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Fractional Cross Product Applied inRadiation Characteristic of Micro-strip Antenna: A simulation Approach

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Abstract
Fractional Calculus Technique (αth order) is already used in different fields of electronics. In this paper, we have derived the E and H plane equation of micro-strip antenna by using fractional cross product and simulated it to see the effect in the radiation pattern which gives an interesting result and different values of 'α' are simulated and compared. This effect can be exploited in future by comparing with the radiation pattern with including fraction order electronics component in the micro-strip antenna.

Keywords: *Fractional cross product, Micro-strip Antenna, MATLAB simulation*

I. INTRODUCTION

The fractional cross product and curl arriving by the generalization of fractional calculus. Fractional calculus has applications in especially electromagnetic theory[1-6], which is based on Maxwell's equations, areheavily depended on calculus. In 90s, some explorations onfractional calculus applications in electromagnetic theory were taken up [7-12]. Fractional calculus is a branch of mathematics where integration and differentiation operations are done with non-integer orders [13-28].

The non-integer-order, that are fractional or complex order integration and differentiation in mathematics, has always been a subject of complexity and interest for several researchers/mathematicians over the years because of their interesting mathematical properties[13-30]. In the bibliography by B. Ross, a historical review of fractional calculus is available which is reprinted in the article by Oldham and Spanier [15], and also in [16]. Mellin transforms, differential equations, complex analysis, and generalized functions etc are few applications of Fractional calculus [15-26], among all these, Abel, in his study of the tautochrone problem, in 1823 gave one of the earliest appears to be application. Furthermore, Lutzen has mentioned in [28] that probably by inspired from the fundamental force law in Ampere's electrodynamics problem, Liouville, who was one of the leadoff of fractional calculus, treated a fractional differential equation in that problem. The operational fractional calculus is introduced and used by Heaviside in his work [29].This tool is also used in rheology by Scott Blair [30].

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At present, few investigations on the concept of fractals which are the applications of fractional calculus have already begun[31 - 35]. Also in the fields of applied science [36-40], engineering including signal processing, controls and biological science, neuroscience, researchers have used some aspects of fractional derivatives and integrals in their work. Very recently an article is published by H G Sun et al, related to application of fractional calculus in real world problem [41]. There has been much work executed on another operator in mathematics called fractionalization, by several researchers, such as Namias [42], Lohmann([43], [44]), Mendlovic and Ozaktas ([45], [46]) and Shamir and Cohen [47], to name a few. These works mostly on fractionalization of the Fourier transform. This fractional derivative integrals concept first applied in electromagnetic by Engheta[7 - 12] which include concept of "fractional" multiples, electrostatic "fractional" image methods for perfectly conducting wedges and cones, the mathematical link between the electrostatic image methods for the conducting sphere and the dielectric sphere, and "fractional" solutions for the standard scalar Helmholtz equation. There are also many other works in the electro magnetics literature on fractional order research, such as plasma physics ([47], [48], [49]) and electromagnetic dynamics [50].The stable power-law statistics and fractional order calculus links [51],the fractals and fractional order behaviour link [52] are also well known. A. A. Potapov has published fractional order radio ideas on both antenna and circuitry in [53], [54], and [55]. In those works, only fractal order radio receiver components were discussed, radio frequency systems overall performance characteristics were not exactly analysed, but he was very close in finding the performance characteristic in 2009 [56]. In the literature we have different fractal antennas, out of which [57], [58], [59], [60], are just a few. All these papers deal with a fractal antenna design using integer-order traditional radio receiver or transmitter. After many attempts by the researchers capacitive-style [61], [62], [63] and inductive-style [64] devices have been made. A research from India [65], indicates the possibility of manufacture, and usability of fractional order devices in electronic parts in future. Fractional order and memristive elements links as well have been drawn [64], [66]. In the year 2013 a research from Utah state university [67] used fractional order components in the driving circuit of a simple radio frequency system, consisting of a single circular loop antenna, instead of integer order components and interesting result obtained which contrast the strength of the radiation pattern of the antenna by simulating. But after 2013 fractional order components has not been used in any other antenna.

However, we have elected to study the radiation of micro-strip antenna using fractional cross product to analyse the outcome and compare the fractional radiation with the existing one. With a larger number of controllable antenna elements, the circuit can be extended into a phased array antenna system. And later we can use the fractional components to compare the radiation pattern. In the present paper, we concentrate on how by introducing the fractional cross product in the derivation of E and H plane equation, alters the antenna radiation pattern. In section-II, we present definition of fractional cross product (taken from [68] and Kindergarten of fractional calculus book by Santanu Das, which is under publication at Jadavpur University, India. presently at limited prints used in courses of Fractional Calculus at Jadavpur University, Calcutta University and StXavier's College, Calcutta, India), and in section-III, the mathematical models of the micro-strip antenna using fractional cross product are presented. The numerical simulations of the models have been obtained by using MATLAB which are presented in section IV. Finally, weconclude with viewpoints and suggestions for future work in section V.

II. FRACTIONAL CROSS PRODUCT:

AXB (A cross B) is a rotation of vector B anticlockwise by fraction of 90 degree ($\alpha 90^0$), about an axis given by A sinϴ that is orthogonal projection component of A on B. This is fractional cross product. Where $0 < \alpha < 1$.

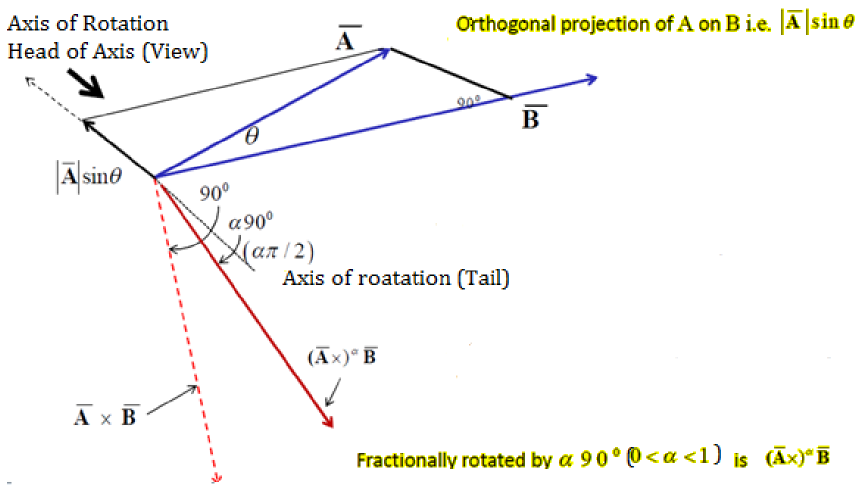
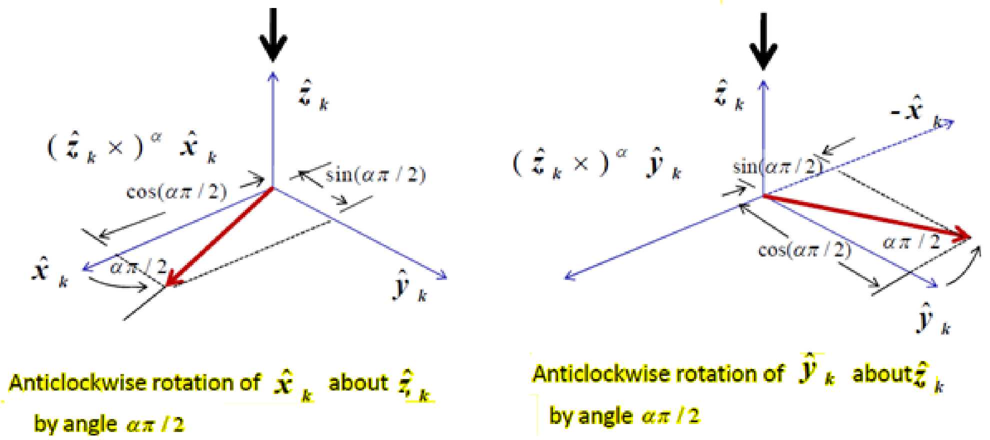


FIGURE 1: CONCEPT OF FRACTIONAL CROSS PRODUCT



$$(\hat{z}_k\times)^{\alpha}\,\hat{x}_k = \left[\cos\left(\tfrac{\alpha\pi}{2}\right)\right]\hat{x}_k + \left[\sin\left(\tfrac{\alpha\pi}{2}\right)\right]\hat{y}_k \qquad (\hat{z}_k\times)^{\alpha}\,\hat{y}_k = \left[-\sin\left(\tfrac{\alpha\pi}{2}\right)\right]\hat{x}_k + \left[\cos\left(\tfrac{\alpha\pi}{2}\right)\right]\hat{y}_k$$

FIGURE 2: FRACTIONAL CROSS PRODUCT OF ARTHOGONAL UNIT VECTOR

The fractional cross product obtained

$$(\hat{z}_k\times)^{\alpha}\,\hat{x}_k = \left[\cos\left(\tfrac{\alpha\pi}{2}\right)\right]\hat{x}_k + \left[\sin\left(\tfrac{\alpha\pi}{2}\right)\right]\hat{y}_k$$
$$(\hat{z}_k\times)^{\alpha}\,\hat{y}_k = \left[-\sin\left(\tfrac{\alpha\pi}{2}\right)\right]\hat{x}_k + \left[\cos\left(\tfrac{\alpha\pi}{2}\right)\right]\hat{y}_k$$
$$(\hat{z}_k\times)^{\alpha}\,\hat{z}_k = 0$$

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The other combinations similarly

$$(\hat{x}_k \times)^\alpha \hat{y}_k = \left[\cos\left(\tfrac{\alpha\pi}{2}\right)\right]\hat{y}_k + \left[\sin\left(\tfrac{\alpha\pi}{2}\right)\right]\hat{z}_k$$

$$(\hat{x}_k \times)^\alpha \hat{z}_k = \left[-\sin\left(\tfrac{\alpha\pi}{2}\right)\right]\hat{y}_k + \left[\cos\left(\tfrac{\alpha\pi}{2}\right)\right]\hat{z}_k$$

$$(\hat{y}_k \times)^\alpha \hat{z}_k = \left[\sin\left(\tfrac{\alpha\pi}{2}\right)\right]\hat{x}_k + \left[\cos\left(\tfrac{\alpha\pi}{2}\right)\right]\hat{z}_k$$

$$(\hat{y}_k \times)^\alpha \hat{x}_k = \left[\cos\left(\tfrac{\alpha\pi}{2}\right)\right]\hat{x}_k + \left[-\sin\left(\tfrac{\alpha\pi}{2}\right)\right]\hat{z}_k$$

FIGURE 3: SIMILAR EQUATIONS

The above figures 1, 2, 3 show the concept of fractional cross product. By using the above equations from figure 3 we can write our required equation in below section.

III. MATHEMATICAL MODELLING OF MICRO-STRIP ANTENNA:

In cavity model of micro-strip antenna[69] side walls employ wall boundary condition, which sets the tangential H components at the slots equal to zero.
Therefore $J_s = \hat{n}$ X H = 0
Only equivalent magnetic current density $M_s = -\hat{n}$ X E has substantial contribution to the radiated field.
The influence of the infinite ground plane is accounted for by the image theory, according to which the currents M*s* in the presence of the infinite plane radiate as if magnetic currents of double strength radiate into free space.

$M_s = -2\hat{n}$ X E -------------------------------------- 1

Ex field at the slots corresponds to M*s* density vector, which is tangential to the ground plane. Thus, its image is of the same direction.

Applying fraction cross product instead of integer cross product in equation 1 with reference to figure 3 equations,

$M_s = (-2\hat{n}\,\overrightarrow{ay})^\alpha$X E $\overrightarrow{ax}$
= $-2E_0$ {[cos (απ/2)] a_x – [sin (απ/2)] a_z } ------------ 2

$$L = \iint Ms\, e^{j k o r' \cos\Psi} ds'$$

$$= \iint_{-h/2}^{h/2} Ms \quad e^{jko(x' \sin\theta\cos\phi + z' \cos\theta)} dx'\, dz' \quad \text{------------------------------------ 3}$$

By substituting equation (2) in (3), we have

$$L = \iint_{-\frac{w}{2}\,-\frac{h}{2}}^{\frac{w}{2}\,\frac{h}{2}} -2E_0 \{[\cos(\alpha\pi/2)]\, a_x - [\sin(\alpha\pi/2)]\, a_z\}\, e^{jkox' \sin\theta\cos\phi}\, dx'\, e^{jkoz' \cos\theta}\, dz'$$

$$= \iint_{-\frac{w}{2}\,-\frac{h}{2}}^{\frac{w}{2}\,\frac{h}{2}} [-2E_0\, e^{jkox' \sin\theta\cos\phi}\, dx'\, e^{jkoz'\cos\theta}\, dz' \cos(\alpha\pi/2)\, a_x + 2E_0\, e^{jkox' \sin\theta\cos\phi}\, dx'\, e^{jkoz'\cos\theta}\, dz' \sin(\alpha\pi/2)\, a_z]$$

By integrating the above equation,

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L = -2 E_0whcos (απ/2) $a_x\frac{sin\ (Z)}{Z}\frac{sin\ (X)}{X}$ + 2 E_0whsin(απ/2) $a_z\frac{sin\ (Z)}{Z}\frac{sin\ (X)}{X}$ ----4

Where X = $cos\phi\ sin\Theta\frac{k0h}{2}$

Z = $cos\Theta\frac{k0w}{2}$

L = L_x + L_z--- 5

Where L_x = -2 E_0whcos (απ/2) $a_x\frac{sin\ (Z)}{Z}\frac{sin\ (X)}{X}$ --------- 6

L_z = 2 E_0whsin(απ/2) $a_z\frac{sin\ (Z)}{Z}\frac{sin\ (X)}{X}$ ----------- 7

$L_\Theta = L_x cos\Theta\ cos\phi + L_y\ cos\Theta\ sin\phi - L_z sin\Theta$ ---------- 8

$L_{\boldsymbol{\phi}} = 0$

Therefore $E_\Theta = \frac{-jk0e^{-jk0r}}{4\pi r}L_{\boldsymbol{\phi}} = 0$ ---------------------- 9

$E_{\boldsymbol{\phi}} = j\frac{k0e^{-jk0r}}{4\pi r}L_\Theta$

$= -j\frac{k0hwE0e^{-jk0r}}{2\pi r}\frac{sin\ (Z)}{Z}\frac{sin\ (X)}{X}$ [$cos\Theta\ cos\phi$cos (απ/2) + sinΘsin(απ/2)] ------------ 10

$E_r = E_\Theta = 0$

The array factor (AF) of two elements spaced a distance L_{eff} apart along the y direction is given by,

AF = 2 cos $(\frac{k0\ Leff\ sin\Theta\ sin\phi}{2})$ ------------------------------ 11

Where $L_{eff}$is the effective distance

Therefore $E_\Phi^t = E_{\boldsymbol{\phi}}$ (AF)

$= E_{\boldsymbol{\phi}}\ [2\cos(\frac{k0\ Leff\ sin\Theta\ sin\phi}{2})]$

$E_\Phi^t = -j\frac{k0\ w\ V0\ e^{-jk0r}}{\pi r}\frac{sin\ (cos\Theta\frac{k0w}{2})}{cos\Theta\frac{k0w}{2}}\frac{sin\ (cos\phi\ sin\Theta\frac{k0h}{2})}{cos\phi\ sin\Theta\frac{k0h}{2}}\cos\ (\frac{k0\ Leff\ sin\Theta\ sin\phi}{2})$ [$cos\Theta\ cos\phi$cos (απ/2) + sinΘsin(απ/2)] --- 12

E-Plane: Θ= 90^0 , $0^0 <= \boldsymbol{\phi} <= 90^0$, $270^0 <= \boldsymbol{\phi} <= 360^0$

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Therefore Equation (12) becomes,

$$E_{\phi}^{t} = -j\frac{k0\ w\ V0\ e^{-jk\ 0r}}{\pi r}\frac{sin\ (\frac{k0h}{2}cos\phi)}{\frac{k0h}{2}cos\phi}\cos(\frac{k0\ Leff\ \ sin\phi}{2})\ \sin(\alpha\pi/2) \text{ ------- } 13$$

At H-Plane: $\boldsymbol{\phi} = 0^0$, $0^0 <= \Theta <= 180^0$

Therefore Equation (12) becomes,

$$E_{\phi}^{t} = -j\frac{k0\ w\ V0\ e^{-jk\ 0r}}{\pi r}\frac{sin\ (\frac{k0w}{2}cos\Theta)}{\frac{k0w}{2}cos\Theta}\frac{sin\ (\frac{k0h}{2}\ sin\Theta)}{\frac{k0h}{2}\ sin\Theta}\ [cos\Theta\cos\ (\alpha\pi/2) + \sin\Theta\sin(\alpha\pi/2)] \text{ -------- } 14$$

IV.SIMULATION RESULTS

For the simulation we have chosen, dielectric constant of 2.2, height of the dielectric 0.1588 cm, and position of the recessed feed point relative to the leading radiating edge of the rectangular patch of 0.3126cm. Resonant frequency for all the simulation chosen is 10GHz.we have simulated the equations from section III by using MATLAB and got the below results.
From the below simulation results we can note that, by comparing figure-4 (α = 1) with figure-5(α = 0.99), the figure5radiation patternextends through azimuthal $\boldsymbol{\phi}$ direction. If α decreases from 0.99 to 0.9 (figure-6),the radiation pattern still increases through the same direction. If α decreases to 0.5(figure-7), still the radiation increases and so on.

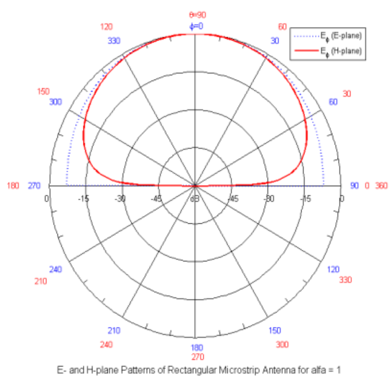

FIGURE 4

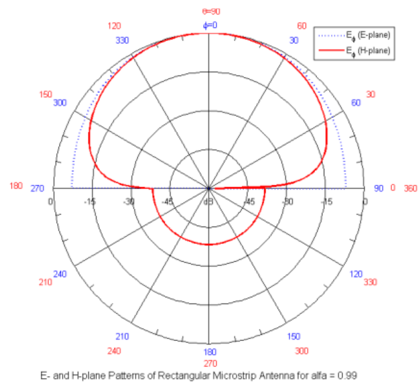

FIGURE 5

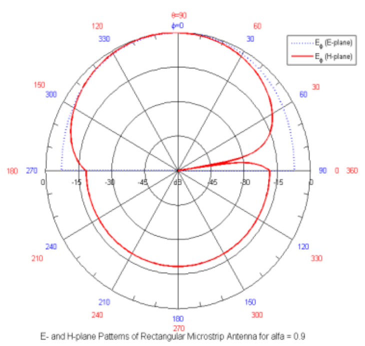

FIGURE 6

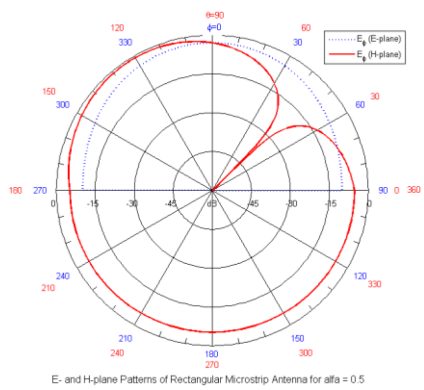

FIGURE 7

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V. CONCLUSION

In this paper, the motivation, back ground, equation derivation with fractional cross product and simulation of micro-strip antenna is presented. From the simulation result we can conclude that, as we decrease the value of α, the pattern keeps on increasing in the azimuthal ϕ direction. Further this effect can be exploited in future by comparing with the radiation pattern with including fraction order electronics components (fractors) in the micro-strip antenna and the input impedance can also be controlled.

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