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Dominator Coloring of Central and Middle Graph of Closed Helm Graph

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Abstract:
Graph coloring and domination are the two fields of graph theory that have numerous applications in the field of computer science and biological networks. An area attained by merging the graph coloring and domination known as dominator coloring of a graph. It is well-defined as a proper coloring of vertices where each vertex of graph governs all vertices in an atleast 1 color class. The least number of colors is essential for a dominator coloring of a graph known as dominator chromatic number. The dominator chromatic number for central and middle graph of the closed Helm graph is obtained and a relationship between them is expressed in this paper.

Keywords: *domination; coloring; dominator coloring; central graph; middle graph; closed Helm graph.*

1. INTRODUCTION

A dominating set is a subset D_s of the vertex or node set of graph G *in* which each node either belongs to D_s or has a neighbour in D_s [1]. The domination number $\gamma(G)$ is the cardinality of a smallest dominating set of G[1]. It is applied for routing and sending data among the nodes in the network (which include military communications, emergency systems and disaster recovery etc.). Domination in graphs has applications in facility location problems, where the number of facilities has to be fixed (e.g., hospitals, fire stations) for optimizing minimum transportation cost, providing equitable service to customers etc.. The dominating sets play a significant role in an area of the human Protein-Protein Interaction (PPI) network in which it identify the proteins set that are contained in significant biological methods and mechanisms vital for cell vitality and drug target.

A proper coloring of a graph G is a function $f: V \to Z_+$ such that for $u, v \in V,\ f(u) \neq f(v)$ whenever u and v adjacent nodes in G. One of the applications of graph coloring in real life problems is allocation of gates for flights without time conflict. Graph coloring techniques are used in biological networks in specific to PPI networks. The vertex coloring information helps to improve the quality i.e., homogeneity and separation of initial protein complexes and this finding help to improve existing protein complex detection methods. Graph coloring techniques are also applied to biological networks specifically to protein-protein interaction (PPI) networks.

A dominator coloring of a graph G is a proper coloring of graph such that every node or vertex of G dominates all nodes of at least one color class. The minimum cardinality of colors used in the graph for dominator coloring is called the dominator coloring number denoted by χ_d *(G).* [2]. Since dominator coloring is mixture of domination and coloring graphs which is used in PPI networks, facility location problems etc..

January - February 2020
ISSN: 0193 - 4120 Page No. 6228 - 6232

Gera in 2006 introduced the concept dominator coloring [2].The relationship between domination number, chromatic number and dominator chromatic number of various graphs were shown in [3], [4], [5]. The dominator coloring of prism graph, m-splitting graph and m-shadow graph of path graph, closed Sun graph, closed Helm graph, generalized Flower Snark, Triangular belt, Alternate Triangular belt, central graph, middle and total graphs, etc. were also studied in various papers[6], [7], [8], [9], [10], [11].

A closed Helm graph denoted by $CH_n$is constructed from the Helm graph by joining its outer vertices. It has 2n+1 vertices and 4n edges. The middle graph of a closed Helm graph $M(CH_n)$ is obtained by the subdivision of each edge of CH_n exactly once and connected by edge of all newly merged nodes of adjacent edges of CH_n. It have 6n+1 vertices and 8n edges.

The dominator coloring number of middle and central of closed Helm graph are obtained and a relationship between them is expressed in this paper.

2. DOMINATOR COLORING NUMBER OF MIDDLE AND CENTRAL GRAPH OF CLOSED HELM GRAPH

Proposition 2.1: Every closed Helm graph denoted by $CH_n$where $n \geq 4$ and n is even, has dominator chromatic number$\chi_d(CH_n) = \left\lceil \frac{n}{3} \right\rceil + 3.$

Proposition 2.2: Every closed Helm graph denoted by $CH_n$where $n \geq 5$ and n is odd, has dominator chromatic number $\chi_d(CH_n) =$

$$\begin{cases} \left\lceil \frac{n}{3} \right\rceil + 3 & when\, n(mod\ 3) \equiv 1 \\ \left\lceil \frac{n}{3} \right\rceil + 4 & when\, n(mod\ 3) \equiv 0\ or\ 2 \end{cases}.$$

Theorem 2.3: If $M(CH_n)$ is the middle graph of closed Helm graph CH_n then for $n \geq 5$ its dominator coloring number is $\chi_d\left(M(CH_n)\right) = \begin{cases} 3\left\lceil \frac{n}{2} \right\rceil + 3 & when\ n\ is\ even \\ 3\left\lceil \frac{n}{2} \right\rceil + 1 & when\ n\ is\ odd \end{cases}$

Proof:
Let the node set and edge set of the closed Helm graph CH_n be
$V = \{w\} \cup \{v_i, u_i\ /\ 1 \leq i \leq n\}$
$E = \{v_1 v_n, u_1 u_n\} \cup \{v_i v_j,\ u_i u_j /\ 1 \leq i \leq n-1,\} \cup \{u_i v_i, w v_i /\ 1 \leq i \leq n\}$.
The middle graph of a closed Helm graph $M(CH_n)$ is obtained by the subdivision of each edge of CH_n exactly once and connecting by an edge of all the newly added nodes of adjacent edges of CH_n.
Let the new nodes obtained by the subdivision of edges $E(CH_n)$ be $\{x_i, y_i, z_i, t_i\ /\ 1 \leq i \leq n\}$.
The node set and edge set of the middle graph of the closed Helm graph $M(CH_n)$are
$V(M(CH_n)) = \{w\} \cup \{u_i, v_i, x_i, y_i, z_i, t_i / 1 \leq i \leq n\}$
$E(M(CH_n))$
$= \{u_i t_i,\ u_i z_i, z_i v_i, v_i y_i, v_i x_i, x_i w, x_i z_i, x_i y_i, z_i t_i, z_i y_i,\ /\ 1 \leq i \leq n\}$
$\cup \{t_n u_1, t_1 t_n, y_n v_1, y_1 y_n, x_1 x_n, t_n z_1, y_n z_1\}$
$\cup \{t_i u_{i+1}, t_i z_{i+1}, t_i t_{i+1}, y_i v_{i+1}, y_i z_{i+1}, y_i y_{i+1}, x_i x_{i+1}\ /\ 1 \leq i \leq n-1\}$
The procedure below explains the dominator coloring of nodes for middle graph of closed Helm graph
Case 1: When $n(mod\ 3) \equiv 0$

For $1 \leq i \leq n$, the nodes w, u_i, v_i are painted with color 1and the nodes x_i are given color $i+1$ respectively. For $1 \leq i \leq 3$, the nodes t_i, z_i are painted with color $i+1$ and $n+i+1$ respectively.For $1 \leq i \leq \left\lceil \frac{n}{3} \right\rceil$, the nodes $y_{3i-2}, y_{3i-1}, y_{3i}$ are painted with color 4, 2, 3respectively. For $1 \leq i \leq \left\lceil \frac{n}{3} \right\rceil - 1$, the nodes $z_{3i+1}, z_{3i+2}, z_{3i+3}$ are painted with color 2, 3, 4respectively. For $1 \leq i \leq \left\lceil \frac{n}{3} \right\rceil - 1$, the nodes $t_{3i+1}, t_{3i+2}, t_{3i+3}$ are painted with color $n + 3\left\lceil \frac{i}{2} \right\rceil + 2, 2, n + 3\left\lceil \frac{i}{2} \right\rceil + 3$ respectively when i is odd and with color $4, n + 3\left\lceil \frac{i}{2} \right\rceil + 4, 3$ respectively when i is even .

The nodes w, x_i aredominated by color class$n+1$. Then for $1 \leq i \leq 3$ the nodes$u_i, v_i, t_i, y_i, z_i$are

January - February 2020
ISSN: 0193 - 4120 Page No. 6228 - 6232

dominated by color class $n+i+1$. For $4 \leq i \leq n$ the nodes v_i, y_i, z_i are dominated by color class $i+1$. And for $1 \leq i \leq \left\lceil \frac{n}{2} \right\rceil - 2$ the nodes $u_{2i+2}, u_{2i+3}, t_{2i+2}, t_{2i+3}$ are dominated by color class $n+4+i$. The nodes u_n, t_n dominate color class $3\left\lceil \frac{n}{2} \right\rceil + 3$ when n is even.

Every neighbouring node is given different color and also it is observed that every node of the graph dominates all the nodes of atleast one color class. Thus it is a dominator coloring of nodes.

Case 2: When $n(mod\ 3) \equiv 1$

For $1 \leq i \leq n$, the nodes w, u_i, v_i are painted with color 1and the nodes x_i are given color $i+1$ respectively. For $1 \leq i \leq 3$, the nodes t_i, z_i are painted with color $i+1$ and $n+i+1$ respectively. For $1 \leq i \leq \left\lceil \frac{n}{3} \right\rceil$-1, the nodes $y_{3i-2}, y_{3i-1}, y_{3i}$ are painted with color 4, 2, 3respectively. The nodes y_{n-2}, y_n are painted with color 2, 3respectively and the nodes y_{n-3}, y_{n-1} are painted with color 4. For $1 \leq i \leq \left\lceil \frac{n}{3} \right\rceil - 2$, the nodes $z_{3i+1}, z_{3i+2}, z_{3i+3}$ are painted with color 2, 3, 4respectively. The nodes z_{n-3}, z_n are painted with color 2 and the nodes z_{n-2}, z_{n-1} are painted with color 3 respectively. For $1 \leq i \leq \left\lceil \frac{n}{3} \right\rceil - 2$, the nodes $t_{3i+1}, t_{3i+2}, t_{3i+3}$ are painted with color $n+3\left\lceil \frac{i}{2} \right\rceil + 2, 2, n+3\left\lceil \frac{i}{2} \right\rceil + 3$ respectively when i is odd and with color $4, n+3\left\lceil \frac{i}{2} \right\rceil + 4, 3$ respectively when i is even. The nodes $t_{n-3}, t_{n-2}, t_{n-1}, t_n$ are painted with color$3\left\lceil \frac{n}{2} \right\rceil, 2, 3\left\lceil \frac{n}{2} \right\rceil + 1, 4$ respectively when n is odd and with color $4, 3\left\lceil \frac{n}{2} \right\rceil + 2,\ 4, 3\left\lceil \frac{n}{2} \right\rceil + 3$ respectively when n is even.

The nodes w, x_i aredominated by color class$n+1$. Then for $1 \leq i \leq 3$ the nodes$u_i, v_i, t_i, y_i, z_i$are dominated by color class $n+i+1$. For $4 \leq i \leq n$ the nodesv_i, $y_i, z_i$are dominated by color class $+1$. And for $1 \leq i \leq \left\lceil \frac{n}{2} \right\rceil - 2$ the nodes $u_{2i+2}, u_{2i+3}, t_{2i+2}, t_{2i+3}$are dominated by color class $n+4+i$. The nodes u_n, t_n dominate color class $3\left\lceil \frac{n}{2} \right\rceil + 3$ when n is even.

Every neighbouring node is given different color and also it is observed that every node of the graph dominates all the nodes of atleast one color class. Thus it is a dominator coloring of nodes.

Case 3: When $n(mod\ 3) \equiv 2$

For $1 \leq i \leq n$, the nodes w, u_i, v_i are painted with color 1and the nodes x_i are given color $i+1$ respectively. For $1 \leq i \leq 3$, the nodes t_i, z_i are painted with color $i+1$ and $n+i+1$ respectively. For $1 \leq i \leq \left\lceil \frac{n}{3} \right\rceil$-1, the nodes $y_{3i-2}, y_{3i-1}, y_{3i}$are painted with color 4, 2, 3respectively. The nodes $y_{n-1}, y_n$are painted with color 4, 3respectively. For $1 \leq i \leq \left\lceil \frac{n}{3} \right\rceil - 1$, the nodes $z_{3i+1}, z_{3i+2}, z_{3i+3}$ are painted with color 2, 3, 4respectively. The nodes $z_{n-1}, z_n$are painted with color 2 respectively. For $1 \leq i \leq \left\lceil \frac{n}{3} \right\rceil - 2$, the nodes $t_{3i+1}, t_{3i+2}, t_{3i+3}$ are painted with color $n+3\left\lceil \frac{i}{2} \right\rceil + 2, 2, n+3\left\lceil \frac{i}{2} \right\rceil + 3$ respectively when i is odd and with color $4, n+3\left\lceil \frac{i}{2} \right\rceil + 4, 3$ respectively when i is even. The nodes $t_{n-1}, t_n$are painted with color $3\left\lceil \frac{n}{2} \right\rceil + 1, 4$ respectively when n is odd and with color $4, 3\left\lceil \frac{n}{2} \right\rceil + 3$ respectively when n is even.

The nodes w, x_i aredominated by color class$n+1$. Then for $1 \leq i \leq 3$ the nodes$u_i, v_i, t_i, y_i, z_i$are dominated by color class $n+i+1$. For $4 \leq i \leq n$ the nodesv_i, $y_i, z_i$are dominated by color class $+1$. And for $1 \leq i \leq \left\lceil \frac{n}{2} \right\rceil - 2$ the nodes $u_{2i+2}, u_{2i+3}, t_{2i+2}, t_{2i+3}$are dominated by color class $n+4+i$. The nodes u_n, t_n dominate color class $3\left\lceil \frac{n}{2} \right\rceil + 3$ when n is even.

Every neighbouring node is given different color and also it is observed that every node of the graph dominates all the nodes of atleast one color class. Thus it is a dominator coloring of nodes.

January - February 2020
ISSN: 0193 - 4120 Page No. 6228 - 6232

Thus the dominator coloring number of middle graph of CH_n is $\chi_d\left(M(CH_n)\right) = \begin{cases} 3\left\lceil\frac{n}{2}\right\rceil + 3 & when\ n\ is\ even \\ 3\left\lceil\frac{n}{2}\right\rceil + 1 & when\ n\ is\ odd \end{cases}$.

Theorem 2.4: If $C(CH_n)$ is the central graph of closed Helm graph CH_n then its dominator coloring number is $\chi_d\left(C(CH_n)\right) = \begin{cases} 2n & when\ n \geq 4 \\ 2n+1 & when\ n = 3 \end{cases}$

Proof:

Let the node set and edge set of the closed Helm graph CH_n be

$V(CH_n) = \{w\} \cup \{v_i, u_i \ / \ 1 \leq i \leq n\}$

$E(CH_n) = \{v_1v_n, u_1u_n\} \cup \{v_iv_j,\ u_iu_j / \ 1 \leq i \leq n-1, \cup uivi, wvi / \ 1 \leq i \leq n$.

The central graph of closed Helm graph denoted by $C(CH_n)$ is obtained by the subdivision of each edge of CH_n exactly once and connecting by an edge all the non-adjacent nodes of CH_n in the central graph of closed Helm graph $C(CH_n)$.

Let the new nodes obtained by the subdivision of edges $E(CH_n)$ be $\{x_i, y_i, z_i, t_i \ / \ 1 \leq i \leq n\}$.

The node set and edge set of the central graph of the closed Helm graph $C(CH_n)$ are

$V(C(CH_n)) = \{w\} \cup \{u_i, v_i, x_i, y_i, z_i, t_i / \ 1 \leq i \leq n\}$

$$E(C(CH_n) = \{u_it_i,\ u_iz_i, z_iv_i, v_iy_i, v_ix_i, wx_i \ / \ 1 \leq i \leq n\} \cup \{t_nu_1, y_nv_1\} \cup \{t_iu_{i+1}, y_iv_{i+1} / \ 1 \leq i \leq n-1\} \cup \ E(CH_n)^c$$

The procedure below explains the dominator coloring of nodes for central graph of closed Helm graph

Case 1: When $n \geq 4$

The node w is painted with color $n+1$. For $1 \leq i \leq n$, the nodes x_i, y_i are painted with color 1 and the nodes v_i are painted with color $n+i$ respectively. For $1 \leq i \ \leq \left\lfloor\frac{n}{2}\right\rfloor$, the nodes $t_{2i-1},\ t_{2i},\ z_{2i-1},\ z_{2i}$ are painted with color 2 when i is odd and color 1 when i is even. When n is odd, the nodes $t_n,\ z_n$ are given color 2. For $1 \leq i \ \leq \left\lceil\frac{n}{2}\right\rceil$, the nodes u_{2i-1} are painted with color $\left\lfloor\frac{n}{2}\right\rfloor + i$ and for $1 \leq i \leq \left\lfloor\frac{n}{2}\right\rfloor$, the nodes u_{2i} are painted with color i.

The node w dominates color class n. For $1 \leq i \ \leq \left\lceil\frac{n}{2}\right\rceil$, the nodes u_{2i-1} dominate color class $\left\lfloor\frac{n}{2}\right\rfloor + i$ and for $1 \leq i \leq \left\lfloor\frac{n}{2}\right\rfloor$, the nodes u_{2i} dominate color class $2n-i$ when i is odd and color class $2n-i-1$ when i is even. For $1 \leq i \ \leq n-1$, the nodes $v_{i+1}, y_{i+1}, z_{i+1}$ dominate color class $n+i+1$. For $1 \leq i \ \leq \left\lceil\frac{n}{2}\right\rceil - 1$, the nodes t_{2i}, t_{2i+1} dominate color class $\left\lfloor\frac{n}{2}\right\rfloor + i + 1$. The nodes v_1, y_1 dominate color class $2n\ , n+2$ respectively. And the nodes z_1, t_1 dominate color class $\left\lfloor\frac{n}{2}\right\rfloor + 1$. For $1 \leq i \leq n$, the nodes x_i dominate color class $n+i$.

Every neighbouring node is given different color and also it is observed that every node of the graph dominates all the nodes of atleast one color class. Hence it is a dominator coloring of nodes and the dominator coloring number of central graph of closed Helm graph is $\chi_d\left(C(CH_n)\right) = 2n$.

Case 2: When $n = 3$

The node w is painted with color 1. For $1 \leq i \leq 3$, the nodes t_i, x_i, y_i, z_i are painted with color 7 and the nodes $u_i,\ v_i$ are painted with color $i,\ i+3$ respectively.

The node w dominates color class n. For $1 \leq i \ \leq 3$, the nodes v_i, x_i, y_i, z_i dominate color class $i+3$ and for $1 \leq i \leq 2$, the nodes t_i, u_{i+1} dominate color class $i+1$ respectively. The nodes t_3, u_1 dominate color class $3, 6$ respectively.

Every neighbouring node is given different color and also it is observed that every node of the graph dominates all the nodes of atleast one color class. Hence it is a dominator coloring of nodes and the dominator coloring number of central graph of closed Helm graph is $\chi_d\left(C(CH_n)\right) = 2n+1$.

Hence the dominator coloring number of central graph of closed Helm graph is $\chi_d\left(C(CH_n)\right) = \begin{cases} 2n & when\ n \geq 4 \\ 2n+1 & when\ n = 3 \end{cases}$

Corollary 2.5: If $M(CH_n)$ is the middle graph of closed Helm graph CH_n thenfor every$n \geq 4$ the closed Helm graph satisfies the relation

$$\chi_d\left(M(CH_n)\right) = \begin{cases} \chi_d(CH_n) + n + \left\lfloor\frac{n}{6}\right\rfloor - 1 & when\ n\ is\ odd \\ \chi_d(CH_n) + n + \left\lfloor\frac{n}{6}\right\rfloor & when\ n\ is\ even \end{cases}$$

Corollary 2.6:If $C(CH_n)$ is the central graph of closed Helm graph CH_n thenfor every$n \geq 4$ the closed Helm graph satisfies the relation

$$\chi_d\left(C(CH_n)\right) = \begin{cases} \chi_d(CH_n) + n + 4\left\lfloor\frac{n}{6}\right\rfloor - 3 & when\ n(mod\ 6) \equiv 0\ or\ 1 \\ \chi_d(CH_n) + n + 4\left\lfloor\frac{n}{6}\right\rfloor - 2 & when\ n(mod\ 6) \equiv 2\ or\ 3 \\ \chi_d(CH_n) + n + 4\left\lfloor\frac{n}{6}\right\rfloor - 1 & when\ n(mod\ 6) \equiv 4\ or\ 5 \end{cases}$$

Corollary 2.7: If $M(CH_n)$, $C(CH_n)$ are the middle graph and the central graph of the closed Helm graph CH_n thenfor every$n \geq 4$ $\chi_d\left(C(CH_n)\right) =$ $\chi dMCHn+n-12-2$

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