January - February 2020
ISSN: 0193 - 4120 Page No. 5970 - 5975

Ant Colony Optimization for Intractable Problems: A step by step Approach.

Bijaya Kumar Nanda and Satchidananda Dehuri,
Department of Information and Communication Technology,
Fakir Mohan University, Vyasa Vihar,
Balasore-756019, Odisha, INDIA.
Emails:{nanda_bijaya@yahoo.co.in, *satchi.lapa@gmail.com}*

Article Info
Volume 82
Page Number: 5970 - 5975
Publication Issue:
January-February 2020

Article History
***Article Received**: 18 May 2019*
***Revised**: 14 July 2019*
***Accepted**: 22 December 2019*
***Publication**: 29 January 2020*

Abstract:
This paper discusses the usage of basic ant colony optimization technique for solving an intractable problem. Ant colony optimization technique is inspired by the observation of natural ant colonies. Ant colonies are distributed systems and highly structured social organization that can accomplish complex tasks like solving TSP, an intractable problem where a polynomial time algorithms take a large amount of time to be of practical use. Although the paper is focused on intractable problem in general and its optimal or near optimal solution through ACO, but we restrict our self with a travelling sales man problem and their solution through ACO by illustrative example.

***Keywords**: Biological control, Functional response, D. opuntiae, Hyperaspis campestris, Cactus*

Introduction: In computational complexity theory, there are many problems, which are regarded as inherently different of its solution requires significant resources (e.g. time and storage) whatever the algorithm used. One such problem is Traveling salesman problem (TSP) which has been studied extensively in past decades with a considerable amount of research effort. [2, 3]

Intuitively the TSP is the problem of a salesperson who, starting from his hometown, wants to find a shortest tour that takes him through a given set of customer cities and then back home, visiting each customer city exactly once. Graphically it can be represented by a complete weighted graph G=<V,E>, where V being the set of vertices representing cities, and E being the set of edges, each are associated with a value d_{ij} (length), which is the distance, between cities i and j , with $(i,j) \in E$. In asymmetric TSP, the distance between a pair of vertices i , j is dependent in the direction of traversing the arc, that is, there is at least one arc (i,j) for which $d_{ij} \neq d_{ji}$. In the symmetric TSP, $d_{ij} = d_{ji}$ holds for all arcs in E. This article illustrates the usage of ACO on symmetric TSP. The goal in the TSP is to find a minimum length Hamiltonian circuit of the graph, where a Hamiltonian circuit is a closed path visiting each of the n vertices of G exactly once. Thus an optimal solution to the TSP is a permutation π of the node {1,2,---,n} such that the length f(π) is minimal where $f(\pi) = \sum_{i=1}^{n=1} d_{\pi(i)\pi(i+1)} + d_{\pi(n)\pi(1)}$.
Let us realize how different this problem if n tends to a very very big number. For example, if n=4, then 6 possible tours needs to examine, similarly if n=10, then 91 number of tours need to examine, hence it is difficult for human being to enumerate and hence it is also difficult for computer to solve the same in a tolerable amount of time. One of the ways to overcome this problem by approximation

January - February 2020
ISSN: 0193 - 4120 Page No. 5970 - 5975

algorithm. In addition Meta heuristic approach are also approximate the solution and ACO is such an approach.[1]

The inspiring source of ACO algorithms are natural ant colonies [4, 5]. In specific, ACO is inspired by the ants forging behavior. At the core of this problem is the direct communication between the ants by means of chemical pheromone trails, which enables them to find optimal or near optimal paths between their nest and food sources. This characteristic of real ant colonies is exploited in ACO for solving the aforesaid symmetric TSP [6, 7, 8]. The rest of the sections are organized as follows. In section 2, the fundamentals of ant colony is described. In section 3 illustrative usage of ACO for solving the TSP is carried out through an easy to understand instance of TSP. Section 4 concludes the article.

2. Ant colony optimization:

Macro Dorigo and team mates have introduced the first ACO algorithm in the early 1990s [1]. The development of this algorithm was inspired by the real ant colonies. The main ideas that the self organizing principles which allow three highly coordinated behaviors of real ants can be exploited to coordinate populations of artificial agents that collaborate to solve the intractable problems. Several different aspects of the behavior of ant colonies have inspired different kinds of ant algorithms. Examples are foraging, division of labor, brood sorting and cooperative transport. In all these examples, ants coordinate their activities via stigmergy, a form indirect communication mediated by modifications of the environment. For example, a foraging ant deposits a chemical on the ground which increases the probability that other ants will follow the same path.
ACO algorithms are based on the following ideas.

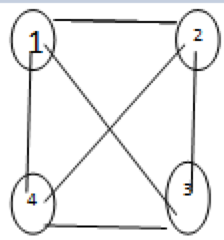
i) Each path followed by an ant is associated with a candidate solution for a given problem.
ii) When an ant follows a path, the amount of pheromone deposited on that path is proportional to the quality of the corresponding candidate solution for the target problem.
iii) When an ant has to choose between two or more paths, the paths with a larger amount of pheromone have a greater probability of being chosen by the ant

3. ACO for TSP: A step by step illustration.

In essence, the design of an ACO algorithm involves the specification of:
i)-An appropriate representation of the problem, which allows the ants to incrementally construct/modify solutions through the use of a probabilistic transition rule, based on the amount of pheromone in the trail ad local, problem dependent heuristic.
ii)-A method to enforce the construction of valid solutions that is solutions that are legal in the real world situation corresponding to the problem definition.
iii)-A problem dependent heuristic function (η) that measures the quality of items that can be added to the current partial solution.
iv)-A rule for pheromone updating, which specify how to modify the pheromone trail (τ).
A probabilistic transition rule based on the value of the heuristic function (η) and on the contents of the pheromone trail (τ) that is used to iteratively construct a solution. The algorithmic form of ACO is illustrated as follows:

Ant Colony Optimization-TSP()
1. Setting of parameters.
2. Initialize pheromone trails.
3. While (terminator condition not met)do
4. Construct ant solutions
5. Apply local research/*optional*/
6. Update pheromones
7. End of the while
8. End of the algorithm

Let us solve the following instance of an asymmetric TSP. consider the complete graph of 4 vertices and associated edges representing TSP with the cost matrix



$$C = \begin{bmatrix} 0 & 10 & 15 & 20 \\ 5 & 0 & 9 & 10 \\ 6 & 13 & 0 & 12 \\ 8 & 8 & 9 & 0 \end{bmatrix}$$

In ACO, m (artificial) ants concurrently build a form of the TSP. Initially, ants are

January - February 2020
ISSN: 0193 - 4120 Page No. 5970 - 5975

put on randomly chosen cities. At each construction step, ant k applies a probability action choice rule, called randomly proportional rule, decide which city to visit next. In particular, the probability with which ant k, currently at city 'i', chooses to go to city 'j' is

$$P_{ij}^{(k)} = \frac{\tau_{ij}^{\alpha} \quad \eta_{ij}^{\beta}}{\sum_{l \in N_i^k} \tau_{ij}^{\alpha} \quad \eta_{ij}^{\beta}}, \quad \text{if } j \in N_i^k \text{,---(1)}$$

Where $\eta_{ij} = \frac{1}{d_{ij}}$ is a heuristic value that is available a prior. α and β are two parameters which determines the relative influence of the pheromone trail and the heuristic information, and N_i^k is the feasible neighborhood of ant k, when being at city 'i', that is, the set of cities that ant k has not visited yet (the probability of choosing a city outside N_i^k *is* 0). By this probabilistic rule, the probability of choosing a particular arc (i, j) increase with the value of the associated pheromone trail τ_{ij} and if the heuristic information valueη_{ij}. The role of the parameters α and β is the following.

If $\alpha = 0$, the closest cities are more likely to be selected. If $\beta = 0$, only pheromone amplification is at work, that is only pheromone is used, without any heuristic bias. This generally leads to rather poor results, and, in particular, for values of $\alpha > 1$ it leads to the rapid emergence of a stagnation situation, which in general, is strongly suboptimal.

Each ant k maintains a memory m^k which contains the cities already visited in the order they are visited. This memory is used to define the feasible neighborhood N_i^k in the construction rule given by equation (1).

Update of pheromone trail

After all the ants have completed their turns, the pheromone trails are updated as follows

$$\tau_{ij} = \tau_{ij} + \sum_{k=1}^{m} \Delta\tau_{ij}^{k} \quad \forall (i,j) \in \text{ L ---------- (2)}$$

Where $\Delta\tau_{ij}^{k}$ is the amount of pheromone ant k deposits on the arcs it has visited? It is defined as follows

$$\Delta\tau_{ij}^{k} = \begin{cases} \frac{1}{c^k} & if\ arc(i,j) \in T^k \\ 0 & otherwise \end{cases} \text{ ----------- (3)}$$

Where c^k, the length of the tour T^k built by the k^{th} ant, is completed as the sum of the lengths of the arcs belonging toT^k. By means of the equation (3), the better an ant tour is, the more pheromone the arcs belonging to this tour receives

Let us see the steps of solution to TSP.

Initialization of parameters:

Assume that α=β=1. And m=4 (i.e. number of ants) and neighborhood size is complete graph.

Initialization of pheromone:

$$\tau_{ij} = \begin{cases} 1 & if\ (i,j) \in E \\ 0 & otherwise \end{cases}$$

$$\tau_{ij}^{0} = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

Iteration 1

Initially put a single ant at each node of the graph, say ant 1 is placed in node 1 i.e. partial tour T^i decides to choose next city e.g. {2,3,4} based on the transition rule

$$P_{12}^{1} = \frac{(\tau_{12})(\eta_{12})}{(\tau_{12})(\eta_{12})+(\tau_{13})(\eta_{13})+(\tau_{14})(\eta_{14})}$$

$$= \frac{1*(\frac{1}{10})}{1*\left(\frac{1}{10}\right)+1*\left(\frac{1}{15}\right)+1*(\frac{1}{20})} = 0.46$$

$$P_{13}^{1} = \frac{1*(\frac{1}{15})}{1*\left(\frac{1}{10}\right)+1*\left(\frac{1}{15}\right)+1*(\frac{1}{20})} = 0.30$$

$$P_{14}^{1} = \frac{1*(\frac{1}{20})}{1*\left(\frac{1}{10}\right)+1*\left(\frac{1}{15}\right)+1*(\frac{1}{20})} = 0.23$$

Based on highest probability value, the next city 2 is added in partial tour T^1= {1, 2}. Now it decides next city from the unvisited list {3, 4}, through the following transition rule.

January - February 2020
ISSN: 0193 - 4120 Page No. 5970 - 5975

$$P^1_{23} = \frac{1*(\frac{1}{9})}{1*(\frac{1}{9})+1*(\frac{1}{10})} = 0.52$$

$$P^1_{24} = \frac{1*(\frac{1}{10})}{1*(\frac{1}{9})+1*(\frac{1}{10})} = 0.47$$

Hence the next city 3 is added in the current partial list T^1={1, 2, 3}. Finally, it includes only city 4 in the partial list T^1 = {1, 2, 3, 4}. Therefore the length of the tour is 39.

Tour of ant 2

Assume, for ant 2 from node 2, the tour includes cities in order as T^2={2,1,3,4} with length 40.

Tour of ant 3

Assume that ant 3 is placed in node, then the complete tour from node 3 is T^3={3,1,2,4} with length 35

Tour of ant 4

Let us assume that, ant 4 is placed at node, the tour is numerated as {4,2,1,3,4}with length 40.

Now since all tour have been completed, update pheromone table by equation (2)and(3).

$$\tau^1_{12} = \tau^0_{12} + (\Delta\tau^1_{12} + \Delta\tau^2_{12} + \Delta\tau^3_{12} + \Delta\tau^4_{12}) = 1+ (\frac{1}{39} + 0 + \frac{1}{35} + 0) = 1.054$$

$$\tau^1_{13} = \tau^0_{13} + (\Delta\tau^1_{13} + \Delta\tau^2_{13} + \Delta\tau^3_{13} + \Delta\tau^4_{13}) = 1+ (0 + \frac{1}{40} + 0 + \frac{1}{40}) = 1.05$$

$$\tau^1_{14} = \tau^0_{14} + (\Delta\tau^1_{14} + \Delta\tau^2_{14} + \Delta\tau^3_{14} + \Delta\tau^4_{14}) = 1+0 = 1.0$$

$$\tau^1_{21} = \tau^0_{21} + (\Delta\tau^1_{21} + \Delta\tau^2_{21} + \Delta\tau^3_{21} + \Delta\tau\, 21\ 4 = 1.05$$

$$\tau^1_{23} = \tau^0_{23} + (\Delta\tau^1_{23} + \Delta\tau^2_{23} + \Delta\tau^3_{23} + \Delta\tau^4_{23})=1+\frac{1}{39}=1.025$$

$$\tau^1_{24} = \tau^0_{24} + (\Delta\tau^1_{24} + \Delta\tau^2_{24} + \Delta\tau^3_{24} + \Delta\tau^4_{24})=1+\frac{1}{35}=1.028$$

$$\tau^1_{31} = \tau^0_{31} + (\Delta\tau^1_{31} + \Delta\tau^2_{31} + \Delta\tau^3_{31} + \Delta\tau^4_{31})=1+\frac{1}{35}=1.028$$

$$\tau^1_{32} = \tau^0_{32} + (\Delta\tau^1_{32} + \Delta\tau^2_{32} + \Delta\tau^3_{32} + \Delta\tau^4_{32})=1+0 =1.00$$

$$\tau^1_{34} = \tau^0_{34} + (\Delta\tau^1_{34} + \Delta\tau^2_{34} + \Delta\tau^3_{34} + \Delta\tau^4_{34})=1+\frac{1}{39}+\frac{1}{40}+\frac{1}{40} =1.075$$

$$\tau^1_{41} = \tau^0_{41} + (\Delta\tau^1_{41} + \Delta\tau^2_{41} + \Delta\tau^3_{41} + \Delta\tau\, 41\ 4 = 1+139 = 1.025$$

$$\tau^1_{42} = \tau^0_{42} + (\Delta\tau^1_{42} + \Delta\tau^2_{42} + \Delta\tau^3_{42} + \Delta\tau\, 42\, 4 = 1+240 = 1.05$$

$$\tau^1_{43} = \tau^0_{43} + (\Delta\tau^1_{43} + \Delta\tau^2_{43} + \Delta\tau^3_{43} + \Delta\tau^4_{43})= 1.028$$

Hence the updated pheromone table is shown bellow

Table-1

$$\tau^1_{ij} = \begin{bmatrix} 0 & 1.054 & 1.05 & 1 \\ 1.05 & 0 & 1.025 & 1.028 \\ 1.028 & 1 & 0 & 1.075 \\ 1.025 & 1.05 & 1.028 & 0 \end{bmatrix}$$

From the table-1 using pheromone values and starting from each node we have the following paths

	Path covers	Total length
Ant1from node1	1-2-3-4-1	10+9+12+8 = 39
Ant2 from node 2	2-1-3-4-2	5+15+12+8 = 40
Ant3 from node 3	3-1-2-4-3	6+10+10+9 = 35
Ant4 from node 4	4-2-1-3-4	8+5+15+12 = 40

So the optimal path is 3-1-2-4-3 with length 35

Iteration 2

Similarly using above computations like iteation-1, the following updated pheromone table as well as table that ant covers path can be obtained.

January - February 2020
ISSN: 0193 - 4120 Page No. 5970 - 5975

$$\tau_{ij}^{2} = \begin{bmatrix} 0 & 1.10 & 1.075 & 1.023 \\ 1.075 & 0 & 1.07 & 1.076 \\ 1.07 & 1 & 0 & 1.125 \\ 1.05 & 1.09 & 1.05 & 0 \end{bmatrix}$$

	Path covers	Total length
Ant1from node1	1-2-4-3-1	10+10+9+6 = 35
Ant2 from node 2	2-4-3-1-2	10+9+6+10 = 35
Ant3 from node 3	3-4-2-1-3	12+8+5+15 = 40
Ant4 from node 4	4-2-1-3-4	8+5+15+12 = 40

So the optimal paths are. 1-2-4-3-1 and 2-4-3-1-2 with lengths 35

Iteration 3

Similarly using above computations in iteration-3, we can get the following pheromone table as well as table that ant covers path.

$$\tau_{ij}^{3} = \begin{bmatrix} 0 & 1.54 & 1.095 & 1.043 \\ 1.1 & 0 & 1.181 & 1.189 \\ 1.121 & 1 & 0 & 1.175 \\ 1.075 & 1.138 & 1.07 & 0 \end{bmatrix}$$

	Path covers	Total length
Ant1from node1	1-2-4-3-1	10+10+9+6 = 35
Ant2 from node 2	2-4-3-1-2	10+9+6+10 = 35
Ant3 from node 3	3-4-2-1-3	12+8+5+15 = 40
Ant4 from node 4	4-2-1-3-4	8+5+15+12 = 40

So the optimal paths are. 1-2-4-3-1 and 2-4-3-1-2 with lengths 35

Iteration 4

Finally in iteration-4, we have the following updated pheromone table that determines the optimal path of ants which is shown in last table

$$\tau_{ij}^{4} = \begin{bmatrix} 0 & 1.208 & 1.12 & 1.06 \\ 1.125 & 0 & 1.22 & 1.109 \\ 1.17 & 1 & 0 & 1.22 \\ 1.10 & 1.186 & 1.09 & 0 \end{bmatrix}$$

	Path covers	Total length
Ant1from node1	1-2-4-3-1	10+10+9+6 = 35
Ant2 from node 2	2-4-3-1-2	10+9+6+10 = 35
Ant3 from node 3	3-1-2-4-3	6+10+10+9 = 35
Ant4 from node 4	4-2-1-3-4	8+5+15+12 = 40

Hence after a fixed number of iteration or some other termination criterion is satisfied, the following optimum paths are enumerated. from node 1: 1-2-4-3-1 , from node 2: 2-4-3-1-2 and from node-3: 3-1-2-4-3 each with length 35 which is shortest distace. However from node 4 till 4th iteration, the optimum path is 4-2-1-3-4 with length 40. But after some more iteration from all nodes shortest distance will be covered.

The TSP is the oldest and most studied intractable problems in both operation research and computer science. Therefore, a large number of diversified algorithms have been

January - February 2020
ISSN: 0193 - 4120 Page No. 5970 - 5975

developed e.g. iterative improvement and exact method like branch and bound or cutting plane, etc. An in depth overview of these early approaches are given in [9] (Lawlen et.al, 1985) since the beginning of the 80i, more and more Meta heuristic algorithms have been developed to solve TSP with a remarkable success

From the previous study, it has been concluded that, in real ant colonies the emergence of high level patterns like shortest paths is only possible through the interaction of a large member of individuals [10]. Additionally, pheromone updates based on solution quality are important for fast convergence, large values for parameters α lead to a strong emphasis of initial, random fluctuation, and to bad algorithm behaviors, the larger the member of ants, the better the convergence behaviors of the algorithms, although this comes at the cost of larger simulation times, and pheromone evaporation is important when trying to solve more complex problems.

Conclusion: It is worth mentioning that the available results obtained by ACO algorithms applied to the TSP are not competitive with the exact approaches however, by adding more sophisticated local search algorithms like the implementation of Lin Kernighan heuristic or Helsgaun's variant of the Lin Kernighan heuristic, ACO's computational results on TSP can certainly be strongly improved. In fact, the best performing variants of ACO algorithms on the TSP often reach world class performance on many other problems. Hence, we argue that, as an alternating optimization tool, ACO can be recommended to use more real life intractable problems without observing the algorithm behaviors by many technicalities.

It is further to highlight that, trail pheromone is a specific type of pheromone that some ant species, such as Lasius Nigo, on the Aagentine ant Iridimyrmex humities use for making paths on the ground. However in India, recently it has been studied that ant become "tandeon leaders". Therefore, any future study includes exploration this idea for solving complex problems.

References:

1. Dorigo, M., Maniezzo, V., & Colorni, A. (1996). Ant system: optimization by a colony of cooperating agents. IEEE Transaction on Systems, Man and Cybernetics, Part B, 26(1), 29-41.
2. Croes, G. A., A Method for Solving Traveling Salesman Problems, Operation Research, 6, 791-812, 1958.
3. Lin, S., Computer Solutions for the Traveling Salesman Problem, Bell Systems Technology Journal, 44, 2245-2269, 1965.
4. Dorigo, M. and Stutzle, T., Ant Colony Optimization, Prentice Hall of India, 2005.
5. Dorigo, M. & Di Caro, G. (1999). The Ant Colony Optimization Meta-Heuristic. In Corne, D., Dorigo, M. , and Glover, F., (Eds.), New Ideas in Optimization, pp. 11–32, Mc Graw-Hill.
6. Gambardella, L. M. & Dorigo, M. Solving symmetric and asymmetric TSPs by ant colonies. In : Proceedings of the 1996 IEEE International Conference on Evolutionary Computation (ICEC'96), pages 622–627. IEEE Press, Piscataway, NJ, 1996.
7. Gambardella, L. M. & Dorigo, M. : Ant Colony System: A cooperative learning approach to the travelling salesman problem, IEEE trans,on evolutionary computation,1(1), 53-66,1997.
8. Gambardella, L. M. & Dorigo, M. :Ant-Q, A reinforcement learning Approach to the travelling salesman problem, In: proc.of 12th int. conf. of machine learning,pp-252-260,1995.
9. Dorigo, M. Di Caro, G.& Gambardella, L. M. Ant algorithms for discrete optimization. Artificial Life, 5(2):137–172, 1999.
10. Dorigo, M. & Blum,C. Ant colony optimization theory: A survey. Theoretical Computer Science, 344(2-3),243-278,2005.