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Non Markovian Priority Queue for Existing Customer with Breaks in Call Center

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Abstract:
This paper actually links the queuing model in a random way with other disciplines like amenity priority, thereby playing an important role in maintaining the call center. The Priority is always given to the customer who already exists. Here an M/G/1 queue with additional location of the server based on Bernoulli timing. It just has only one type of server but it provides multiple types of amenities. Already present client distribution with prospect and new customer amenity with prospect with amenity time follows common distribution. The new client luxury will hindered once an old client lands in the framework. The time subordinate possibility producing capacity have been obtained regarding Laplace change and relating consistent formal grades are acquired unequivocally for this type. Additionally selected exhibition estimates, for example, mean line length and mean sitting tight time are processed for both line. The legitimacy of this model is featured by certain methods for a speculative circumstance. By utilizing the beneficial variable procedure, the consistent state prospect creating capacities can be acquired for the framework/circle. Some significant framework execution quantifies, the mean occupied period and the mean occupied cycle are also talked about.

Keywords: *Poisson arrival, prospect generation function, stable state, non-dynamic state, queue length, waiting time*

I. INTRODUCTION

In this paper, the queueing in a call center and the effects revolving around can be discussed. The first and foremost thing to notice is the waiting time of the customers, which are unavoidable in most cases. But, we have arrived at certain conclusions by intense surveys and multiple efforts. This is done by applying prospect theories at various states in the server. The various states in which these are applied at are static and dynamic as well. Despite the efforts, if still the condition persists then a solution is not a support for it. This solution is far more permanent and applicable to multiple constraints rather than the previous works or literature that is published in this area. A suitable method was always needed to regularise the call center waiting process. In this context, some of these equations are discussed in detail and sufficient depth to understand its intensities and usages at various levels and they are all discussed.

There are limitations in these methods which could be compensated with the help of sufficient number of workers. The increase in the number of workers will in turn increase the cost of the project. But it is anyway better than having a wrong review on the company. Regardless of the past and previous history, an efficient way to handle the callers could be implemented with the help of all these constraints. The reverse process which is also known as the retrial in various instances can also be applied into this relevant space. The time which is limited and valuable in each of the given cases of the call center

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mathematics is also taken into account. With that taken into the consideration into the retrial process, which is basically the call divert process. This function is an automatic in our situation and does not depend on the arrival of the calls.

For this queuing framework, the server isn't ceaselessly accessible. From time to time, the server breaks down and an irregular repair time is required to reestablish appropriate server conduct. In specific, it is permitted to break down whereas a client gets convenience. The breakdown teach at that point decides how the enhancement goes on after the comparing repair. The breakdown disciplines considered incorporate rehashing and proceeding enhancement after the repair, as well as more perplexing combinations of these disciplines, which are presented underneath. The third queuing model may be a need queuing framework. In a need line with two classes, say a tall- and a low-priority lesson, clients of the high-priority course are served at whatever point such clients are present within the framework. In specific, when a low-priority client is being served upon the entry of a high-priority client, the previous instantly takes off the server, making room for the last mentioned. Consequently, for the needed frameworks, the nearness of low-priority customer is preferred.

II. LITERATURE REVIEW

The current clients who are framed in line or not in line are served by control of preemptive or non-preemptive. Blocked pool of clients, new clients (called as conventional clients), leave the framework and join the retrial gathering to retry its comfort after some time when the server is free. Additionally, in a portion of the frameworks, a showing up higher need client may push out the lower need clients whose luxury is continuous to the line or the circle. For a complete examination of need queuing models the peruse may allude [1] Liu et al., [2] Liu and Gao , [3]Senthilkumar et al.(2013), [4] Wu and Lian, related work on preemptive need lines in this area, and on lines with breaks in the following one have talked about. Preemptive need queueing models discover applications in various fields, including media transmission systems and generation frameworks.

Recently, [5] Arivudainambi et al. presented M/G/1 retrial line with single working excursion. Besides, during the working excursion time frame, if there are clients at a lower convenience consummation moment, the server can stop the travel and return to the typical occupied state.

Consistently many queuing cases in which luxury isn't incite and clients need to stand by to get comfort. The preemptive and non-preemptive need idea assumes a significant job in need discipline. In such manner, Choi and Park [6] gave a non-preemptive need retrial line through which clients appreciate non-preemptive need instead of ordinary clients. At the point when all high-need clients have left the framework, the server either continues the pleasantry of the low-need client or rehashes the convenience of the client. In the last case, the pleasantry time might be resembled, or may continue as before. These queuing frameworks can be portrayed as pursues. So also, by distinguishing the bustling times of high-need clients with the fix times of the queuing framework with breakdowns, the framework with breakdowns can be utilized to survey execution of low-need clients. Breakdown teaches at that point promptly identify with need disciplines. For a broad assessment of need queuing models, the peruse may allude to Gao [7] and Peng [8].

Previously, queuing models with various courtesy rates were contemplated by a few creators. The activity of these models nearly made a change the courtesy rate contingent upon the circumstance of the framework, for example, lines in an irregular domain, lines through breakdown, working breakdown, models with breaks and working getaways. In 2002, Servi and Finn [9] exhibited a single server Markovian queuing framework with occupied breaks. The single server Markovian queuing line with breaks to an

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M/G/1/WV line was proceeded by Wu and Takagi [10]. Further, Tien Van Do [11] built up a model in M/M/1/WV with retrial idea.

A short review note on working excursion queuing models was composed by Chandrasekaran et al. [12]. Creators like Gao et al. [13], Rajadurai et al. [14], Gao and Liu [15], Zhang and Liu [16] investigated the ideas of single server retrial queuing frameworks with working breaks.

In 2012, Kalidass and Ramanath [17] exhibited working breakdowns in M/M/1 lines, i.e., the framework may confront disappointment brought about by fiascos whenever the ordinary occupied server is in enhancement and the framework should be prepared with another (reserve) server in readiness for conceivable prime server disappointments. The substitute server gives pleasantry to clients while the significant server is fixed. The pace of pleasantry for the substitute server is various from (lower than) that of the significant server. Right now of the fix finish, the significant server returns to the framework and is available once more. Accordingly, the working breakdown courtesy is a logically reasonable fix approach for temperamental queuing frameworks. As of late, Kim and Lee [18] considered a model M/G/1 queuing framework with calamities and working breakdown civilities. Using the framework geometric strategy, Ma et al. [19] acquired the stationary circulation of an M/M/1 line with different getaways and working breakdowns. The preemptive-continue discipline is likewise applied to evaluate execution of psychological radio systems where auxiliary clients get to the remote channel when it isn't utilized by its essential clients. The need model enables us to represent channel access by essential clients, detecting mistakes of auxiliary clients, just as for heterogeneous channel limit Recently Deepa and Kalidass [20], Jiang and Liu [21], Liu and Song [22] built up a model calamity with working breakdowns. B.Balamurugan, M.Mullai [23] and [24] talked about input, dismissal and breaks in call focuses. Sherif I et al [25] have broke down Preemptive need retrial queueing framework.

To the creators best of information, there are numerous works accessible in the idea of retrial queueing framework with working get-away by utilizing the strategy for lattice geometry examination, yet there is no work distributed in the queueing writing with the blend of need queueing framework with general occasions, quick Bernoulli input and breaks by utilizing the strategy for strengthening variable system.

III. MATHEMATICAL DESCRIPTION OF THE MODEL

Current models are regularly confined to a straightforward arrangement of activities, e.g., little scale enterprises, creation businesses, industrial facilities, bank divisions which needs just a less measure of pleasantry. Things being what they are, these rearrangements don't reflect reality in a suitable manner. If there should arise an occurrence of huge scale enterprises, Big Production businesses and correspondence systems where the enhancements are more out of luck, the phases of civilities and periods of conveniences and discretionary courtesies are expanded.

In context of all the above research work, another lining model has been wrapped for the above versatile correspondence system. The organization time takes after a general apportionment. Server takes a mandatory trip of sporadic length when the organization towards the last customer in the need unit gets over. If the server involved or amidst some entertainment a showing up non need customer get the line together with the possibility υ .notwithstanding that, not all the showing up customers are allowed to join the line. A basic thought of kept tolerability is considered over the arrival of customers. For the above orchestrated model, the time subordinate producing capacity for both need

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unit and non-expect units to the degree Laplace changes have chosen.

In this paper coming up next are assumptions

- Appearance procedure of current occurs when new clients pursues Poisson appropriation.
- Amenity time pursues general appropriation.
- Queue discipline adheres to preemptive recurrent need rule.
- A existing client is served in the request for appearance in the line
- A new client is conceded into the luxury in the request for appearance in the line.
- Further expect when another client is in comfort and on the off chance that a current Customer lands in, at that point the courtesy to the new client is hindered and existing client is served.

- After the finish of courtesy to existing client, the new client whose convenience is hindered can get his luxury rehashed with prospect and he can withdraw from the framework with prospect if the pleasantry happened to be fruitful given there is no client to existing enhancement.
- Customers land at the framework individually as per a Poisson stream with appearance rate.

$$\mu_j(t_1) = \frac{g_j(t_1)}{1 - G_j(t_1)}, \quad j = 1,2 \tag{1}$$

$$g_j(v) = \mu_j(v) e^{-\int_0^a \mu_j(t_1) dt_1}, \quad j = 1,2 \tag{2}$$

- After the consummation of each sort of courtesy, the server may enjoy a reprieve with prospect or may keep remaining in the framework with prospect.
- Proceeding coming back after get-away the server immediately begins overhauling the client. The bury appearance interval, the convenience periods of all caring of courtesy and break times are free of one another.
- Various stochastic procedures associated with the framework are autonomous of one another.

IV. MATHEMATICAL ASSUMPTIONS OF THE MODEL

Customers go to the middle by individually in a way as indicated by a Poisson procedure with an appearance rate.

- Every customer runs over more than one phase of non-homogeneous enhancement given by a solitary server on a progressive system premise. The courtesy time of the two phases pursue diverse general circulations with appropriation work and the thickness work
- After fulfillment of second phase of luxury, if the client is disappointed with its pleasantry for certain explanation or on the off chance that it got un effective comfort, the client may promptly join the tail of the first line with prospect. Generally the client may withdraw always from the framework with prospect.
- Assume that at whatever point the server gets away, it is of a consistent span.
- Later the fulfillment of both sort 1 or 2 enhancement, uncertainty the client stays disappointed by its convenience, it can promptly connection the end of the first line by way of a criticism client aimed at getting additional luxury by prospect . Generally the client could leave perpetually after the framework through prospect

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- The client both recently showed up and individuals that are criticism are attended in the request wherein they join the end of the first line.

Notations and equations governing the system

$P_{n_E,1}(t_1,t)$ is denote the prospect that at time t, the call center is active providing first Stage of amenity and there are $n_E\,(\geq 0)$ existing consumers in the queue without the single being served and the forgotten amenity interval for this consumer is t_1.

Consequently $P_{n_E,1}(t)=\int_0^\infty P_{n_E,1}(t_1,t)\,dt_1$ represents the prospect that on time t around are n_E consumers in the queue without the one consumer in the leading stage of amenity nevertheless of the value of t_1.

$P_{n_N,2}(t_1,t)$ is the prospect that at time t, the call center is active providing second stage of amenity and there are $n_N\,(\geq 0)$ consumers in the queue excluding the one being served and the elapsed amenity time for this consumer is t_1.

Consequently $P_{n_N,2}(t)=\int_0^\infty P_{n_N,2}(t_1,t)\,dt_1$ represents the prospect that at time t there are consumers in the queue excluding the one consumer in the second stage of amenity irrespective of the n_N value of t_1.

$E(t)$ is the prospect that at time t, there are no consumers in the system and the call center is idle but available in the system.

B_{n_E} Prospect that at time t, there are $n_E \geq 0$ customers in the existing customer queue q_E and the server is on break.

B_{n_N} Prospect that at time t, there are $n_N \geq 0$ customers in the new customer queue q_N and the server is on break.

By the usage of birth and death procedure, the equations are framed.

Here, the X represents the population of that particular time of incoming calls whereas E represents the general prospect of the calls that arrive every time the lapse is created and the minimum threshold for the calls is exceeded.

Let $X(t)$ be a birth and death procedure and let $X(t)=n$ be the incident with $P_n(t)=P(X(t)=n)$ be the corresponding prospect. Then $P_n(t+\Delta t)=P\left(X(t+\Delta t)=n\right)$ is the prospect that the size of the populace is n at time $(t+\Delta t)$

Relating to amenity,

$$P_{n_E,1}(t_1+\Delta t_1,t)=P_{n_E,1}(t_1)\left[1-\mu_1(t_1)\Delta t_1\right]$$

$$\frac{P_{n_E,1}(t_1+\Delta t_1,t)-P_{n_E,1}(t_1)}{\Delta t_1}=P_{n_E,1}(t_1)\,\mu_1(t_1) \qquad (3)$$

$$\frac{\partial}{\partial t_1}P_{n_E,1}(t_1,t)=-\mu_1(t_1,t)\,P_{n_E,1}(t_1,t)$$

Relating to arrival,

$$P_{n_E,1}(t_1+\Delta t_1,t)=P_{n_E,1}(t_1,t)[1-\lambda\Delta t]+P_{n_E,1}(t_1,t)[\lambda\Delta t]$$

$$\frac{P_{n_E,1}(t_1+\Delta t_1,t)-P_{n_E,1}(t_1,t)}{\Delta t}=-\lambda P_{n_E,1}(t_1,t)+\lambda P_{n_E-1,1}(t_1,t) \qquad (4)$$

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$$\frac{\partial}{\partial t_1} P_{n_E,1}(t_1,t) = -\lambda P_{n_E,1}(t_1,t) + \lambda P_{n_E-1,1}(t_1,t)$$

Adding equation (3) and (4)

$$\frac{\partial}{\partial t_1} P_{n_E,1}(t_1,t) + \frac{\partial}{\partial t} P_{n_E,1}(t_1,t) + (\lambda + \mu_1(t_1)) P_{n_E,1}(t_1,t) = \lambda P_{n_E-1}(t_1,t), \quad n = 1,2,\dots$$

(5) $$\frac{\partial}{\partial t_1} P_{0,1}(t_1,t) + \frac{\partial}{\partial t} P_{0,1}(t_1,t) + (\lambda + \mu_1(t_1)) P_{0,1}(t_1,t) = 0,$$

(6)

$$\frac{\partial}{\partial t_1} P_{n_N,2}(t_1,t) + \frac{\partial}{\partial t} P_{n_N,2}(t_1,t) + (\lambda + \mu_2(t_1)) P_{n_N,2}(t_1,t) = \lambda P_{n_N-1,2}(t_1,t) \tag{7}$$

$$\frac{\partial}{\partial t_1} P_{0,2}(t_1,t) + \frac{\partial}{\partial t} P_{0,2}(t_1,t) + (\lambda + \mu_2(t_1)) P_{0,2}(t_1,t) = 0, \tag{8}$$

$$\frac{d}{dt} B_{n_E}(t) = (-\lambda + \alpha) B_{n_E}(t) + \lambda B_{n_E-1}(t) + p_b \int_0^\infty P_{n_E,1}(t_1,t)\mu_1(t_1)\,dt_1 \tag{9}$$

$$\frac{d}{dt} B_0(t) = (\lambda + \alpha) B_0(t) + p_b \int_0^\infty P_{0,1}(t_1,t)\mu_1(t_1)\,dt_1 \tag{10}$$

$$\frac{d}{dt} B_{n_N}(t) = (-\lambda + \alpha) B_{n_N}(t) + \lambda B_{n_N-1}(t) + p_b \int_0^\infty P_{n_N,2}(t_1,t)\mu_2(t_1)\,dt_1$$

(11)

$$\frac{d}{dt} B_0(t) = (\lambda + \alpha) B_0(t) + p_b \int_0^\infty P_{0,2}(t_1,t)\mu_2(t_1)\,dt_1 \tag{12}$$

For q_E and q_N

$$\frac{d}{dt} E(t) = -\lambda E(t) + \alpha B_0(t) + (1-p_b)\int_0^\infty P_{0,1}(t_1,t)\mu_1(t_1)\,dt_1 + (1-p_b)q\int_0^\infty P_{0,2}(t_1,t)\mu_2(t_1)\,dt_1 \tag{13}$$

Equations (5)-(13) are to be explained with the next margin constraints

$$P_{0,1}(0,t) = \lambda p_E E_1(t) + \alpha p_E B_1(t) + p_E(1-p_b)\int_0^\infty P_{0,1}(t_1,t)\mu_1(t_1)\,dt_1 \tag{14}$$

$$P_{n_E,1}(0,t) = \alpha p_E B_{n_E+1}(t) + p_E(1-p_b)\int_0^\infty P_{n_E,1}(t_1,t)\mu_1(t_1)\,dt_1 \tag{15}$$

$$P_{0,2}(0,t) = \lambda p_N E_2(t) + \alpha p_N B_1(t) + p_N(1-p_b)\int_0^\infty P_{0,2}(t_1,t)\mu_2(t_1)\,dt_1 + p_N q(1-p_b)\int_0^\infty P_{1,2}(t_1,t)\mu_2(t_1)\,dt_1 \tag{16}$$

$$P_{n_N,2}(0,t) = \lambda p_N B_{n_N+1}(t) + p_N(1-p_b)\,p\int_0^\infty P_{n_N,2}(t_1,t)\mu_2(t_1)\,dt_1 + p_N q(1-p_b)\int_0^\infty P_{n_N+1,2}(t_1,t)\mu_2(t_1)\,dt_1 \tag{17}$$

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$$B_0(0) = B_n(0) = 0, \quad E(0) = 0 \quad \text{and } P_{n,i}(0) = 0 \;\; for \;\; n = 0,1,2,\ldots \quad ,i = 1,2 \tag{18}$$

V. GENERATING FUNCTIONS OF THE LINE SIZE

Here look at the main equations for the determination of the length of the queue and also the property of convergence and then the transforming functions of the generation and then their respective solutions with respect to the circle of elements in a server which influences the distribution of the calls in a call center.

$$\begin{aligned}
P_{q_E,1}(z,t_1,t) &= \sum_{n_E=0}^{\infty} z^{n_E} P_{n_E,1}(t_1,t), \\
P_{q_E,1}(z,t) &= \sum_{n_E=0}^{\infty} z^{n_E} P_{n_E,1}(t), \\
P_{q_N,2}(z,t_1,t) &= \sum_{n_N=0}^{\infty} z^{n_N} P_{n_N,2}(t_1,t), \\
P_{q_N,2}(z,t) &= \sum_{n_N=0}^{\infty} z^{n_N} P_{n_N,1}(t), \\
B_{q_E}(z,t) &= \sum_{n_E=0}^{\infty} z^{n_E} B_{n_E}(t), \\
B_{q_N}(z,t) &= \sum_{n_N=0}^{\infty} z^{n_N} B_{n_N}(t),
\end{aligned} \tag{19}$$

$$\bar{f}(b) = \int_0^{\infty} e^{-bt} f(t)\,dt, \quad \mathcal{Real}(b) > 0 \tag{20}$$

Apply the Laplace transform of (5) to (14) and using (15),

$$\frac{\partial}{\partial t_1}\bar{P}_{n_E,1}(t_1,b) + \big(b+\lambda+\mu_1(t_1)\big)\bar{P}_{n_E,1}(t_1,b) = \lambda\bar{P}_{n_E-1}(t_1,b) \tag{21}$$

$$\frac{\partial}{\partial t_1}\bar{P}_{0,1}(t_1,b) + \big(b+\lambda+\mu_1(t_1)\big)\bar{P}_{0,1}(t_1,b) = 0 \tag{22}$$

$$\frac{\partial}{\partial t_1}\bar{P}_{n_N,2}(t_1,b) + \big(b+\lambda+\mu_2(t_1)\big)\bar{P}_{n_N,2}(t_1,b) = \lambda\bar{P}_{n_N-1,2}(t_1,b) \tag{23}$$

$$\frac{\partial}{\partial t_1}\bar{P}_{0,2}(t_1,b) + \big(b+\lambda+\mu_2(t_1)\big)\bar{P}_{0,2}(t_1,b) = 0 \tag{24}$$

$$(s+\lambda+\alpha)\bar{B}_{n_E}(t) + \lambda\bar{B}_{n_E-1}(t) + p_b\int_0^{\infty} P_{n_E,1}(t_1,t)\,\mu_1(t_1)\,dt_1 \tag{25}$$

$$(s+\lambda+\alpha)\bar{B}_0(t) = p_b\int_0^{\infty} \bar{P}_{0,1}(t_1,t)\,\mu_1(t_1)\,dt_1 \tag{26}$$

$$(s+\lambda+\alpha)\bar{B}_{n_N}(t) = \lambda\bar{B}_{n_N-1}(t) + p_b\int_0^{\infty} \bar{P}_{n_N,2}(t_1,t)\,\mu_2(t_1)\,dt_1 \tag{27}$$

$$(s+\lambda+\alpha)\bar{B}_0(t) = p_b\int_0^{\infty} \bar{P}_{0,2}(t_1,t)\,\mu_2(t_1)\,dt_1 \tag{28}$$

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$$(b+\lambda)\bar{E}(b)=1+\alpha\bar{B}_0(b)+(1-p_b)\int_0^\infty \bar{P}_{0,1}(t_1,t)\mu_1(t_1)dt_1+q(1-p_b)\int_0^\infty \bar{P}_{0,2}(t_1,t)\mu_2(t_1)dt_1 \quad (29)$$

$$\bar{P}_{0,1}(0,b)=\lambda p_E\bar{E}_1(b)+\alpha p_E\bar{B}_1(b)+p_E(1-p_b)\int_0^\infty \bar{P}_{0,1}(t_1,b)\mu_1(t_1)dt_1 \quad (30)$$

$$\bar{P}_{n_E,1}(0,b)=\lambda p_E\bar{E}_1(b)+p_E(1-p_b)\int_0^\infty \bar{P}_{n_E,1}(t_1,b)\mu_1(t_1)dt_1 \quad (31)$$

$$\begin{aligned}\bar{P}_{0,2}(0,b)&=\lambda p_N\bar{E}_2(b)+\alpha p_N\bar{B}_1(b)+\\ &p_N(1-p_b)p\int_0^\infty \bar{P}_{0,2}(t_1,b)\mu_2(t_1)dt_1+p_N(1-p_b)q\int_0^\infty \bar{P}_{1,2}(t_1,b)\mu_2(t_1)dt_1\end{aligned} \quad (32)$$

$$\begin{aligned}\bar{P}_{n_N,2}(0,b)&=\alpha p_N\bar{B}_{n_N+1}(b)+\\ &p_N(1-p_b)p\int_0^\infty \bar{P}_{n_N,2}(t_1,b)\mu_1(t_1)dt_1+p_N(1-p_b)q\int_0^\infty \bar{P}_{n_N+1,2}(t_1,b)\mu_2(t_1)dt_1\end{aligned} \quad (33)$$

From (21) and (22) by appropriate powers of z and summation over n_E from 1 to ∞, add (19)

$$\frac{\partial}{\partial t_1}\bar{P}_{q_N,1}(z,t_1,b)+\left(b+\lambda-\lambda z+\mu_1(t_1)\right)\bar{P}_{q_N,1}(z,t_1,b)=0 \quad (34)$$

Performing similar operations on (23) and (24)

$$\frac{\partial}{\partial t_1}\bar{P}_{q_N,2}(z,t_1,b)+\left(b+\lambda-\lambda z+\mu_2(t_1)\right)\bar{P}_{q_N,2}(z,t_1,b)=0 \quad (35)$$

From (25) and (26) by appropriate supremacies of z and summing over n_E from 1 to ∞, add (19)

$$(b+\alpha+\lambda-\lambda z)\bar{B}_{q_E}(z,b)=p_b\int_0^\infty \bar{P}_{q_E,1}(z,t_1,t)\mu_1(t_1)dt_1 \quad (36)$$

From (27) and (28) by appropriate supremacies of z and summing over n_N from 1 to ∞, add (19)

$$(b+\alpha+\lambda-\lambda z)\bar{B}_{q_N}(z,b)=p_b\int_0^\infty \bar{P}_{q_N,2}(z,t_1,t)\mu_2(t_1)dt_1 \quad (37)$$

For the boundary constraints, multiply z on equation (30) , z^{n+1} on (31) sum over n_E from 1 to ∞, and get equation (38) by using (19) for q_E

$$z\bar{P}_{q_E,1}(z,0,b)=\lambda zp_E\bar{E}(b)+\alpha p_E\bar{B}(z,b)+zp_E(1-p_b)\int_0^\infty \bar{P}_{q_E,1}(t_1,z,b)\mu_1(t_1)dt_1 \quad (38)$$

$$z\bar{P}_{q_N,2}(z,0,b)=\lambda zp_N\bar{E}(b)+\alpha p_N\bar{B}(z,b)+p_N(q+pz)(1-p_b)\int_0^\infty \bar{P}_{q_N,2}(t_1,z,b)\mu_2(t_1)dt_1 \quad (39)$$

Integrating equation (34) and (35) from 0 to t_1 yields

$$\bar{P}_{q_E,1}(z,t_1,b)=\bar{P}_{q_E,1}(z,0,b)e^{-(b+\lambda-\lambda z)t_1-\int_0^{t_1}\mu_1(t)dt} \quad (40)$$

$$\bar{P}_{q_N,2}(z,t_1,b)=\bar{P}_{q_N,2}(z,0,b)e^{-(b+\lambda-\lambda z)t_1-\int_0^{t_1}\mu_2(t)dt} \quad (41)$$

Where $\bar{P}_{q_E,1}(z,0,b)$ and $\bar{P}_{q_E,1}(z,0,b)$ are given by equation (38) and (39).

Integrating equation (40) by parts w.r.t. t_1

$$\bar{P}_{q_E,1}(z,b)=\bar{P}_{q_E,1}(z,0,b)\left[\frac{1-\bar{G}_1(b+\lambda-\lambda z)}{(b+\lambda-\lambda z)}\right] \tag{42}$$

Where

$$\bar{G}_1(b+\lambda-\lambda z)=\int_0^\infty e^{-(b+\lambda-\lambda z)t_1}dG_1(t_1) \tag{43}$$

$$\int_0^\infty \bar{P}_{q_E,1}(z,t_1,b)\mu_1(t_1)dt_1=\bar{P}_{q_E 1}(z,0,b)\bar{G}_1(b+\lambda-\lambda z) \tag{44}$$

0

Similarly, on integrating equations (41) from 0 to t_1,

$$\bar{P}_{q_N,2}(z,b)=\bar{P}_{q_N,2}(z,0,b)\left[\frac{1-\bar{G}_2(b+\lambda-\lambda z)}{(b+\lambda-\lambda z)}\right] \tag{45}$$

Where $\bar{G}_2(b+\lambda-\lambda z)=\int_0^\infty e^{-(b+\lambda-\lambda z)t_1}dG_2(t_1)$ (46)

$$\int_0^\infty \bar{P}_{q_N,2}(z,t_1,b)\mu_2(t_1)dt_1=\bar{P}_{q_N,2}(z,0,b)\bar{G}_2(b+\lambda-\lambda z) \tag{47}$$

$$z\bar{P}_{q_E,1}(z,0,b)=\lambda z p_E\bar{E}_1(b)+\alpha p_E\bar{B}(z,b)+zp_E(1-p_b)\bar{P}_{q_E,1}(z,0,b)\bar{G}_1(b+\lambda-\lambda z) \tag{48}$$

$$z\bar{P}_{q_N,2}(z,0,b)=\lambda z p_N\bar{E}_2(b)+\alpha p_N\bar{B}(z,b)+p_N(q+pz)(1-p_b)\bar{P}_{q_N,2}(z,0,b)\bar{G}_2(b+\lambda-\lambda z) \tag{49}$$

From (48)

$$z\bar{P}_{q_E,1}(z,0,b)\left(1-p_E(1-p_b)\bar{G}_1(b+\lambda-\lambda z)\right)=\lambda z p_E\bar{E}_1(b)+\alpha p_E\bar{B}(z,b) \tag{50}$$

From (49)

$$\bar{P}_{q_N,2}(z,0,b)\left(z-p_N(q+pz)(1-p_b)\bar{G}_2(b+\lambda-\lambda z)\right)=\lambda z p_N\bar{E}_2(b)+\alpha p_N\bar{B}(z,b) \tag{51}$$

From (50)

$$z\bar{P}_{q_E,1}(z,0,b)\left(1-p_E(1-p_b)\bar{G}_1(b+\lambda-\lambda z)\right)=\lambda z p_E\bar{E}_1(b)+\frac{\alpha p_E p_b\bar{P}_{q_E,1}(z,0,b)\bar{G}_1(b+\lambda-\lambda z)}{\alpha+b+\lambda-\lambda z}$$

$$z\bar{P}_{q_E,1}(z,0,b)\left(1-p_E(1-p_b)\bar{G}_1(b+\lambda-\lambda z)\right)-\frac{\alpha p_E p_b\bar{P}_{q_E,1}(z,0,b)\bar{G}_1(b+\lambda-\lambda z)}{\alpha+b+\lambda-\lambda z}=\lambda z p_E\bar{E}_1(b)$$

$$\bar{P}_{q_E,1}(z,0,b)=\frac{\lambda z p_E\bar{E}_1(b)(b+\alpha+\lambda-\lambda z)}{z(b+\alpha+\lambda-\lambda z)-p_E\bar{G}_1(b+\lambda-\lambda z)\left(z(b+\alpha+\lambda-\lambda z)(1-p_b)+\alpha p_b\right)} \tag{52}$$

From (51)

$$z\bar{P}_{q_N,2}(z,0,b)\left(z-(q+pz)p_N(1-p_b)\bar{G}_2(b+\lambda-\lambda z)\right)=\lambda z p_N\bar{E}_2(b)+\frac{\alpha p_N p_b\bar{P}_{q_N,2}(z,0,b)\bar{G}_2(b+\lambda-\lambda z)}{\alpha+b+\lambda-\lambda z}$$

$$z\bar{P}_{q_N,2}(z,0,b)\left(z-(q+pz)p_N(1-p_b)\bar{G}_2(b+\lambda-\lambda z)\right)-\frac{\alpha p_N p_b\bar{P}_{q_N,2}(z,0,b)\bar{G}_2(b+\lambda-\lambda z)}{\alpha+b+\lambda-\lambda z}=\lambda z p_N\bar{E}_2(b)$$

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$$\bar{P}_{q_N,2}(z,0,b)=\frac{\lambda z p_N\bar{E}_2(b)(b+\alpha+\lambda-\lambda z)}{z(b+\alpha+\lambda-\lambda z)-p_N\bar{G}_2(b+\lambda-\lambda z)\big((q+pz)(b+\alpha+\lambda-\lambda z)(1-p_b)+\alpha p_b\big)} \quad (53)$$

From (36)

$$\bar{B}_{q_E}(z,b)=\frac{p_b\int_0^\infty \bar{P}_{q_E,1}(z,t_1,t)\mu_1(t_1)dt_1}{(b+\alpha+\lambda-\lambda z)}=\frac{p_b\bar{P}_{q_E,1}(z,0,t)\bar{G}_1(b+\lambda-\lambda z)}{(b+\alpha+\lambda-\lambda z)} \quad (54)$$

From (37)

$$\bar{B}_{q_N}(z,b)=\frac{p_b\int_0^\infty \bar{P}_{q_N,2}(z,t_1,t)\mu_2(t_1)dt_1}{(b+\alpha+\lambda-\lambda z)}=\frac{p_b\bar{P}_{q_N,2}(z,0,t)\bar{G}_2(b+\lambda-\lambda z)}{(b+\alpha+\lambda-\lambda z)} \quad (55)$$

Using (50) in (42),

$$\bar{P}_{q_E,1}(z,b)=\frac{\lambda z p_E\bar{E}(b)+\alpha p_E\bar{B}(z,b)}{z-zp_E(1-p_b)\bar{G}_1(b+\lambda-\lambda z)}\left[\frac{1-\bar{G}_1(b+\lambda-\lambda z)}{(b+\lambda-\lambda z)}\right] \quad (56)$$

Using (51) in (45),

$$\bar{P}_{q_N,2}(z,b)=\frac{\lambda z p_N\bar{E}(b)+\alpha p_N\bar{B}(z,b)}{z-p_E(q+pz)(1-p_b)\bar{G}_1(b+\lambda-\lambda z)}\left[\frac{1-\bar{G}_2(b+\lambda-\lambda z)}{(b+\lambda-\lambda z)}\right] \quad (57)$$

VI. THE STEADY STATE OUTCOMES

$$\lim_{b\to 0}\bar{h}(b)=\lim_{t\to\infty}h(t) \quad (58)$$

$$P_{q_E,1}(z)=\frac{\lambda z p_E E_1(\alpha+\lambda-\lambda z)}{z(\alpha+\lambda-\lambda z)-p_E\bar{G}_1(\lambda-\lambda z)\big(z(\alpha+\lambda-\lambda z)(1-p_b)+\alpha p_b\big)}\left[\frac{1-\bar{G}_1(\lambda-\lambda z)}{(\lambda-\lambda z)}\right] \quad (59)$$

$$P_{q_N,2}(z)=\frac{\lambda z p_N E_2(\alpha+\lambda-\lambda z)}{z(\alpha+\lambda-\lambda z)-p_N\bar{G}_2(\lambda-\lambda z)\big((q+pz)(\alpha+\lambda-\lambda z)(1-p_b)+\alpha p_b\big)}\left[\frac{1-\bar{G}_2(\lambda-\lambda z)}{(\lambda-\lambda z)}\right] \quad (60)$$

$$B_{q_E,1}(z)=\frac{\bar{G}_1(\lambda-\lambda z)\lambda z p_b p_E E_1}{z(\alpha+\lambda-\lambda z)-p_E\bar{G}_1(\lambda-\lambda z)\big(z(\alpha+\lambda-\lambda z)(1-p_b)+\alpha p_b\big)} \quad (61)$$

$$B_{q_N,1}(z)=\frac{\bar{G}_2(\lambda-\lambda z)\lambda z p_b p_N E_2}{z(\alpha+\lambda-\lambda z)-p_N\bar{G}_1(\lambda-\lambda z)\big((q+pz)(\alpha+\lambda-\lambda z)(1-p_b)+\alpha p_b\big)} \quad (62)$$

For $P_{q_E,1}(z)$

$$Nr=\lambda z p_E E_1(\alpha+\lambda-\lambda z)\big(1-\bar{G}_1(\lambda-\lambda z)\big)$$

$$Nr'=\lambda p_E E_1\Big[1.(\alpha+\lambda-\lambda z)\big(1-\bar{G}_1(\lambda-\lambda z)\big)+z(-\lambda)\big(1-\bar{G}_1(\lambda-\lambda z)\big)+z(\alpha+\lambda-\lambda z)\big(-\bar{G}_1'(\lambda-\lambda z)\big)(-\lambda)\Big]$$

$$\bar{G}_j(0)=1;-\bar{G}_j'(0)=\mathrm{E}(v_j);\bar{G}_j''(0)=\mathrm{E}(v_j^{\,2});\ j=1,2$$

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$$Nr'(1)=\lambda p_E E_1\left[1.(\alpha)\left(1-\bar{G}_1(0)\right)+z(-\lambda)\left(1-\bar{G}_1(0)\right)+z(\alpha)\left(-\bar{G}_1'(0)\right)(-\lambda)\right]$$
$$=\lambda p_E E_1\alpha \mathrm{E}\left(v_j\right)(-\lambda)=-\lambda^2 p_E E_1\alpha \mathrm{E}\left(v_1\right)$$

Similarly $Dr'(1)=-\alpha\lambda\left(1-p_E\right)$

$$P_{q_E,1}(1)=\frac{-\lambda^2 p_E E_1\alpha \mathrm{E}\left(v_1\right)}{-\alpha\lambda\left(1-p_E\right)}=\frac{\lambda p_E E_1 \mathrm{E}\left(v_1\right)}{\left(1-p_E\right)} \tag{63}$$

Similarly

$$P_{q_N,2}(1)=\frac{\lambda p_N E_2 \mathrm{E}\left(v_2\right)}{\left(1-p_N\right)} \tag{64}$$

Proceeding on similar lines,

$$B_{q_E}(1)=\frac{\lambda p_E p_b E_1\left(1+\lambda \mathrm{E}\left(v_1\right)\right)}{(\alpha-\lambda)-p_E\left(1-p_b\right)(\alpha-\lambda)-\alpha\lambda p_E \mathrm{E}\left(v_1\right)} \tag{65}$$

$$B_{q_N}(1)=\frac{\lambda p_N p_b E_2\left(1+\lambda \mathrm{E}\left(v_2\right)\right)}{(\alpha-\lambda)-p_N\left(1-p_b\right)(p\alpha-\lambda)-\alpha\lambda p_N \mathrm{E}\left(v_2\right)} \tag{66}$$

Normalized condition for existing customer queue q_E is given by

$$P_{q_E,1}(1)+B_{q_E}(1)+E_1=1 \tag{67}$$

$$\frac{\lambda p_E E_1 \mathrm{E}\left(v_1\right)}{\left(1-p_E\right)}+\frac{\lambda p_E p_b E_1\left(1+\lambda \mathrm{E}\left(v_1\right)\right)}{(\alpha-\lambda)-p_E\left(1-p_b\right)(\alpha-\lambda)-\alpha\lambda p_E \mathrm{E}\left(v_1\right)}+E_1=1$$

$$E_1=\frac{\left(1-p_E\right)\left((\alpha-\lambda)-p_E\left(1-p_b\right)(\alpha-\lambda)-\alpha\lambda p_E \mathrm{E}\left(v_1\right)\right)}{\left(1-p_E\left(1-\lambda \mathrm{E}\left(v_1\right)\right)\right)\left((\alpha-\lambda)-p_E\left(1-p_b\right)(\alpha-\lambda)-\alpha\lambda p_E \mathrm{E}\left(v_1\right)\right)+\left(1-p_E\right)\lambda p_E p_b\left(1+\lambda \mathrm{E}\left(v_1\right)\right)} \tag{68}$$

$$\rho_1=1-E_1 \tag{69}$$

Normalized condition for existing customer queue q_N is given by

$$P_{q_E,2}(1)+B_{q_N}(1)+E_2=1 \tag{70}$$

$$E_2=\frac{\left(1-p_N\right)\left((\alpha-\lambda)-p_N\left(1-p_b\right)(p\alpha-\lambda)-\alpha\lambda p_N \mathrm{E}\left(v_2\right)\right)}{\left(1-p_N\left(1-\lambda \mathrm{E}\left(v_2\right)\right)\right)\left((\alpha-\lambda)-p_N\left(1-p_b\right)(p\alpha-\lambda)-\alpha\lambda p_N \mathrm{E}\left(v_2\right)\right)} \tag{71}$$
$$+\left(1-p_N\right)\lambda p_N p_b\left(1+\lambda \mathrm{E}\left(v_2\right)\right)$$

$$\rho_2=1-E_2 \tag{72}$$

VII. THE AVERAGE QUEUE SIZE AND WAITING TIME

$$L_q=\left[\frac{d}{dz}W_q(z)\right]_{z=1} \tag{73}$$

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$$L = L_q + \rho \tag{74}$$

The mean waiting time

Let W_q and W represent the average waiting time. Then using little's formula,

$$W_q = \frac{L_q}{\lambda} \tag{75}$$

$$W = \frac{L}{\lambda} \tag{76}$$

VIII. CONCLUSION

In this paper, a non-markovian system that has the issues in terms of waiting time, non-working time and various lapses in the process is explained. In this context, suitable conclusions for managing the various phenomena revolving the call center and its solutions. The relations between the remote servers are also mentioned along with their respective breakdowns. The classifications of such servers as effective and in-effective based on the customer support are discussed in detail alongside the matters of the absolute reality in management. This work further goes on telling about the depths of the subject to an extent which even the customers are distributed to different executive based on their history and certain orbits are designed to assist this process. Integrated it into a whole function that explains the aspects of variable distribution of calls in a busy server in critical conditions and the higher order parameters that are used in governing it.

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IX. REFERENCES

[1]. Liu, Z., Wu, J., and Yang, G. 2009. An M/G/1 retrial G-queue with preemptive resume and feedback under N-policy subject to the server breakdowns and repairs. Computers and Mathematics with Applications. 58 (9), 1792–1807.

[2]. Liu, Z. and Gao, S. 2011. Discrete-time retrial queue with two classes of customers and feedback. Mathematical and Computer Modelling. 53(5-6), 1208-1220.

[3]. Senthilkumar, M., Chakravarthy, S.R., and Arumuganathan, R. 2013. Preemptive resume priority retrial queue with two classes of MAP arrivals. Applied Mathematical Sciences. 7(52), 2569-2589.

[4]. Wu, J. and Lian, Z. 2013. A single-server retrial G-queue with priority and unreliable server under Bernoulli vacation schedule. Computers and Industrial Engineering. 64(1), 84-93

[5]. Arivudainambi, D.; Godhandaraman, P.; Rajadurai, P. Performance analysis of a single server retrial queue with working vacation. Opsearch 2014, 51, 434–462.

[6]. Choi, B.D.; Park, K.K. The M/G/1 retrial queue with Bernoulli schedule. Queueing Syst. 1990, 7, 219–228.

[7]. Gao, S. A preemptive priority retrial queue with two classes of customers and general retrial times. Oper. Res.2015, 15, 233–251.

[8]. Peng, Y. On the discrete-time Geo/G/1 retrial queueing system with preemptive resume and Bernoulli feedback. Opsearch 2016, 53, 116–130.

[9]. Servi, L.D.; Finn, S.G. M/M/1 queues with working vacations. Perform. Eval. 2002, 50, 41–52.

[10].Wu, D.; Takagi, H. M/G/1 queue with multiple working vacations. Perform. Eval. 2006, 63, 654–681.

[11].Van Do, T. M/M/1 retrial queue with working vacations. Acta Inform. 2010, 47, 67–75.

[12]. Chandrasekaran, V.M.; Indhira, K.; Saravanarajan, M.C.; Rajadurai, P. A survey on working vacation queueing models. Int. J. Pure Appl. Math. 2016, 106, 33–41.

[13]. Gao, S.; Wang, J.; Li, W. An M/G/1 retrial queue with general retrial times, working vacations and vacation interruption. Asia-Pac. J. Oper. Res. 2014, 31, 06–31.

[14]. Rajadurai, P.; Chandrasekaran, V.M.; Saravanarajan, M.C. Analysis of an unreliable retrial G-queue with working vacations and vacation interruption under Bernoulli schedule. Ain Shams Eng. J. 2018, 9, 56–580.

[15]. Gao, S.; Liu, Z. An M/G/1 queue with single working vacation and vacation interruption under Bernoulli schedule. Appl. Math. Model. 2013, 37, 1564–1579.

[16]. Zhan, M.; Liu, Q. An M/G/1 G-queue with server breakdown, working vacations and vacation interruption. Opsearch 2015, 52, 256–270.

[17]. Kalidass, K.; Ramanath, K. A queue with working breakdowns. Compute. Ind. Eng. 2012, 63, 779–783.

[18]. Kim, B.K.; Lee, D.H. The M/G/1 queue with disasters and working breakdowns. Appl. Math. Model. 2014, 38, 1788–1798.

[19]. Ma, Z.; Cui, G.; Wang, P.; Hao, Y. M/M/1 vacation queueing system with working breakdowns and variable arrival rate. J. Compute. Inf. Syst. 2015, 11, 1545–1552.

[20]. Deepa, B.; Kalidass, K. An M/M/1/N Queue with Working Breakdowns and Vacations. Int. J. Pure Appl. Math.2018, 119, 859–873.

[21].Jiang, T.; Liu, L. The GI/M/1 queue in a multi-phase amenity environment with disasters and working breakdowns. Int. J. Compute. Math. 2017, 94, 707–726.

[22].Liu, Z.; Song, Y. The MX/M/1 queue with working breakdown. Rairo Oper. Res. Rech. Opér. 2014, 48, 399–413.

[23]. B.Balamurugan, M.Mullai (2019) “Rejection and Breaks in Pharmaceutical Call Center” International Journal of Pharmaceutical Research, Volume 11, Issue 4, 221-230.

[24]. B.Balamurugan, M.Mullai (2019) “Non Markovian Feedback Queue with Two Stage different Amenity and constant Server breaks” International Journal of Research and Analytical Reviews, Volume 6, Issue 2, 634-644

[25]. Sherif I. Ammar and Rajadurai P(2019) Performance Analysis of Preemptive Priority Retrial Queueing System with Disaster under Working Breakdown Amenities, Symmetry, MDPI, Volume 11, Issue 419,1-15