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Common Fixed Point Theorems in Intuitionistic Fuzzy Metric Spaces

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Abstract

In this paper, we obtain common fixed point theorems in intuitionistic fuzzy metric spaces using occasionally weakly compatible maps. Our results are the intuitionistic fuzzy version of some fixed point theorems for occasionally weakly compatible mappings on different metric spaces. The binary operator, intuitionistic fuzzy metric space, modified intuitionistic fuzzy metric space, and compatible mapping are only a few of the more fundamental notions and theorems that have been borrowed from other recent scientific articles. we define the notion of intuitionistic fuzzy metric spaces as a natural generalization of fuzzy metric spaces due to George and Veeramani [Fuzzy Sets Syst. 64 (1994) 395] and prove some known results of metric spaces including Baire's theorem and the Uniform limit theorem for intuitionistic fuzzy metric spaces.

Keywords: Intuitionistic fuzzy metric space, weakly compatible mappings, Common fixed point

1. INTRODUCTION

In this paper, using the idea of intuitionistic fuzzy sets, we define the notion of intuitionistic fuzzy metric spaces with the help of continuous t-norms and continuous t-conorms as a generalization of fuzzy metric space due to George and Veeramani. In 2004, Park defined the notion of intuitionistic fuzzy metric space with the help of continuous t-norms and continuous t-conorms. Recently, in 2006, Alaca et al. using the idea of intuitionistic fuzzy sets, defined the notion of tuitionistic fuzzy metric space with the help of continuous t-norm and continuous t-conorms as a generalization of fuzzy metric space due to Kramosil and Michalek .We define the notion of intuitionistic fuzzy metric spaces as a natural generalization of fuzzy metric spaces due to George and Veeramani [Fuzzy Sets Syst. 64 (1994) 395] and prove some known results of metric spaces including

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Baire's theorem and the Uniform limit theorem for intuitionistic fuzzy metric spaces.

Zadeh was the first to use the phrase ”fuzzy sets”. In many circumstances, the nature of uncertainty in the behavior of specified system processes is fuzzy rather than stochastic. Numerous academics have lately shown an interest in investigating fuzzy initial value concerns in their theoretical framework. Chang and Zadeh first suggested fuzzy derivatives, which are currently frequently used. Dubosi and Prade first proposed the extension notion. Recent work introduced differential and integral calculus for fuzzy-set-valued functions

2. PRELIMINARIES

The concepts of triangular norms (t-norms) and triangular conorms (t-conorms) are known as the axiomatic Skelton that we use are characterization fuzzy intersections and union respectively. These concepts were originally introduced by Menger [8] in study of statistical metric spaces.

DEFINITION2.1. A binary operation *: [0, 1] x [0, 1] → [0, 1] is continuous t-norm if * satisfies the following conditions:

(i) * is commutative and associative;

(ii) * is continuous;

(iii) $a * 1 = a$ for all $a \in [0, 1]$;

(iv) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$ for all $a, b, c, d \in [0, 1]$.

DEFINITION 2.2. A binary operation ⋄: [0, 1] x [0, 1] → [0, 1] is continuous t-conorm if ⋄ satisfies the following conditions:

(i) ⋄ is commutative and associative;

(ii) ⋄ is continuous;

(iii) $a \diamond 0 = a$ for all $a \in [0, 1]$;

(iv) $a \diamond b \leq c \diamond d$ whenever $a \leq c$ and $b \leq d$ for all $a, b, c, d \in [0, 1]$.

Alaca et al. [1] using the idea of intuitionistic fuzzy sets, defined the notion of intuitionistic fuzzy metric space with the help of continuous t-norm and continuous t-conorms as a generalization of fuzzy metric space due to Kramosil and Michalek [5] as :

DEFINITION 2.3. A 5-tuple (X, M, N, ∗, ⋄) is said to be an intuitionistic fuzzy metric space if X is an arbitrary set, ∗ is a continuous t-norm, ⋄ is a continuous t-conorm and M, N are fuzzy sets on X2×[0, ∞) satisfying the following conditions:

(i) $M(x, y, t) + N(x, y, t) \leq 1$ for all $x, y \in X$ and $t > 0$;

(ii) $M(x, y, 0) = 0$ for all $x, y \in X$;

(iii) $M(x, y, t) = 1$ for all $x, y \in X$ and $t > 0$ if and only if $x = y$;

(iv) $M(x, y, t) = M(y, x, t)$ for all $x, y \in X$ and $t > 0$;

(v) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$ for all $x, y \in X$ and $s, t > 0$;

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(vi) for all x, y ∈X , M(x, y, .) : [0, ∞) → [0, 1] is left continuous;

(vii) limt→∞ M(x, y, t) = 1 for all x, y ∈ X and t > 0;

(viii) N(x, y, 0) = 1 for all x, y ∈ X ;

(ix) N(x, y, t) = 0 for all x, y ∈ X and t > 0 if and only if x = y;

(x) N(x, y, t) = N(y, x, t) for all x, y ∈ X and t > 0;

(xi) N(x, y, t)⋄ N(y, z, s) ≥ N(x, z, t + s) for all x, y ∈ X and s, t > 0;

(xii) for all x, y ∈ X, N(x, y, .) : [0, ∞) → [0, 1] is right continuous;

(xiii) limt→∞ N(x, y, t) = 0 for all x, y ∈ X.

Then (M, N) is called an intuitionistic fuzzy metric space on X. The functions M(x, y, t) and N(x, y, t) denote the degree of nearness and the degree of non-nearness between x and y w.r.t. t respectively.

DEFINITION 2.4. A continuous t-norm F on L∗ is called continuous trepresentable if and only if there exist a continuous t-norm ∗ and continuous t-conorm ♦ such that

∀ ζ = (ζ1, ζ2), η = (η1, η2) ∈ L∗, F (ζ, η) = (ζ1 ∗ η1, ζ2♦η2)

DEFINITION 2.5. Let M is complete if and only if every Cauchy sequence in X is convergent where [44]

1. ∀ t > 0 and α, β ∈ αo, ∃ a sequence {Sα} in X is a Cauchy sequence if for any 0 << 1 there exists αo ∈ N where MV, N (Sα, Sβ, t) >L∗ (Ns (),).

2. ∀ t > 0, ∃ a sequence {Sα} in X is a convergent to a point s ∈ X where limα−→∞ MV, N (Sα, s, t) = 1L∗

Remark 2.1. Let (X , MV ,N , F) is an modified intuitionistic fuzzy metric space, where V(η1 , η2 , .) is non-decreasing and N (η1 , η2 , .) is non-increasing ∀ η1 , η2 ∈ X . Then (X, M V, N , F) is non-decreasing function ∀ η1 , η2 ∈ X.

DEFINITION 2.6.A pair of self-mappings (f, g) of intuitionistic fuzzy metric space is said to be weakly compatible if they commute at the coincidence points i.e., if fu = gu for some u ∈ X, then fgu = gfu.

DEFINITION 2.7. A pair of self-mappings (f, g) of a metric space is said to be weakly compatible if they commute at the coincidence points i.e. fu = gu for some u ∈ X, then fgu = gfu.

DEFINITION 2.8. Two self-mappings f and g of intuitionistic fuzzy metric space are said to be occasionally weakly compatible (owc) if there is a point x in X which is coincidence point of f and g at which f and g commute.

Lemma 2.1. Let (X, M, N, ∗, ⋄) be an intuitionistic fuzzy metric space. f and g be self-maps on X and let f and g have a unique

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point of coincidence, $w = fx = gx$, then w is the unique common fixed point of f and g.

PROOF.

Since f and g are owc, there exists a point x in X such that $fx = gx = w$ and $fgx = gfx$. Thus, $ffx = fgx = gfx$, which says that fx is also a point of coincidence of f and g. Since the point of coincidence $w = fx$ is unique by hypothesis, $gfx = ffx = fx$, and $w = fx$ is a common fixed point of f and g. Moreover, if z is any common fixed point of f and g, then $z = fz = gz = w$ by the uniqueness of the point of coincidence.

Alaca [1] proved the following results:

Lemma 2.2. Let $(X, M, N, *, \diamond)$ be intuitionistic fuzzy metric space and for all $x, y \in X$, $t > 0$ and if for a number $k \in (0, 1)$ such that $M(x, y, kt) \geq M(x, y, t)$ and $N(x, y, kt) \leq N(x, y, t)$. Then, $x = y$.

3. MAIN RESULTS

THEOREM3.1. Let $(X, M, N, *, \diamond)$ be an intuitionistic fuzzy metric space with continuous t-norm * and continuous t-conorm $\diamond$. Let A, B, S and T be self-mappings of X. Let the pairs (A, S) and (B, T) be owc. If there exist $k \in (0, 1)$ such that

M(Ax , By , kt) ≥ min {M(Sx , Ty ,t), M(By, Sx ,t), M(Sx , Ax ,t).

M(By ,Ty , t), M (Ax ,Ty, t).(1+ M(Sx, Ax ,t) 1+ M(By ,T y, t))}

and

N (Ax, By, kt) ≤ max {N (Sx, Ty, t), N (By, Sx, t), N (Sx, Ax, t).

N(By, Ty ,t),N(Ax ,Ty, t).(1+N(Sx , Ax, t) 1+N(By ,T y, t))} for all $x, y \in X$ and $t > 0$. Then, there is a unique common fixed point of A, B, S and T.

PROOF.

As the pairs (A, S) and (B, T) are owc, so there are points $x, y \in X$ such that $Ax = Sx$ and $By = T y$. We claim that $Ax = By$. By (3.1), we have,

M(Ax,By,kt) ≥ min{M(Sx, Ty, t),M(By ,Sx ,t),M(Sx , Ax, t).

M(By,Ty,t), M(Ax,Ty,t).(1+M(Sx,Ax,t) 1+M(By ,T y, t))}

M(Ax,By,kt) ≥ min{M(Ax, By, t), M(By, Ax ,t), M(Ax, Ax, t).

M (By, By, t),M(Ax, By, t).(1+M(Ax, Ax, t) 1+M(By, By, t))}

M(Ax,By,kt) ≥ min {M(Ax, By, t),M(By, Ax, t),1,M(Ax, By, t)}= M(Ax, By, t)

And

N(Ax,By,kt) ≤ max {N(Sx ,Ty ,t),N(By, Sx, t),N(Sx ,Ax ,t).

N (By, Ty, t), N (Ax, Ty, t).(1+N(Sx, Ax, t) 1+N(By, Ty, t))}

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N(Ax,By,kt) ≤ max{N(Ax, By, t),N(By, Ax, t),N(Ax, Ax, t).

N (By, By, t), N (Ax, By, t).(1+N(Ax, Ax, t) 1+N(By, By, t))}

N(Ax,By,kt) ≤ max {N(Ax, By, t),N(By, Ax, t),0,N(Ax, By, t)} = N(Ax, By, t)

therefore,

by lemma 2.2,

Ax = By i.e., Ax = Sx = By = T y.

Suppose that, there is another point z such that Az = Sz then by inequality (3.1),

we have Az = Sz = By = T y so Ax = Az and w = Ax = Sx is the unique point of coincidence of A and S. By lemma 2.1, w is the only common fixed point of A and S. Similarly, there is a unique point z in X such that z = Bz = T z. We now show that z = w. By (3.1), we have,

M(w, z, kt)= M(Aw, Bz, kt) ≥ min {M(Sw, Tz ,t), M(Bz, Sw, t),M(Sw, Aw ,t).

M(Bz,Tz,t),M(Aw,Tz,t).(1+M(Sw,Aw,t) 1+M(Bz,Tz,t))}

= min {M(w,z,t),M(z,w,t),M(w,w,t).M(z,z,t),M(w, z,t).(1+M(w,w,t) 1+M(z,z,t))}

= min {M(w,z,t),M(z,w,t),1,M(w,z,t)}= M(w,z,t)

And

N(w, z, kt)= N(Aw, Bz, kt) ≤ max {N(Sw, Tz ,t), N(Bz, Sw, t),N(Sw, Aw, t).

N(Bz,Tz,t), N(Aw,Tz,t).(1+ N(Sw,Aw,t) 1+ N(Bz, Tz, t))}

= max {N (w, z, t), N(z, w, t), N(w, w, t). N(z, z, t), N(w ,z, t). (1+ N(w,w,t) 1+ N(z,z,t))}

= max {N(w,z,t), N(z,w,t),0, N(w,z,t)}= N(w,z,t).

Therefore,

By lemma 2.2,

We have w = z, hence z is a common fixed point of A, B, S and T. For uniqueness, let u be another common fixed point of A, B, S and T. Then, by (3.1), we have

M(z,u,kt)= M(Az,Bu,kt) ≥ min {M(Sz,Tu,t), M(Bu,Sx,t), M(Sz,Az,t).

M(Bu, Tu, t), M(Az, Tu ,t).(1+ M(Sz , Az, t) 1+ M(Bu, T u ,t))}

= min {M (z, u, t), M (u, z, t), M (z, z, t). M(u,u,t), M(z,u,t).(1+ M(z,z,t) 1+ M(u,u,t))}

= min {M (z, u, t), M (u, z, t), 1, M (z, u, t)} = M (z, u, t)

And

N(z,u,kt)= N(Az,Bu,kt) ≤ max { N(Sz,Tu,t), N(Bu,Sx,t), N(Sz,Az,t).

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N(Bu, Tu, t), N(Az, Tu ,t).(1+ N(Sz,Az,t) 1+N(Bu, Tu, t))}

= max {N(z, u, t),N(u, z, t),N(z, z, t).N(u ,u ,t),N(z, u, t).(1+N(z, z, t) 1+N(u, u, t))}

= max {N (z, u, t), N (u, z, t), 0, N (z, u, t)} = N (z, u, t).

Therefore, by lemma 2.2, we have z = u. Hence, z is unique common fixed point of A, B, S and T.

THEOREM3.2. Let (X, M, N, ∗, ⋄) be an intuitionistic fuzzy metric space with continuous t-norm * and continuous t-conorm ⋄. Let A, B, S and T is self-mappings of X. Let the pairs (A, S) and (B, T) be owc. If there exist k ∈ (0, 1) such that

M(Ax,By,kt) ≥ φ (min {M(Sx, Ty, t),M(By, Sx, t),M(Sx, Ax ,t).

M (By, Ty, t), M (Ax, Ty, t).(1+ M(Sx, Ax, t) 1+ M(By, Ty, t))})

And

N (Ax, By, kt) ≤ ψ (max {N (Sx, Ty, mt), N (By, Sx, t), N (Sx, Ax, t).

N (By, Ty, t), N (Ax, Ty, t). (1+ N (Sx, Ax, t) 1+ N (By, Ty, t))})

for all x, y ∈ X, t > 0 and φ, ψ:[0,1]⟶ [0,1] such that φ(t)> t, ψ(t) for all t ∈ (0, 1) . Then, there is a unique common fixed point of A, B, S and T.

PROOF. The proof follows on the lines of theorem 3.1.

THEOREM3.3.

Let (X, M, N, ∗, ⋄) be an intuitionistic fuzzy metric space with continuous t-norm * and continuous t-conorm ⋄ . Let A, B, S and T be self-mappings of X. Let the pairs (A, S) and (B, T) be owc. If there exist k ∈ (0, 1) such that

M (Ax, By, kt) ≥ φ (M (Sx, Ty, t), M (By, Sx, t), M (Sx, Ax, t).

M (By, Ty, t), M (Ax, Ty, t). (1+ M (Sx, Ax, t) 1+ M (By, Ty, t))) And

N (Ax, By, kt) ≤ ψ (N (Sx, Ty, t), N (By, Sx, t), N (Sx, Ax, t).

N(By,Ty,t), N(Ax,Ty,t).(1+ N(Sx, Ax, t) 1+ N(By ,T y ,t)))

for all x, y ∈ X, t > 0 and φ, ψ:[0,1]4⟶ [0,1] such that φ(t,t,1,t)> t, ψ(t,t,0,t) < t for all t ∈ (0, 1) . Then, there is a unique common fixed point of A, B, S and T.

PROOF. The proof follows on the lines of theorem 3.1.

THEOREM3.4

Let (X, M, N, ∗, ⋄) be an intuitionistic fuzzy metric space with a*b = min {a, b} and a⋄ b = max {a, b} for all a, b ∈ [0, 1]. Let A, B, S and T be self-mappings of X. Let the pairs

(A, S) and (B, T) be owc. If there exist k ∈ (0, 1) such that

M (Ax, By, kt) ≥ M (Sx, Ty, t)* M (By, Sx, t)* M (Sx, Ax, t).

M (By, Ty, t)*M (Ax, Ty, t). (1+M (Sx, Ax, t) 1+M (By, Ty, t)) and

N(Ax,By,kt) ≤ N(Sx, Ty, t)⋄ N(By, Sx, t)⋄ N(Sx, Ax, t).

N (By, Ty, t) ⋄ N (Ax, Ty, t). (1+N (Sx, Ax, t) 1+N (By, T y, t))

for all x, y ∈ X, t > 0. Then, there is a unique common fixed point of A, B, S and T.

PROOF. The proof follows on the lines of theorem 3.1

CONCLUSION

Metric spaces play a vital role in functional analysis and its related concepts. Modern and latest developments are due to fuzzy theory, fuzzy logic, and its vast applications in almost all fields of research. This motivated us to define metric-type spaces in fuzzy version. Since fixed-point techniques have a lot of wonderful applications in science and technology, so in this research article, we intended to put our efforts in obtaining coincidence points and common fixed points in IFMS .In such a way, many useful, present, and conventional results are presented as consequences of our results. Furthermore, as an application we have proved an implicit function theorem with the help of our main result. This activity will definitely motivate researchers to do further work in these spaces and fixed-point theory.

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