcification
CIRC Well-defined/circumscribed masses
SPIC Spiculated masses
MISC Other, ill-defined masses
ARCH Architectural distortion
ASYM Asymmetry
NORM Normal
Fourth Column:
Severity of abnormality;
B Benign
M Malignant
Fifth and Sixth Column:
x,y image-coordinates of centre of abnormality.
Seventh Column:
Approximate radius (in pixels) of a circle enclosing the abnormality.

The size of *all* the images is 1024 pixels x 1024 pixels. The images have been centered in the matrix. When calcifications are present, centre locations and radii apply to clusters rather than individual calcifications. Coordinate system origin is the bottom-left corner. In some cases calcifications are widely distributed throughout the image rather than concentrated at a single site. In these cases centre locations and radii are inappropriate and have been omitted.

7. Fractals in Mammogram Analysis

In science, fractals have a variety of applications due to the existence of its property of self similarity everywhere. They can be used to model plants, blood vessels, nerves, clouds, mountains, turbulence, etc. Fractal geometry also has an application to biological analysis. The concept of fractals and chaos which are specific to non-linear systems are widely observed in biological systems too [22]. For instance, variation in the development of the dendrite stage can be evaluated with a fractal dimension. There are several applications of fractals in computer science namely data mining, automatic object classification, texture characterization, shape generation, image compression, etc. The application of fractals in the mammogram analysis, which helps in the prognosis of breast cancer, is discussed in detail as follows: Different mammogram images are collected from the Mini-Mammographic database of the Mammographic Image Analysis Society from the Pilot European Image Processing Archive (PEIPA) [23] at the University of Essex. The developed algorithms are implemented using Matlab.

7.1 Fractals in the Suppression of Pectoral Muscle

The important pre-processing step in the mammogram analysis is the suppression of pectoral muscle from the original mammogram image. The region of pectoral muscles containing the brightest pixels has an adverse effect, during the detection of masses/microcalcification clusters in a mammogram. This pectoral muscle can be segmented using the principle of Fractal dimension of the image, which is image-dependent, along with the application of morphological operations [24]. This Fractal technique based pectoral muscle segmentation is proved to produce quite promising results.

For an input mammogram image, initially the fractal dimension FD is calculated by box counting method, using Eqn.(1) and the image is enhanced by mapping the intensity values of the input gray scale image I to new values in EI. The enhancement process being used saturates 1% of data at low and high intensities of I, thus increasing the contrast of the input image. Then Sobel operator is used to detect the edges of the enhanced input image EI and those edge intensities are stored in ED which are compared with the Fractal dimension of the image I. Now, the pixels in ED for which the intensities are less than the Fractal dimension value are marked in

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another image EI that gives the brightest pixels of I which represents both the masses and clusters of microcalcifications together with the pectoral muscle region.

The pectoral muscle may be the intensities of bright white pixels located either at the left upper or right upper corner of the breast region. To detect this region, the morphological operation dilation and erosion are applied on the image EI followed by the morphological opening operation. The structuring element of the morphological opening operation to be used is a disk with radius 2 pixels. This gives the projection of pectoral muscle region alone that results in exact segmented image ES.The segmentation results clearly indicate that the suppression of pectoral muscle using Fractal dimension is more precise and robust. The entire process of segmentation of pectoral muscle from a digital mammogram is depicted in Fig. 1 (a)-(e).

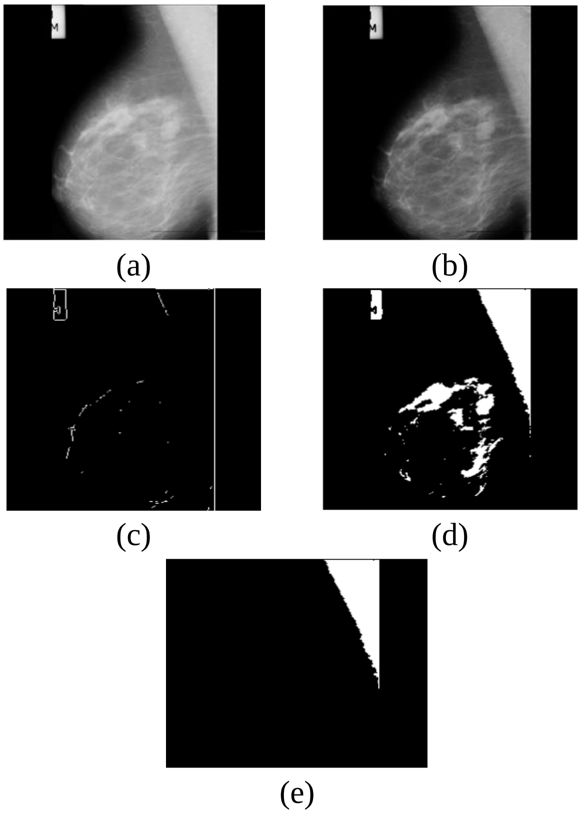

Fig.1: (a) Original Image of mdb019.pgm, (b) Enhanced Image of (a), (c) Intensity Values of Edges, (d) Detected brightest white pixels regions (e) Segmented Pectoral muscle

7.2 Fractals in the Detection of Mass Boundary

For a given input mammogram image initially the fractal dimension, FD is found by box counting method using Eqn.(1). Then as preprocessing step, the morphological operations dilation and erosion are applied on the input mammogram image. Images are inherent of randomness. As the fractal analysis is sensitive to noise, application of morphological operations tend to suppress noise if any, in addition to image enhancement on the input image. Using the fractal dimension FD, find the Hurst coefficient FH using Eqn.(2). Now, using Hurst coefficient as one of the factors along with thresholding value in the Sobel edge detection, gradient mask is obtained. To smoothen the image, post processing events dilation and erosion are applied again and the interior gaps are filled, which leads to the edge detection in an image. Finally from the segmented image masses are detected by outlining the border of the region containing masses [25, 26]. The procedure is explained with the help of a mammogram in Fig 2.

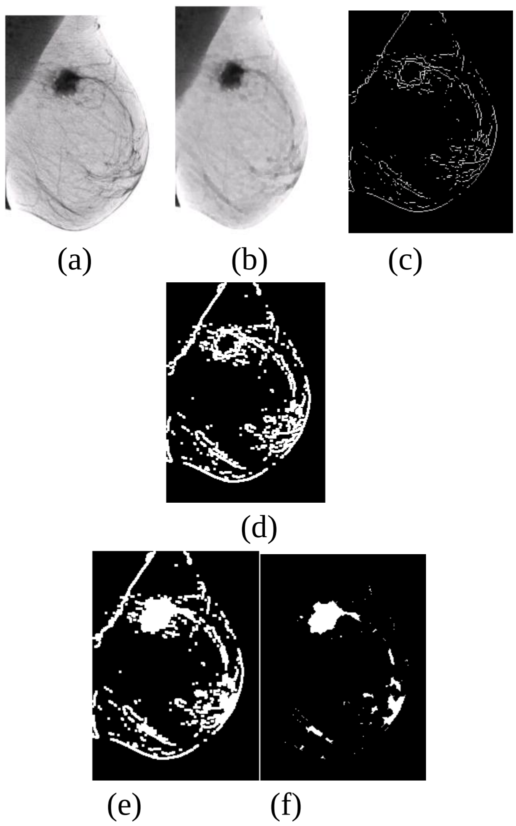

Fig 2. (a) Original Image (b) Pre-processed image (c) Edges detected using Sobel operator with Hurst

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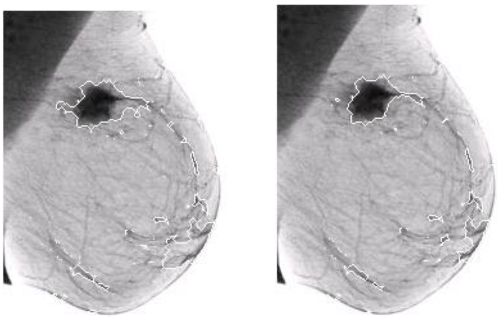
co-efficient (d) Dilated image of (c) (e) Image (d) with filled holes (f) Segmented Ouput Image.

To ascertain the merit of the fractal hurst based edge detection, as a comparative study, the edge detection using Sobel is done, using Fudge factor in addition to Hurst coefficient factor. Both the results are analysed and are depicted in Fig 3. The edges constructed using Hurst coefficient is far accurate and are found to be confined to the masses than the one obtained with Fudge factor edges.



(a) (b)

Fig 3. Edge detection using (a) Sobel with Fudge factor (b) Sobel with Hurst co-efficient

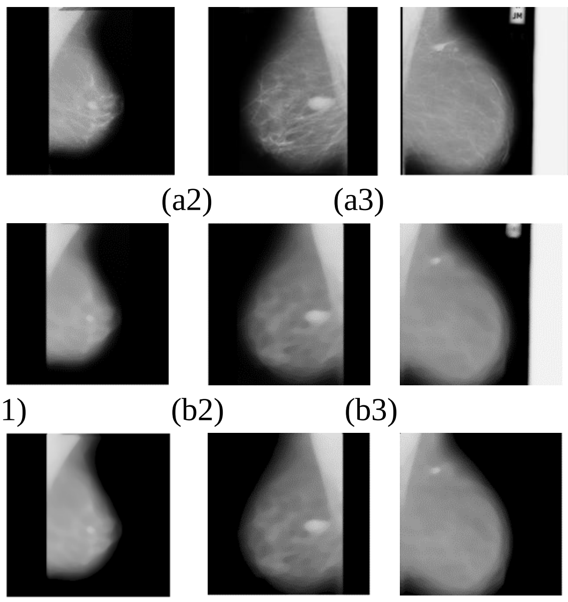
From the images (a) and (b) of Fig 3, it is clearly noticed that the image segmentation and masses detected are far better in Fig. 3(b) whereas Fig.3(a) gives some unwanted additional edges. Hence it is clearly understood that the Mass detection using Fractal Hurst coefficient factor based edge detection has an edge over the conventional Sobel approach.

7.3 Fractal Thresholding for Artifact Removal and Mass Segmentation

Compute the values of fractal dimension FD and fractal hurst FH and also find out the statistical measure standard deviation S for the input mammogram image. Now two fractal thresholding values T1 and T2 are formulated by calculating T1 = (H*H)/S and T2 = (H/S)*D. Filtering is then applied on the image I using median filtering technique to acquire I_{FM}. The significance of using median filtering is that it removes the noise without disturbing the edges as the edges play an important role in the segmentation of mammogram.

Then the film artifacts such as labels and x-ray marks are removed from I_{FM} by making the pixels as ones whose intensity values are greater than fractal thresholding value T1 and else pixels as zeros along with the usage of morphological operations to produce the image I_{LABEL}. Further, the label removed image I_{LABEL} is enhanced, in order to increase the variation in brightness and to improve the computational consistency, by framing a look up table with respect to intensity value ranges and converting the values in I_{LABEL} based on the look up table to acquire I_E, thus increasing the contrast of the filtered label removed image. Now, the mass is detected from I_E by considering only those pixels whose intensity values are greater than the Fractal thresholding value T2 which is represented by the image I_{DETECT}. Application of morphological operations dilation, erosion and reconstruction again on the image I_{DETECT} suppresses the pectoral muscle and results in the final segmented mass image I_{MASS} from the input mammogram image [27].

For illustrative purpose of this method, the results of 3 mammograms (mdb010.pgm, mdb025.pgm and mdb132.pgm) with circumscribed masses are depicted in Fig.2 (a1)-(f1), (a2)-(f2), (a3)-(f3) respectively.



(a1) (a2) (a3)

(b1) (b2) (b3)

(c1) (c2) (c3)

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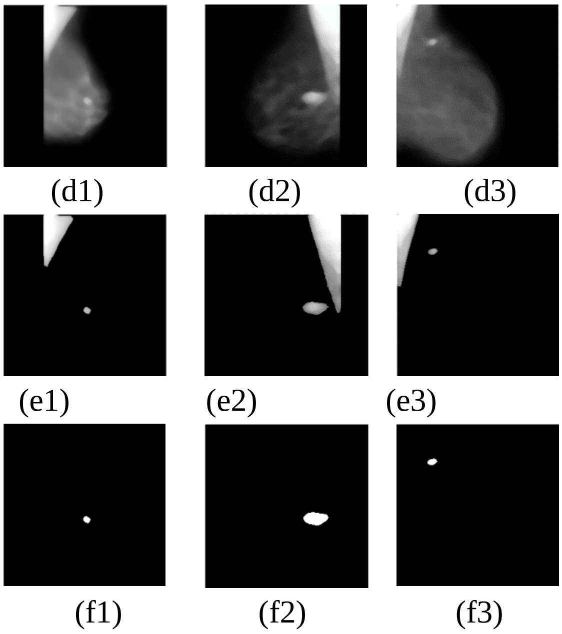

Fig.4: Original Image of (a1) mdb010.pgm, (a2) mdb025.pgm, (a3) mdb132.pgm; (b1)-(b3) Median Filtered Image of (a1)-(a3); (c1)-(c3) Label removed Image of (b1)-(b3); (d1)-(d3) Contrast Enhanced Image of (c1)-(c3); (e1)-(e3) Detected Image of (d1)-(d3); (f1)-(f4) Final Segmented mass from (e1)-(e4).

8. CONCLUSION

The study of fractals has been utilized in science and research since years. Research studies show that any object that possesses the property of self similarity can be crtically analyzed by the principle of fractals. This provides a platform to utilize the fractals as a feature descriptor in feasible medical diagnostics for screening the disorders in radiological and ultra sound images. The application of fractals in breast cancer screening from digital mammogram varies from suppression of pectoral muscle, boundary detection, segmentation, classification and analysis of suspicious masses. The results of the fractal-based application methods for mammogram analysis obtained over various tested mammograms from miniMIAS database have substantiated the genuenity of fractal applications in diagnosis.

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