

# Mathematical Study of Emphysema and Its Complications Using the Homotopy Perturbation Method

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## Abstract

Mathematical Study of Emphysema And Its Complications Using The Homotopy Perturbation Method” is a paper in which we can determine the increase or decrease in the size of population of Emphysema. Emphysema is a kind of lung disease which makes the breathing process very difficult for a normal person. A deterministic Mathematical Equation of the Emphysema was framed as Non-Linear Differential Equation and Boundary Condition is applied to it. The equation is solved using the power series and integrated with respect to time. The results are represented graphically using Matlab Software. From the Graph we can observe that the level of Emphysema varies for the parameters like mortality rate, possibility of a Emphysema patient developing a complication, the rate at which complications are cured and the rate at which patients with complication become severely disabled. Homotopy Perturbation Method is more efficient and can give solutions with greater accuracy. This Paper can be used to Predict the number of patients with more complications faced due to Emphysema and prevent patients from this disease.

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## INTRODUCTION

Chronic Obstructive Pulmonary Disease is a kind of disease which affects the lungs and makes some troubles to breathe normally. Main types are Emphysema and chronic bronchitis. Emphysema is a type of Chronic Obstructive Pulmonary Disease (COPD), and it has several types including paraseptalcentrilobular, which affects mainly the upper lobes and it generally affects the chain smokers which affects the both paraseptal and centrilobular areas. In emphysema the alveoli in the lungs become harmed and stretched. Most cases of this disease are due to cigarette smoking. However

the treatment does not bring complete recoverment, but it can reduce the risk factors and prevent from the severe attacks. The main goal of this treatment is to prevent complications and to stabilize the conditions through the supportive therapy and medication uses. If left untreated, emphysema can be life threatening. JiHuanwho first proposed the Fundamentals of Homotopy Perturbation Method (HPM) helps in solving many types of linear and nonlinear functional equations.

This technique is a intermingling of homotopy in topology and classic perturbation methods, gives a

best way to get analytic or approximate solutions in many various areas.

### THE MATHEMATICAL MODEL

The Mathematical equation describing the dynamics of Emphysema and its complications is given by the ordinary differential equations (ODEs).

$$\begin{aligned} \frac{dX}{dt} &= I - (\lambda + \mu) X + \gamma Y \\ \frac{dY}{dt} &= I + \lambda X - (\gamma + \mu + \nu + \delta) Y \end{aligned} \quad (1.1)$$

$X = X(t)$  represents the number of emphysema without complications.

$Y = Y(t)$  represents the number of emphysema with complications.

$I = I(t)$  denotes the incidence of emphysema

$\mu$  represents natural mortality rate.

$\lambda$  stands for the probability of a emphysema person developing a complication.

$\gamma$  stands for the rate of recoverment of complications.

$\nu$  stands for the rate at which emphysema patients with complication become oppressively disabled.

$\delta$  stands for the death rate because of complications.

$Z = Z(t) = Y(t) + X(t)$  represents the measure of population of Emphysema in time  $t$ .

when  $Z = X + Y$ , we get,

$$\frac{dY}{dt} = -(\lambda + \theta)Y + \lambda Z, \quad t > 0 \quad (1.2)$$

$$\frac{dZ}{dt} = I - (\nu + \delta)Y - \mu Z$$

where  $\theta = \gamma + \mu + \nu + \delta$

with conditions

$$Y(0) = Y_0, Z(0) = Z_0 \quad (1.3)$$

### SOLUTION BY THE HOMOTOPY PERTURBATION METHOD

To show the general notions of the method, considered the following nonlinear differential equation:

$$R(u) - f(r) = 0, \quad r \in \Omega \quad (2.1)$$

subject to the boundary condition:

$$S(u, \frac{du}{dn}) = 0, \quad r \in \Gamma \quad (2.2)$$

where  $R$  represents general differential operator,  $S$  stands for boundary operator,  $f(r)$  is termed as analytical function and  $\Gamma$  is the boundary of the domain  $\Omega$ . The  $R$  can be classified into two parts  $L$  and  $Z$ , where  $L$  stands for the linear part, and  $Z$  is the nonlinear component. Equation (2.1) may therefore be rewritten as:

$$L(u) + Z(u) - f(r) = 0, \quad r \in \Omega \quad (2.3)$$

The homotopy perturbation structure is shown as follows

$$H(v, p) = (1 - p)[L(v) - Lu_0] + p[R(v) - f(r)] = 0 \quad (2.4)$$

where

$$v(r, p) : \Omega \rightarrow R \quad (2.5)$$

In equation (2.4),  $p \in [0,1]$  is an embedding parameter and  $u_0$  is the first assumption that fulfills the boundary condition. It can be assumed that the solution of equation (2.4) can be written as power series as follows :

$$v = v_0 + P v_1 + P v_2 + \dots \quad (2.6)$$

Let us assume,

$$u = \lim_{p \rightarrow 1} v = v_0 + v_1 + v_2 + \dots \quad (2.7)$$

The series (2.6) is convergence for most cases; but, the convergent rate determine on the nonlinear operator  $R(v)$ .

From (1.2)

$$\frac{dY}{dt} = -(\lambda + \theta)Y + \lambda Z \quad (2.8)$$

$$\frac{dZ}{dt} = I - (\nu + \delta)Y - \mu Z \quad (2.9)$$

By homotopy perturbation to equation (2.8) and (2.9) with

$$e = \lambda + \theta$$

$$w = v + \delta$$

we get,

$$(1 - p) \frac{dY}{dt} + P \left( \frac{dY}{dt} + eY - \lambda Z \right) = 0 \quad (2.10)$$

$$(1 - p) \frac{dZ}{dt} + P \left( \frac{dZ}{dt} - I + wY + \mu Z \right) = 0 \quad (2.11)$$

i.e.,

$$\frac{dY}{dt} + P (eY - \lambda Z) = 0 \quad (2.12)$$

$$\frac{dZ}{dt} + p (-I + wY + \mu Z) = 0 \quad (2.13)$$

Let

$$\left. \begin{aligned} Y &= Y_0 + pY_1 + p^2Y_2 + \dots \\ Z &= Z_0 + pZ_1 + p^2Z_2 + \dots \end{aligned} \right\} \quad (2.14)$$

$$\left. \begin{aligned} \frac{dY}{dt} &= \frac{dY_0}{dt} + \frac{dY_1}{dt} + p^2 \frac{dY_2}{dt} + \dots \\ \frac{dZ}{dt} &= \frac{dZ_0}{dt} + \frac{dZ_1}{dt} + p^2 \frac{dZ_2}{dt} + \dots \end{aligned} \right\} \quad (2.15)$$

Substituting (2.14) and (2.15) into equations (2.12) and (2.13), we have

$$\frac{dY_0}{dt} + p \frac{dY_1}{dt} + p^2 \frac{dY_2}{dt} + p [e(Y_0 + pY_1 + p^2Y_2) + \lambda(Z_0 + pZ_1 + p^2Z_2)] = 0 \quad (2.16)$$

$$\frac{dZ_0}{dt} + p \frac{dZ_1}{dt} + p^2 \frac{dZ_2}{dt} + p [-I + w(Y_0 + pY_1 + p^2Y_2) + \mu(Z_0 + pZ_1 + p^2Z_2)] = 0 \quad (2.17)$$

$$P^0 : \frac{dY_0}{dt} = 0, Y_0(0) = Y \quad (2.18)$$

$$P^1 : \frac{dY_1}{dt} + eY_0 - \lambda Z_0 = 0, Y_1(0) = 0 \quad (2.19)$$

$$P^2 : \frac{dY_2}{dt} + eY_1 - \lambda Z_1 = 0, Y_2(0) = 0 \quad (2.20)$$

$$P^0 : \frac{dZ_0}{dt} = 0, Z_0(0) = Z_0 \quad (2.21)$$

$$P^1 : \frac{dZ_1}{dt} - I + wY_0 + \mu Z_0 = 0, Z_1(0) = 0 \quad (2.22)$$

$$P^2 : \frac{dZ_2}{dt} + wY_1 + \mu Z_1 = 0, Z_2(0) = 0 \quad (2.23)$$

By (2.18) we obtained

$$Y_0(t) = Y_0$$

Likewise from equation (2.21) we get

$$Z_0(t) = Z_0$$

From equation (2.19)

$$\frac{dY_1}{dt} = -eY_0 + \lambda Z_0$$

By Integrating , we get

$$Y_1(t) = (-eY_0 + \lambda Z_0)t + K_1$$

$$Y_1(0) = 0 + K_1 = 0$$

This implies

$$K_1 = 0.$$

Therefore

$$Y_1(t) = (-eY_0 + \lambda Z_0)t;$$

i.e.,

$$Y_1(t) = \alpha t,$$

where

$$\alpha = -eY_0 + \lambda Z_0$$

From equation (2.22)

$$\frac{dZ_1}{dt} = I - wY_0 - \mu Z_0$$

By Integrating , we get

$$Z_1(t) = (I - wY_0 - \mu Z_0)t + K_2$$

At t = 0

$$Z_1(0) = 0 + K_2 = 0$$

This implies

$$K_2 = 0.$$

Therefore

$$Z_1(t) = (I - wY_0 - \mu Z_0)t;$$

i.e.,

$$Z_1(t) = \beta t,$$

where

$$\beta = I - wY_0 - \mu Z_0.$$

From equation (2.20)

$$\frac{dY_2}{dt} + eY_1 - \lambda Z_1 = 0$$

$$\frac{dY_2}{dt} = -e\alpha t + \lambda\beta t$$

By Integrating , we get

$$Y_2(t) = -\frac{1}{2} e\alpha t^2 + \frac{1}{2} \lambda\beta t^2 + K_3$$

At t = 0

$$Y_2(0) = 0 + K_3 = 0$$

This implies

$$K_3 = 0$$

Therefore,

$$Y_2(t) = \frac{1}{2} (\lambda\beta - e\alpha)t^2$$

From equation (2.23)

$$\frac{dZ_2}{dt} + wY_1 + \mu Z_1 = 0$$

$$\frac{dZ_2}{dt} = -w\alpha t - \mu\beta t$$

$$\frac{dZ_2}{dt} = -w\alpha t - \mu\beta t$$

By Integrating , we get

$$Z_2(t) = \frac{1}{2} (-w\alpha - \mu\beta)t^2 + K_4$$

At t = 0

$$Z_2(0) = 0 + K_4 = 0$$

This implies

$$K_4 = 0$$

Therefore

$$Z_2(t) = \frac{1}{2} (-w\alpha - \mu\beta)t^2$$

As stated by HomotopyPerturbation ,

$$Y(t) = \lim_{p \rightarrow 1} Y_0(t) + P^1 Y_1(t) + P^2 Y_2(t)$$

$$Y(t) = Y_0(t) + Y_1(t) + Y_2(t)$$

i.e.,

$$Y(t) = Y_0 + \alpha t + \frac{1}{2}(\lambda\beta - e\alpha)t^2$$

$$Z(t) = \lim_{p \rightarrow 1} Z_0(t) + P^1 Z_1(t) + P^2 Z_2(t)$$

$$Z(t) = Z_0(t) + Z_1(t) + Z_2(t)$$

$$\text{i.e., } Z(t) = Z_0 + \beta t + \frac{1}{2}(-w\alpha - \mu\beta)t^2$$

From (1.2) is given by

$$Y(t) = Y_0 + \alpha t + \frac{1}{2}(\lambda\beta - e\alpha)t^2$$

$$Z(t) = Z_0 + \beta t + \frac{1}{2}(w\alpha - \mu\beta)t^2$$

where

$$\alpha = (-eY_0 + \lambda Z_0),$$

$$\beta = (I - mY_0 - \mu Z_0),$$

$$\theta = \gamma + \mu + \gamma + \delta,$$

$$e = \lambda + \theta$$

$$w = v + \delta$$

## RESULTS

To implement the numerical simulations of Z(t) and Y(t) we used a mathematical software (Matlab) in this section, and the results are shown below in graphical form.

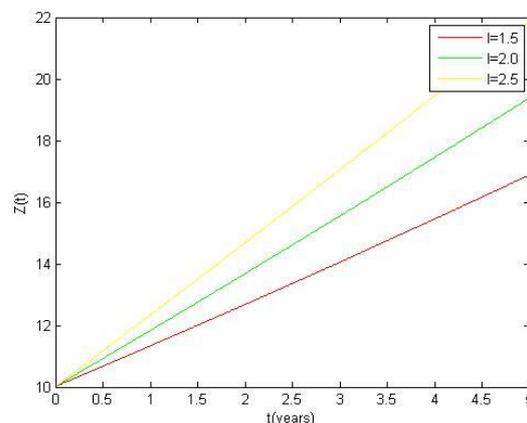


Figure 3.1 :Showing graphs of Z(t) for various values of I when  $v = 0.05$ ,  $\delta = 0.05$ ,  $\mu = 0.02$ ,  $\gamma = 0.08$ ,  $\lambda = 0.02$ .

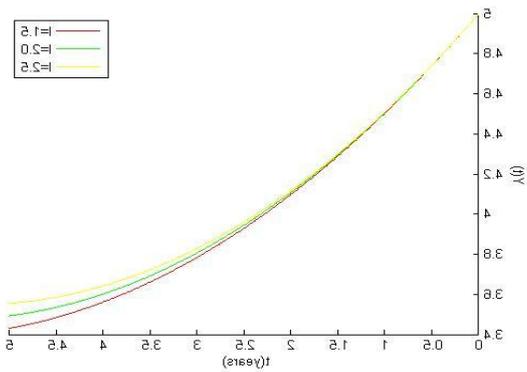


Figure 3.2 :Showing graphs of  $Y(t)$  for various values of  $I$  when  $\nu = 0.05$ ,  $\delta = 0.05$ ,  $\mu = 0.02$ ,  $\gamma = 0.08$ ,  $\lambda = 0.02$

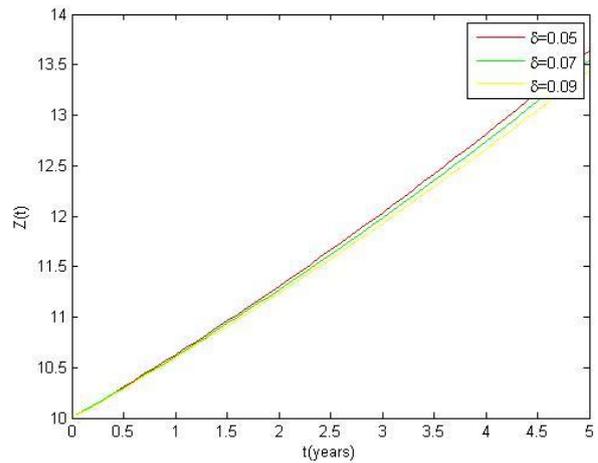
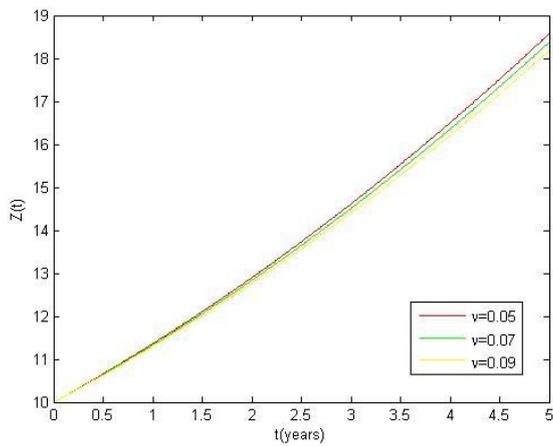


Figure 3.5 :Showing graphs of  $Z(t)$  for different values of  $\delta$  when  $I = 1.5$ ,  $\nu = 0.05$ ,  $\mu = 0.02$ ,  $\gamma = 0.08$ ,  $\lambda = 0.02$



3.3

:Showing graphs of  $Z(t)$  for various values of  $\nu$  when  $I = 1.5$ ,  $\delta = 0.05$ ,  $\mu = 0.02$ ,  $\gamma = 0.08$ ,  $\lambda = 0.02$

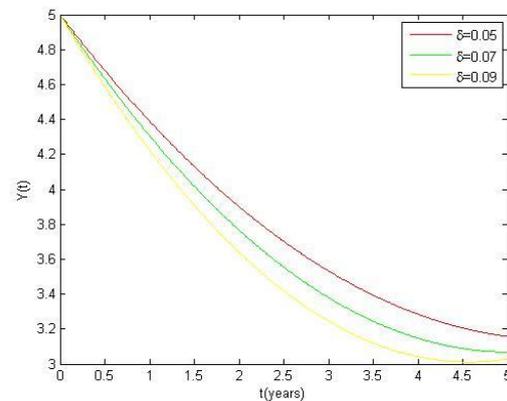


Figure 3.6 :Showing graphs of  $Y(t)$  for different values of  $\delta$  when  $I = 1.5$ ,  $\nu = 0.05$ ,  $\mu = 0.02$ ,  $\gamma = 0.08$ ,  $\lambda = 0.02$

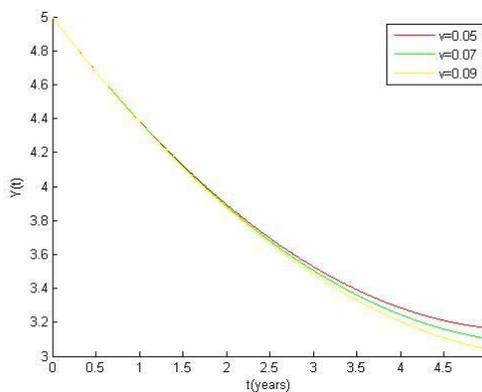


Figure 3.4 : Showing graphs of  $Y(t)$  for various values of  $\nu$  when  $I = 1.5$ ,  $\delta = 0.05$ ,  $\mu = 0.02$ ,  $\gamma = 0.08$ ,  $\lambda = 0.02$

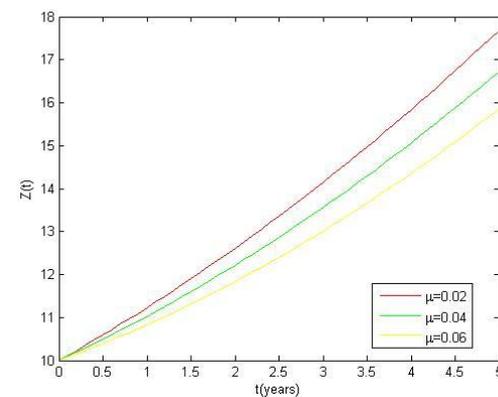


Figure 3.7 :Showing graphs of  $Z(t)$  for different values of  $\mu$  when  $I = 1.5$ ,  $\nu = 0.05$ ,  $\delta = 0.05$ ,  $\gamma = 0.08$ ,  $\lambda = 0.02$

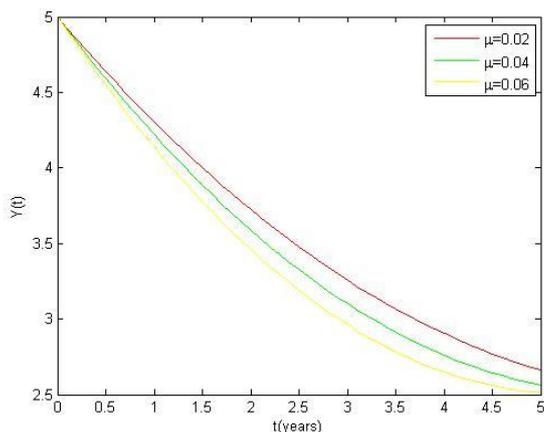


Figure 3.8 :Showing graphs of  $Y(t)$  for different values of  $\mu$  when  $I=1.5, v=0.05, \delta=0.05, \gamma=0.08, \lambda=0.02$

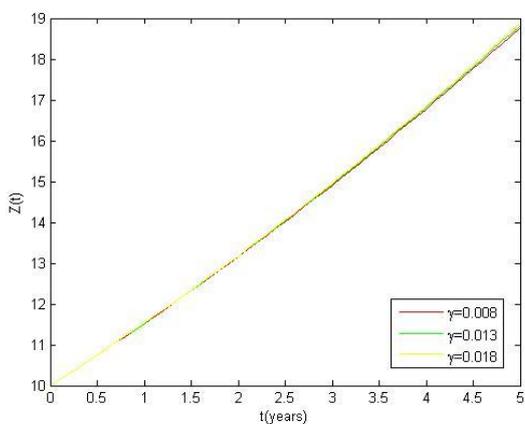


Figure 3.9 :Showing graphs of  $Z(t)$  for various values of  $\gamma$  when  $I=1.5, v=0.05, \delta=0.05, \mu=0.02, \lambda=0.02$

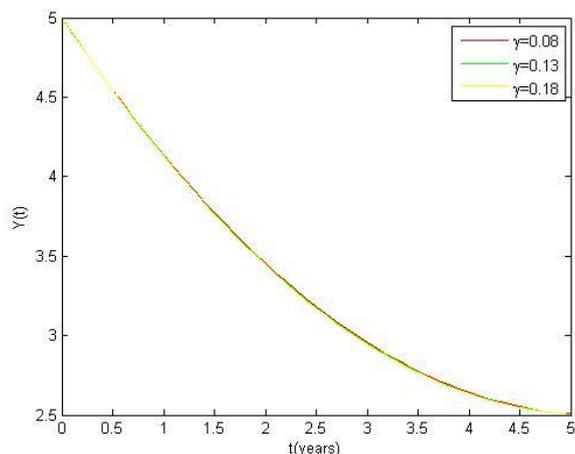


Figure 3.10 :Showing graphs of  $Y(t)$  for various values of  $\gamma$  when  $I=1.5, v=0.05, \delta=0.05, \mu=0.02, \lambda=0.02$

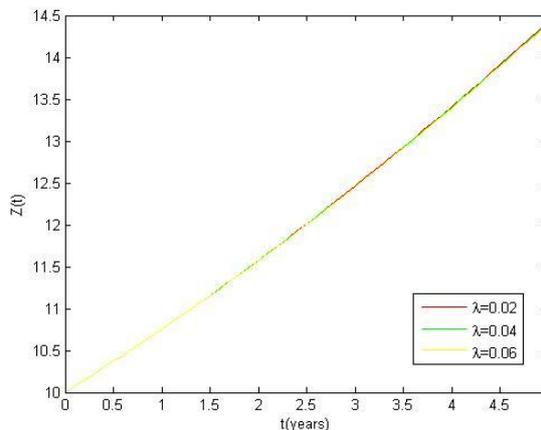


Figure 3.11:Showing graphs of  $Z(t)$  for different values of  $\lambda$  when  $I=1.5, v=0.05, \delta=0.05, \mu=0.02, \gamma=0.08$

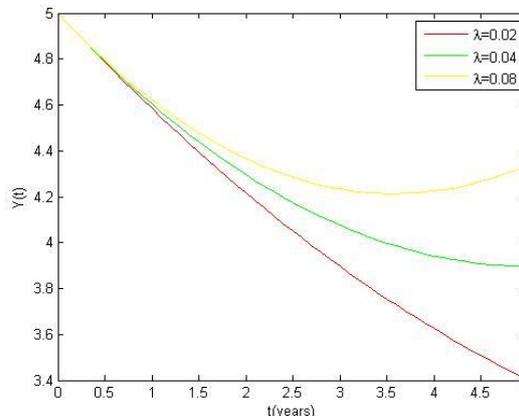


Figure 3.12:Showing graphs of  $Y(t)$  for different values of  $\lambda$  when  $I=1.5, v=0.05, \delta=0.05, \mu=0.02, \gamma=0.08$

## RESULTS AND DISCUSSION

Image 3.1 and 3.2 represents the graph of  $Z(t)$  and  $Y(t)$  against  $t$  respectively for three values of  $I$ . From the graph we can understand that when the value of  $I(t)$  increases,  $Z(t)$  and  $Y(t)$  also increases.

Image 3.3 and 3.4 represents the graph of  $Z(t)$  and  $Y(t)$  against  $t$  respectively for different values of  $v$ . From the graph we can understand that when the value of  $v(t)$  increases,  $Z(t)$  and  $Y(t)$  is also increases.

Image 3.5 and 3.6 represents the graph of  $Z(t)$  and  $Y(t)$  against  $t$  respectively for different values of  $\delta$ . It was observed that when  $\delta(t)$  increases,  $Z(t)$  and  $Y(t)$  decreases and viceversa.

Image 3.7 and 3.8 represents the graph of  $Z(t)$  and  $Y(t)$  respectively against  $t$  for different values of  $\mu$ . From the graph we can understand that  $Z(t)$  increases faster as the  $\mu(t)$  decreases while  $Y(t)$  decreases faster as the natural mortality rate increases.

Image 3.9 and 3.10 represents the graph of  $Z(t)$  and  $Y(t)$  respectively against  $t$  for different values of  $\gamma$ . From the graph we can understand that the rate at which complications are cured had no effect on both  $Z(t)$  and  $Y(t)$ .

Image 3.11 represents the graph of  $Z(t)$  against  $t$  for different values of  $\lambda$ . From the graph we can understand that the rate at which complications are cured had no effect on  $Z(t)$ .

Image 3.12 represents the graph of  $Y(t)$  against  $t$  for different values of  $\lambda$ . From the graph we can understand that the number of Emphysema patients with complication increases faster as the probability of Emphysema persons developing a complication increases.

## CONCLUSION

Through mathematical method we have predicted the growth of emphysema disease and with the graph as a result of our research, we have concluded that the variables that we have used in our research have played a role in determining the size of population of Emphysema at time  $t$ .

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